E2-2005-161

N. P. Konopleva*

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Submitted to the Proceedings of the VIII International Workshop
«Relativistic Nuclear Physics: from Hundreds of MeV to TeV»,
23.05–28.05, 2005, JINR, Dubna, Russia

*Permanent address: All-Russian Scientific Research Institute of Electromechanics, Horomnyi tup. 4, Moscow 101000;
e-mail: nelly@theor.jinr.ru; vniiem@orc.ru; fax: (095)2074962

Коноплева Н. П. E2-2005-161

Инстантонные полевые конфигурации и черные дыры

Обсуждается роль релятивизации вакуума в КХД и теории ядра. Показано, что релятивистский вакуум должен описываться вакуумными уравнениями Эйнштейна. В КХД черные дыры должны появиться благодаря шварцшильдовскому решению этих уравнений. Инстантонные конфигурации любых полей не меняют уравнений Эйнштейна и их решений, поскольку их тензор энергии-импульса равен нулю. Но они делают возможным определение топологии пространства-времени, которая не может задаваться дифференциальными уравнениями Эйнштейна. Поэтому число черных дыр в пространстве-времени можно связать с инстантонными конфигурациями полей и другой материи. Инстантоны не падают в черные дыры и являются именно той материей, которая их окружает.

Работа выполнена в Лаборатории теоретической физики им. Н. Н. Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 2005

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Instanton Field Configurations and Black Holes

The role of vacuum relativization in QCD and nucleus theory is discussed. It is shown that relativistic vacuum must be described by vacuum Einstein equations. Black Holes have to make their appearance in QCD because of Schwarzschildean solution of these equations. Instanton configurations of any fields do not change vacuum Einstein equations and their solutions, because their energy-momentum tensors are zero. But they make it possible to determine a space-time topology, which cannot be defined by differential Einstein equations. Therefore, Black Holes number in space-time is possibly connected with instanton configurations of fields and other matter. Instantons do not fall into Black Holes and are the very matter which surrounds them.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 2005

1. RELATIVIZATION OF VACUUM

Any physical field describing interactions between particles can be regarded as a gauge field (see [1]). In this approach, different Lie groups of symmetry transformations correspond to different kinds of forces. If these groups are global, their transformation parameters do not depend on a choice of space-time point. It means that the domain in which the symmetry transformations are realized is the whole space-time. In this case, vacuum must be also given globally and its properties can not depend on space-time point. Practically it is a nonrelativistic situation. In this sense Special Relativity is only a relativistic theory in 3D space, but in 4D space-time it must be considered as a nonrelativistic one. It is a corollary of the symmetry transformations of SR that are globally carried out in flat Minkowski space-time V_4.

When we go over to 4D relativistic theory, we have to assume that all symmetry transformations are realized locally, i.e. their parameters depend on space-time point. It concerns both internal and space-time symmetries. Localization of internal symmetries leads to gauge field appearance, and localization of space-time symmetries leads to gravitational field appearance. Therefore, localization of vacuum arises [2]. General Relativity does inevitably come to the gauge field theory, both classical and quantum forms of it. GR is connected with localization of space-time point coordinate transformations: $x^{\mu\prime} = f^{\mu}(x^{\nu})$, where f^{μ} are arbitrary continuous functions of point coordinates x^{μ}.

Einstein equations describe world geometry formation process step by step when invariant square form $ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}$ is given to start with. Although we can always choose the local geometry to be flat, the whole space-time will be curved. Its properties are described by Riemannian geometry without torsion.

So, in a real 4D relativistic theory global vacuum should be replaced by infinite set of vacuums, where each vacuum corresponds to some point of 4D Riemannian space-time [2]. Each local space-time is flat and coincides with Minkowski V_4 in infinitesimal neighbourhood of Riemannian V_4. Both the whole 4D space-time and set of the local vacuums are described by vacuum Einstein equations. This relativistic vacuum is universal for all kinds of matter and their interactions. Any field equation system must be supplemented with Einstein equations if vacuum is regarded as a relativistic and local one.

In the next section it will be shown that in General Relativity both global and local vacuums are the solutions of self-duality equations of Riemannian curvature tensor.

2. INSTANTON FIELD CONFIGURATIONS

In the quantum field theory, instantons are classical trajectories connecting vacuums among themselves. They are being used for description of tunnel processes between vacuums [3]. All calculations are usually carried out in Euclidean space-time. But now we are interested in pseudoeuclidean space-time. In such a space with metrics of hyperbolical signature it is possible to define objects which satisfy equations similar to self-duality equations of Euclidean space. Earlier I named similar objects «hyperbolical instantons» [4, 5]. Here for the sake of brevity all solutions of self-duality equations will be named «instantons» independent of metrics signature.

In the Euclidean version of the gauge field theory (GFT) instantons are the solutions of self-duality equation

$$F^a_{\mu\nu} = \mp^* F^a_{\mu\nu}. \tag{1}$$

Here $F^a_{\mu\nu}$ is the gauge field strength tensor and * means dual conjugation.

In Euclidean version of GFT there are examples of nontrivial solutions of self-duality equations. But it is evident that trivial solutions of (1) satisfying equation

$$F^a_{\mu\nu} = 0 \tag{2}$$

exist in both Euclidean and pseudo-Euclidean cases. They are named «pure gauges». Vector-potential of the gauge field A^a_μ in this case is $A^a_\mu = \partial_\mu \epsilon^a(x)$, where $\epsilon^a(x)$ are gauge transformation parameters depending on space-time point x.

When the gauge field is gravitational one in pseudo-Riemannian space-time there is an analog of Eq. (2) having the form

$$R_{\mu\nu}{}^{\tau}_{\lambda} = 0, \tag{3}$$

where $R_{\mu\nu}{}^{\tau}_{\lambda}$ is Riemannian curvature tensor of space-time V_4. It means that space-time is globally flat, i.e. it is Minkowski space-time. It is trivial instanton, and simultaneously *it is global vacuum in GR and GFT.*

Moreover in GR, Eq. (1) has a nontrivial analog [5]

$$R_{\mu\nu}{}^{\tau}_{\lambda} = \mp^* R_{\mu\nu}{}^{\tau *}_{\lambda}. \tag{4}$$

It follows from (4) that

$$R_{\mu\nu} = \mp^* R^*_{\mu\nu} \tag{5}$$

and

$$R = \mp^* R^*. \tag{6}$$

Therefore

$$R_{\mu\nu} = 0. \tag{7}$$

This is vacuum Einstein equations. Their derivation from self-duality equations (4) was carried out in [5]. Hence, all vacuum solutions of Einstein equations are nontrivial instantons (more exactly, hyperbolical instantons). In particular, Schwarzschildean solution is a gravitational instanton.

Very important property of all instanton solutions is vanishing of energy-momentum tensor $T_{\mu\nu}$. It is a corollary of their definition by Eqs. (1) and (4) in the gravity case.

Another important property of all instantons is transmutation of gauge field action integral into a topological constant characterizing number of field singularities.

If nongravitational gauge fields or other kinds of matter form any instanton configuration, i.e. their energy-momentum tensor $T_{\mu\nu}$ is zero, they cannot change Eq. (7). Therefore, all instanton configurations of fields and matter correspond to vacuum Einstein equations. *They are the matter without gravity.* Instantons do not fall into Black Holes and they are the very matter which surround them.

Both Minkowski space-time and vacuum Einsteinian spaces are instanton field configurations (trivial and nontrivial ones, respectively).

So, *vacuum solution of Einstein equations describe relativistic localized vacuum in GR and GFT.*

3. BLACK HOLES

Black Holes are known as objects of radius $r \leqslant r_g$. Gravitational radius r_g is just value of corresponding parameter r in Schwarzschildean metrics which make it singular one. This metrics has the form [6]

$$d\tau^2 = [1 - \frac{2M\gamma}{r}]dt^2 - [1 - \frac{2M\gamma}{r}]^{-1}dr^2 - r^2 d\theta - r^2 \sin^2\theta d\phi^2, \qquad (8)$$

where $d\tau$ is invariant eigentime; r, θ, ϕ are usual spherical polar coordinates; γ is gravitational constant; M is integral constant.

Under $r = r_g = 2M\gamma$ metrics, components g_{rr} become infinite. Therefore, on the Black Holes' surface, gravity becomes so strong that any signal cannot leave it.

Moreover, by Birkhoff theorem, space spherical symmetry of metrics (8) leads to its statical character [7]. Even internal pulses of spherically symmetric object can not generate radiation. This result is valid both in Newtonian theory and GR. Black Holes can be objects both of Newtonian gravity and Einsteinian one. We see that Newtonian gravitational potential appears in Schwarzschildean metrics. This vacuum solution of Einstein equations includes a parameter M, which gets sense of energy of some object when Schwarzschildean metrics is considered with respect to the flat Minkowski one. In Minkowski space-time, one can calculate integral components of energy-momentum vector P_μ corresponding with Schwarzschildean metrics. They are the following: $P_i = 0$ $(i = 1, 2, 3)$; $P_0 = M$ (see [8]). Therefore, usually the parameter M is interpreted as an object mass. But really we have only potential energy of gravity field. In Newtonian mechanics this energy corresponds to gravitational energy of massive body whose center coincides with point $r = 0$ of Schwarzschildean metrics. But in GR we have nobody in our problem! We have only relativistic vacuum and its potential energy relative to flat global vacuum. Only singularity in spherically symmetric metrics exists in Riemannian V_4 but not any massive body.

One of Einsteinian ideas just consists in the fact that elementary particles (in particular electron) are singularities of force lines of gravitational field. Einstein with his collaborators showed that these singularities move along geodesic lines of Riemannian V_4 like real test bodies [9, 10, 11].

In Newtonian mechanics the situation is contrary: gravity is only generated by massive bodies and, hence, in spherical symmetric case, gravitational singularities can only be inside continuous massive body but not by itself. It is necessary to note that inside a spherical shell the solution of Eq. (7) corresponds to flat Minkowski space-time. Similar result is also known in Newtonian mechanics (gravity is absent inside a spherical shell).

In support of Einsteinian point of view one can say that optics knows many examples when singularities of force lines appear and move by itself. Maybe it is a common property of all field theories especially nonlinear ones.

So, how can Black Holes radiate? Because of static character of Schwarzschildean metrics Black Holes cannot radiate. The question is Hawking pseudo-radiation [12]. It is a flow of real particles to the infinite generated by quantum effects of vacuum polarization. Hawking assumed that in gravitational field of Black Hole some virtual particles of pairs can diverge too far from each other and turn into real ones. Then one of them can fall into Black Hole and the other one will fly away to the infinite. Just the flow of such particles on the infinite is implied when Black Holes radiation is under discussion [13]. It is assumed that such a radiation can be experimentally registered. It must have blackbody spectrum and finish by explosion. Temperature of Hawking radiation is correlated with Black Hole mass M_{BH} and is equal to $T \sim 10^{-7}\,\dfrac{M_\odot}{M_{BH}} K$. Small Black Holes must be rapidly disappearing in consequence of Hawking process.

Black Holes with mass $\sim 10^{15}$ g have radius $r \sim 10^{-13}$ cm, i.e. they are similar to elementary particles. Their lifetime is similar to Universe age. The temperature of their radiation is $T \sim 10^{12}$ K or $kT \sim 100$ MeV [14]. If really Hawking process exists, Black Holes can be registered in modern experiments. In this connection one must take into account the relation between instanton field configurations and singularities of corresponding fields.

Acknowledgements. It is a pleasure to thank the Organizers of the International Workshop RNP-2005 for support of this investigation.

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Received on October 19, 2005.

Редактор *Н. С. Скокова*

Подписано в печать 12.01.2006.
Формат $60 \times 90/16$. Бумага офсетная. Печать офсетная.
Усл. печ. л. 0,44. Уч.-изд. л. 0,63. Тираж 415 экз. Заказ № 55172.

Издательский отдел Объединенного института ядерных исследований
141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.
E-mail: publish@pds.jinr.ru
www.jinr.ru/publish/