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NEW TREATMENT OF NUCLEAR STRUCTURE BY MEANS OF ANALYTICAL-EMPIRICAL DEVIATION METHOD

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Новая трактовка структуры ядра с помощью
аналитико-эмпирического метода девиаций

Для лучшего распознавания строения заполненных ядерных оболочек предложен новый метод, в котором используются девиации (отклонения) энергии отделения последнего нуклона от ядра. Метод позволяет исключить плавную зависимость от жидкокапельной энергии и пилообразную зависимость от энергии спаривания. Изменения оболочечной энергии и оболочечной составляющей парной энергии не исключаются. Этот метод позволил открыть новые оболочки и подоболочки с числами нейтронов $N = 65, 64, 56, 39, 15, 14$ и с числами протонов $Z = 100, 64, 39, 15, 14$.

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New Treatment of Nuclear Structure
by Means of Analytical-Empirical Deviation Method

To gain a better insight into the structure of nuclear closed shells, a new method is proposed that uses the deviations of the separation energy of the last nucleon in the nucleus. The method allows the smooth liquid drop energy as well as nucleon pairing staggering to be excluded; the changes in the shell energy and in the pairing energy, as part of the shell energy, are not excluded. This method enabled the discovery of the new shells and subshells with the neutron numbers $N = 65, 64, 56, 39, 15, 14$ and with the proton numbers $Z = 100, 64, 39, 15, 14$.

The investigation has been performed at the Flerov Laboratory of Nuclear Reactions, JINR.

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INTRODUCTION

A number of ways are known of identifying nuclear magic numbers. They use the discontinuities associated with magic numbers in binding energy (the separation energy of one or two last nucleons–neutrons or protons), two-nucleon pairing energy, α- and β-decay energies, 2^+ nuclear state excitation energy, $B(E2)$ value of the 0^+–2^+ rotation transition, (γ, n), (p, d) reaction thresholds, heavy-ion interaction cross sections, some other phenomena being also used. For example, to analyze neutron shell closuring in neutron-rich nuclei with $Z = 10 - 20$ and $A = 28 - 50$, the separation energy of two last neutrons was used in recent work [1]. As is known, two-neutron binding energy is insensitive to the staggering pairing effect and more prominently reflects shell structure. To study more carefully shell closuring, the authors of [1] subtracted the collective macroscopic interaction energy, calculated by the liquid drop model, from the separation energy of two last neutrons, which allowed them to obtain new results.

1. NEW TREATMENT OF NUCLEAR STRUCTURE

Further more detailed analysis of nuclear structure requires advanced approaches. Therefore a new analytical method, which is based on deviations of experimental data on nuclear masses, was proposed [2]. This method enables the regular macroscopic collective interaction energy to be excluded to a greater degree from nucleon binding energy.

In the framework of the method proposed, the deviation of the separation energy of the last nucleon, for example, of a neutron, can be expressed as

$$\Delta S_{1n}(Z,N) = S_{1n}(Z,N) - [S_{1n}(Z,N+2) + S_{1n}(Z,N-2)]/2, \qquad (1)$$

where $\Delta S_{1n}(Z,N)$ is deviation of one-neutron separation energy, $S_{1n}(Z,N)$ is one-neutron separation energy, or

$$\begin{aligned}\Delta S_{1n}(Z,N) = [&M(Z,N+2) - M(Z,N+1) - 2M(Z,N) + \\ &+ 2M(Z,N-1) + M(Z,N-2) - M(Z,N-3)]c^2/2, \qquad (2)\end{aligned}$$

where $M(Z,N)$ is mass or mass excess of the nucleus.

Formulae (1) and (2) give the deviation of one-neutron separation energy from the mean separation energy of two neighbouring neutrons of identical parity. Using the deviation allows the smooth liquid drop energy and nucleon pairing staggering to be excluded. The changes in the shell energy and in the pairing energy, as part of the shell energy, are not excluded.

Similar to (1) and (2), there are also formulae for the deviation of one-proton separation energy.

In the framework of the conception considered, the pairing energy of two nucleons, for example, two neutrons, is

$$S_{\mathrm{pair}}(Z,N) = \{S_{1n}(Z,N) - [S_{1n}(Z,N+1) + S_{1n}(Z,N-1)]/2\}/2 \qquad (3)$$

or

$$S_{\mathrm{pair}}(Z,N) = [M(Z,N+1) - 3M(Z,N) + \\ + 3M(Z,N-1) - M(Z,N-2)]\mathrm{c}^2/4. \qquad (4)$$

The value of the denominator is determined by the choice of zero for the pairing energy scale.

Formula (4) for pairing energy was first given in monograph [3]. Alternative consideration of formulae (3) and (4) as well as another ways of choosing zero for the pairing energy scale is presented in [4].

So, if one-neutron separation energy $S_{1n}(Z,N)$ is expressed as

$$S_{1n}(Z,N) = S_{\mathrm{drop}}(Z,N) + S_{\mathrm{shell}}(Z,N) + S_{\mathrm{pair}}(Z,N)$$

and S_{drop}, S_{shell} and the absolute value of S_{pair} (for the details associated $|S_{\mathrm{pair}}|$ see formulae (3) and (4)) remain constant over the range of a shell, and if S_{shell} and $|S_{\mathrm{pair}}|$, as part of the shell energy, change sharply over $\Delta S_{\mathrm{shell}}$ and $\Delta|S_{\mathrm{pair}}|$ at the shell boundary, then at $\Delta S_{\mathrm{shell}} < 0$ and $\Delta|S_{\mathrm{pair}}| < 0$ there will be two positive deviations where the closed shell ends and there will be two negative deviations where the next shell starts to be occupied

$$\pm\Delta S_{1n} \approx (\Delta S_{\mathrm{shell}} + \Delta|S_{\mathrm{pair}}|)/2 \approx G/2,$$

where G is the energy gap between the two shells or the two subshells.

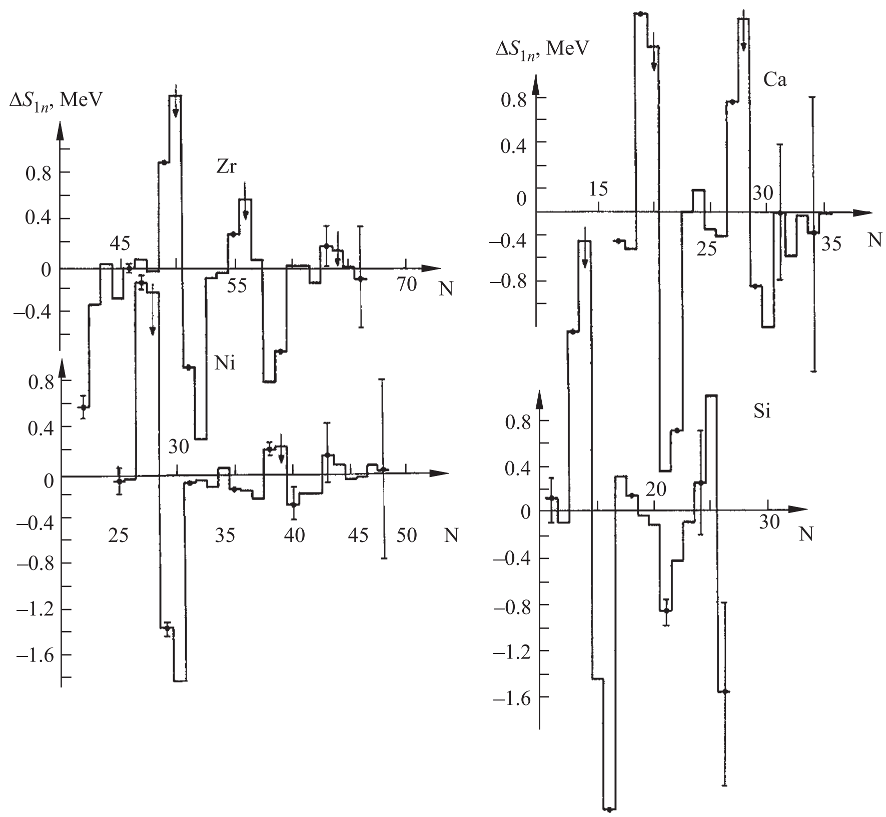
Let us cite monograph [5, p. 59]: «We shall define a nuclear shell as a group of levels which are separated from other levels by reasonably wide energy gaps ... owing to the spin-orbit coupling...». Thus one-neutron separation energy deviation $\Delta S_{1n}(Z,N)$ can be regarded as an indicator of a neutron closed shell (subshell) by means of visualising an intershell (intersubshell) energy gap.

2. RESULT OF CALCULATIONS AND THEIR DISCUSSION

Formula (2) was tested with data on nuclear masses taken from table [6] and supplement to [1]. One-neutron separation energy deviation was calculated for almost each known isotopes of elements of $Z = 102 - 107$. The formula for one-proton separation energy deviation was applied to a number of series of isotones with neutron numbers from $N = 154$ up to $N = 8$.

In particular, by the method proposed, nucleon shell closuring was investigated in light and middle nuclei. In the last few years, those nuclei have aroused considerable interest in theoretical and experimental physics, generated by the possible existence of new shells and subshells.

Figure shows diagrams of one-neutron separation energy deviations for isotopes of elements of Zr, Ni, Ca and Si as four examples of many obtained. Two



The deviation of the one-neutron separation energy $\Delta S_{1n}(Z, N))$ for isotopes of Zr, Ni, Ca and Si versus the neutron number N

large positive deviations (of more than 1 MeV) followed by two large negative deviations (of more than 1 MeV in absolute value) are clearly visible in the case of the known $N = 50$ neutron magic number in the ^{90}Zr nucleus, indicated by an arrow. The intershell energy gap $G = 2.4 \pm 0.2$ MeV.

There are similar gaps at $N = 50$ in the neighbouring isotopes from ^{96}Pd ($G = 1.76 \pm 0.04$ MeV) to ^{84}Se ($G = 2.8 \pm 0.2$ MeV).

Close to the large deviations at the known magic neutron numbers $N = 50$, 28 and 20, there are sometimes less large deviations at $N = 64$, 56, 39 and 14 (see figure).

Figure gives the probable error $\varepsilon[\Delta S_{1n}(Z, N)]$ for some particular deviations, calculated by the formula

$$\varepsilon[\Delta S_{1n}(Z,N)] = \pm 0.6745\{\varepsilon^2[M(Z,N+2)]+ \\ + \varepsilon^2[M(Z,N+1) + 2\varepsilon^2[M(Z,N)]+ \\ + 2\varepsilon^2[M(Z,N-1)] + \varepsilon^2[M(Z,N-2)] + \varepsilon^2[M(Z,N-3)]\}^{1/2}/2,$$

where $\varepsilon[M(Z, N)]$ is absolutely error of nuclear mass taken from the table [6] or from the supplement to [1]. The circles at which the particular error is not indicated show the data region of $\varepsilon[\Delta S_{1n}(Z, N)] \leqslant 0.03$ MeV.

To calculate the probable errors for energy gaps G the following formula was used:

$$r = \pm 0.6745[\Sigma\varepsilon^2/n(n-1)]^{1/2},$$

where ε is the absolute difference of the deviation from the mean deviation; n — the number of deviations found ($n = 4$).

Data on the neutron and proton structure discussed are tabulated in Tables 1 and 2. New results are asterisked.

Nuclear structure of an odd-neutron number and spectroscopic symbols of boundary gap neutron levels were identified following monograph [3]. Let us cite [3, p. 191]: «The separation energy (ionization potential) of the last electron of the neutral atoms is ... a function of the atomic number Z. The shell structure is strikingly exhibited in the decrease of the separation energy after the completion of each major shell. Subshell structure is also discernible... The small maxima at $Z = 7$, 15 and 33 occur at the middle of the filling of the p-shells, and reflect the fact that in these configurations it is possible to achieve a maximum number of antisymmetric bonds between the p electrons, and therefore a minimum in the Coulomb repulsion».

This circumstance is reflected in the fact that the spectroscopic symbol for the boundary neutron levels at the gap following the submagic neutron number 39 is of the (2p1/2)/2 type – a 2p1/2 level of two vacancies when occupied by one neutron – (2p1/2)/2.

Table 1. **Energy gaps in the scheme of neutron levels**

Number of neutrons in layer	Nuclei isotones	Energy of interlayer gap G*, MeV	Spectroscopic symbol of boundary gap levels	$M^* = GA^{1/2}$, MeV
152	^{254}No,^{253}Md,^{252}<u>Fm</u>	0.40±0.06	$1j$15/2 – $3d$5/2*	6.3
126	^{216}Th-^{208}Pb,^{207}<u>Tl</u>	2.1 ±0.3	$1i$13/2 – $2g$9/2	28.8
82	^{152}Yb-^{132}<u>Sn</u>	3.0±0.1	$1h$11/2 – $1h$9/2	34.5
65*	^{115}Sn, ^{110}<u>Rh</u>	0.35±0.05	($3s$1/2)/2 – $2d$3/2*	3.7
64*	^{105}<u>Nb</u>, ^{104}Zr	0.30±0.05	$2d$5/2 – $2d$3/2	3.1
56*	^{106}Sn,^{104}Cd,^{97}Nb,^{96}<u>Zr-</u>^{94}Sr	1.2 ±0.2	$2d$5/2 – $1g$7/2*	11.8
50	^{96}Pd-^{84}<u>Se</u>	2.8±0.2	$1g$9/2 – $1g$7/2	25.7
39*	^{78}Y,^{70}Ga-^{68}Cu,^{67}<u>Ni</u>	0.5 ±0.1	($2p$1/2)/2 – ($2p$1/2)/2*	4.1
39*	^{65}Fe	–0.8 ±0.1	($2p$1/2)/2 – ($2p$1/2)/2*	–6.4
28	^{58}Zn-^{55}Co, ^{52}Cr, ^{50}Ti-^{48}<u>Ca</u>, ^{47}K	3.6±0.4	$1f$7/2 – $2p$3/2	24.9
20	^{42}Ti-^{40}<u>Ca</u>, ^{39}K	2.1±0.1	$1d$3/2 – $1f$7/2	13.3
15*	^{31}S, ^{27}Mg, ^{26}Na, ^{25}<u>Ne</u>, ^{24}F	2.0± 0.1	($2s$1/2)/2 – ($2s$1/2)/2*	10.0
14*	^{29}P, ^{28}<u>Si</u>	4.0 ±0.2	$1d$5/2 – $2s$1/2*	21.2

The spectroscopic symbols of boundary gap neutron levels without an asterisk are taken from monograph [5, p. 58].

The gaps of maximal energy given in column 3 are attributed to the underlined nuclei in column 2.

The negative value of the energy gap in the ^{65}Fe nucleus after the neutron number $N = 39$ suggests that it is the case of there being an intruder layer $N = 40$ similar known intruder shell state $N = 20$ [7]. This assumption is supported by the fact that there are two positive and two negative deviations of an inverse order in the region of $N = 38 - 41$, which the ^{64}Fe–^{67}Fe isotopes belong to.

The author introduced the parameter of reduced magicity $M = GA^{1/2}$, where A is the total number of nucleons in a nucleus. It may be supposed that $M > 10$ corresponds to a closed shell.

There is no $Z = 56$ shell in the proton potential well, whereas there is an $N = 56$ neutron shell there. Further comments in Table 2 are similar to those in Table 1.

Table 2. **Energy gaps in the scheme of proton levels**

Number of protons in layer	Nuclei isotopes	Energy of interlayer gap G*, MeV	Spectroscopic symbol of boundary gap levels	M*=GA$^{1/2}$, MeV
100*	$^{254-\underline{252},251}$Fm	0.25 ± 0.02	2f7/2 – 2f5/2	4.0
82	$^{\underline{208}-202}$Pb	2.3 ± 0.1	1h11/2 – 1h9/2	33.2
64*	^{146}Gd	0.35 ± 0.03	2d5/2 – 2d3/2	4.2
56*	^{138}Ba	< 0.05		
50	$^{\underline{132}-120}$Sn	3.8 ± 1.0	1g9/2 – 1g7/2	43.7
39*	$^{91-\underline{89}-87}$Y	0.9 ± 0.2	(2p1/2)/2 – 2p1/2)/2*	8.5
28	$^{68-\underline{56}}$Ni	3.1 ± 0.1	1f7/2 – 2p3/2	23.2
20	$^{48,41,\underline{40},39}$Ca	3.5 ± 0.1	1d3/2 – 1f7/2	22.1
15*	$^{38,\underline{37}-35,31}$P	1.7 ± 0.3	(2s1/2)/2 – (2s1/2)/2*	10.3
14*	$^{33-\underline{32},31,29,28}$Si	3.9 ± 0.6	1d5/2 – 2s1/2	22.1

Thus, the proposed deviation method is a useful tool for analyzing nuclear structure. This method allowed one to find the gaps at the neutron numbers $N = 65$, 64, 56, 39, 15, 14 and at the proton numbers $Z = 100$, 64, 39, 15, 14.

Possible large gaps of odd numbers seem to be of special importance for the synthesis of new heavy and superheavy nuclei. If half occupancy levels of the $3p1/2$ or $4s1/2$ types are followed by a large energy gap in the range of $Z > 120$ and $N > 162$, this will result in a superheavy nucleus of higher stability owing to a higher fission barrier and a higher hindrance factor for α-decay.

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