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ORTHONORMAL POLYNOMIAL APPROXIMATION OF THERMOMETRIC FUNCTIONS FOR **CERNOX**–RuO_2 COMPOSITION SENSORS

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Ортонормальная полиномиальная аппроксимация термометрических функций для сенсоров CERNOX–RuO_2

Представлено математическое описание термометрических характеристик криогенных термосенсоров CERNOX–RuO_2 на основе ортонормированных полиномов. Вычислены аппроксимирующие функции для сопротивления и температуры в полном температурном интервале $330 \div 1.7$ K и в трех подынтервалах. Результаты численных экспериментов, относящиеся к аппроксимационным отклонениям, среднеквадратическим отклонениям и абсолютной разрешающей способности для температуры и сопротивления, показывают возможности применения предложенного подхода. Математическое моделирование на основе этих полиномов обеспечивает хорошую точность и гибкость описания характеристик как в подынтервалах, так и в полном интервале.

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Orthonormal Polynomial Approximation of Thermometric Functions for CERNOX–RuO_2 Composition Sensors

Mathematical description of thermometric characteristics of CERNOX–RuO_2 composition cryogenics temperature sensors is proposed using orthonormal polynomials. The approximated functions for resistance and temperature are carried out in the whole temperature interval $330 \div 1.7$ K and in three subintervals. The results from numerical experiments, concerning calculated approximated differences, root-mean square deviations and absolute temperature and resistance resolutions demonstrate the applicability of the used approach. The mathematical modeling based on these polynomials ensures good accuracy and flexibility of the description of the characteristics not only in subintervals but in the whole interval.

The investigation has been performed at the Laboratory of Information Technologies, JINR.

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INTRODUCTION

The investigations of the properties of materials applied in magnetic systems at low temperatures define searching for new cryogenic thermometers. It is important to minimize the high magnetic field effects on the used temperature sensors. The ceramic nitride oxides (CERNOX) resistors and thick-film chip resistors, based on RuO_2, appear to be suitable for this application. The studied new cryogenic thermometers CERNOX–RuO_2 are prepared and experimentally calibrated at the Institute of Low Temperature and Structure Research (ILTSR), Poland. They are composition of two parallel connected resistors — CERNOX (CX 1030 Bare Chip, Lake Shore, 0.7×1.0×0.38 mm^3) and one half of commercially available RuO_2 (2×0.6×0.4 mm^3, 2 $k\Omega$), first proposed by E. Bruck [1]. Two soldered resistors (indium was used as a solder for CERNOX and Sn + Pb alloy — for RuO_2 resistor) are glued by BF-2 to a small copper plate 2.3×2.6×0.3 mm^3. Parallel electrical connection is done from platinum wire 0.03 mm in diameter. These composition thermometers have the advantages to be small and to have a higher resistivity.

In this paper an approximation of thermometric functions $R(T)$ and $T(R)$, where R is resistance and T is temperature, for CERNOX–RuO_2 composition temperature sensors is proposed using our approach based on Orthonormal Polynomial Expansion Method (OPEM) [2–4]. Mathematical description of thermometric characteristics in the range of $330 \div 1.7$ K is made in the whole temperature interval and in three subintervals $330 \div 77$, $78 \div 20$, $21 \div 1.7$ K. The presented in the Table and in Figs. 1–5 numerical results are discussed.

1. MATHEMATICAL ALGORITHM

Our Orthonormal Polynomial Method (OPEM) is based on Forsythe [5] three-term relation for generating orthogonal polynomials over the given point set $\{q_i, f_i\}, i = 1, 2 \ldots M$ with given weights $w_i = 1/\sigma^2_{f_i}$. It is a generalization for calculating derivatives and integrals with fourth term. The principal relation for one-dimensional generation of orthonormal polynomials $\Psi_k^{(0)}, k = 0, 1, 2, \ldots$ and their derivatives $\Psi_k^{(m)}, m = 1, 2, \ldots$, in OPEM is

$$\Psi_{k+1}^{(m)}(q) = \gamma_{k+1}[(q-\alpha_{k+1})\Psi_k^{(m)}(q) - (1-\delta_{k0})\beta_k \Psi_{k-1}^{(m)}(q) + m\Psi_k^{(m-1)}(q)], \quad (1)$$

where $\Psi_{-1} = 0$, $\Psi_0 = \gamma_0 = 1/\beta_0$, $\beta_0 = \sqrt{\sum_{i=1}^{M} w_i}$.

The polynomials $\Psi_k^{(0)}$ satisfy the following orthogonality relations:

$$\sum_{i=1}^{M} w_i \Psi_k^{(0)}(q_i)\Psi_l^{(0)}(q_i) = \delta_{kl}$$

over the given point set $\{q_i\}$ with given weights $\{w_i\}$.

The approximation values f^{appr} of the function f and its mth derivative $f^{(m)\mathrm{appr}}$ are given by

$$f^{(m)\mathrm{appr}}(q) = \sum_{k=0}^{N} a_k \Psi_k^{(m)}(q) = \sum_{k=0}^{N} c_k qk. \quad (2)$$

The optimal degree N of the approximating polynomials in Eq. (2) is selected by the algorithm, using the following two criteria. First, the fitting curve should lie in the error corridor of the dependent variable. Second, the minimum χ^2 should be reached. When the first criterion is satisfied, the search of the minimum χ^2 stops. (The new version of the algorithm defining the total variance was published in [6, 7].)

The functions $R(T)$ and $T(R)$ are described by orthonormal polynomials using new type of weights, W^R and W^T. By definition the weighting functions W^R and W^T are $1/\sigma_f^2$, where σ_f^2 is a variance for $R(T)$ and $T(R)$ approximations. In our investigation these variances are accepted to be squares of the absolute resistance resolution and absolute temperature resolution of the investigated sensors ΔR_{arr} and ΔT_{atr}, respectively.

Here the calculated accuracy of our $R(T)$ experimental data is in the range of 0.20–0.50%, on average of 0.35%, giving

$$\Delta R_{\mathrm{arr}} = 0.0035R \;\; [\Omega]. \quad (3)$$

The characteristic ΔT_{atr} is defined by

$$\Delta T_{\mathrm{atr}} = \Delta R_{\mathrm{arr}}/|dR/dT| = 0.0035R/|dR/dT| \; [\mathrm{K}]. \quad (4)$$

Then the weights W_i^R and W_i^T in every point are:

$$W_i^R = 1/(\Delta R_{\mathrm{arr}}^2)_i = 8.10^4/R_i^2 \; [\Omega^{-2}],$$
$$W_i^T = 1/(\Delta T_{\mathrm{atr}})2)_i = 8.10^4 (dR/dT)_i^2/R_i^2 \; [\mathrm{K}^{-2}], \quad (5)$$

i. e. the W^R and W^T are related to the specified operating conditions and to the first derivative dR/dT of the described thermometers.

The deviations ΔR_i between experimental $R_i^{\rm exp}$ and approximating $R_i^{\rm appr}$ values of the resistance as well as their temperature equivalents $(\Delta R_i)_{\rm te}$ are also estimated in every point as follows:

$$\Delta R_i = (R_i^{\rm exp} - R_i^{\rm appr})\ [\Omega],\ \ (\Delta R_i)_{\rm te} = (R_i^{\rm exp} - R_i^{\rm appr})/(dR/dT)_i\ [{\rm K}]. \quad (6)$$

The deviations ΔT_i between experimental $T_i^{\rm exp}$ and approximating $T_i^{\rm appr}$ values of the temperature are given by the differences

$$\Delta T_i = (T_i^{\rm exp} - T_i^{\rm appr})\ [{\rm K}]. \quad (7)$$

The temperature dependences of the evaluated in every point first derivative (dR/dT) of these temperature sensors as well as the temperature behavior of the calculated approximation differences ΔT, ΔR and $\Delta R_{\rm te}$, the root-mean square RMS deviations for $T(R)$ and $R(T)$ functions, the absolute temperature resolutions $\Delta T_{\rm atr}$ and absolute resistance resolution $\Delta R_{\rm arr}$ for studied thermometers are calculated by the OPEM.

2. APPROXIMATION DETAILS

The most interesting results of the approximation of the functions $T(R)$ and $R(T)$ as: the optimal degree of polynomials N and the approximation characteristics — maximum deviation $\Delta_{\rm max}$ [Ω], $\Delta_{\rm max}$ [K], root-mean square deviation RMS [K], absolute mean deviation AMD [K] at M number of points are compared in the Table.

Table. OPEM approximations of $T(R)$ and $R(T)$ for CERNOX–RuO_2 sensor

T, K	M	R, Ω	f, q	N	$\Delta_{\rm max}, \Omega$	$\Delta_{\rm max}$, K	RMS, K	AMD, K
$330 \div 1.7$	239	$67 \div 4616$	$R(T)$	12	5.05(T=18)	–0.86(T=253)	0.202	–
$330 \div 1.7$	239	$67 \div 4616$	$T(R)$	12	–	–0.88(T=253)	0.197	0.095
$330 \div 77$	100	$67 \div 259$	$R(T)$	12	–0.53(T=156)	–1.06(T=253)	0.281	–
$330 \div 77$	100	$67 \div 259$	$T(R)$	12	–	–1.08(T=253)	0.280	0.195
$78 \div 20$	67	$259 \div 851$	$R(T)$	5	–1.59(T=21)	–0.043(T=33)	0.019	–
$78 \div 20$	67	$259 \div 851$	$T(R)$	5	–	0.04 (T=21)	0.018	0.014
$21 \div 1.7$	82	$851 \div 4616$	$R(T)$	6	4.84(T=18)	0.099(T=18)	0.017	–
$21 \div 1.7$	82	$851 \div 4616$	$T(R)$	6	–	0.098 (T=18)	0.016	0.008

Note 1. The $\Delta_{\rm max}$ [K] corresponds to $\Delta T_{\rm max}$ [K] for $T(R)$ approximation or to $(\Delta R_{\rm te})_{\rm max}$ [K] for $R(T)$ approximation. The $\Delta_{\rm max}$ [Ω] corresponds to $\Delta R_{\rm max}$ [Ω] for $R(T)$ approximation.

The optimal degrees are selected from the algorithm between 1st and 12th degrees. For $R = R(T)$ and $T = T(R)$ approximations the polynomial degree is

12 for the whole interval and correspondingly 12, 5, 6 for subintervals $330 \div 77$, $78 \div 20$, $21 \div 1.7$ K. The Table shows that the whole interval approximation has quite good accuracy. The root-mean square RMS deviations for the second and third intervals are about ten times smaller than these for the whole and first intervals.

The temperature dependences of the function $R(T)$, absolute (dR/dT), relative $(1/R)(dR/dT)$ and specific $(T/R)(dR/dT)$ sensitivities for the whole interval approximation are presented in Fig. 1. The steepest slope for the resistance is observed at temperature below 20 K.

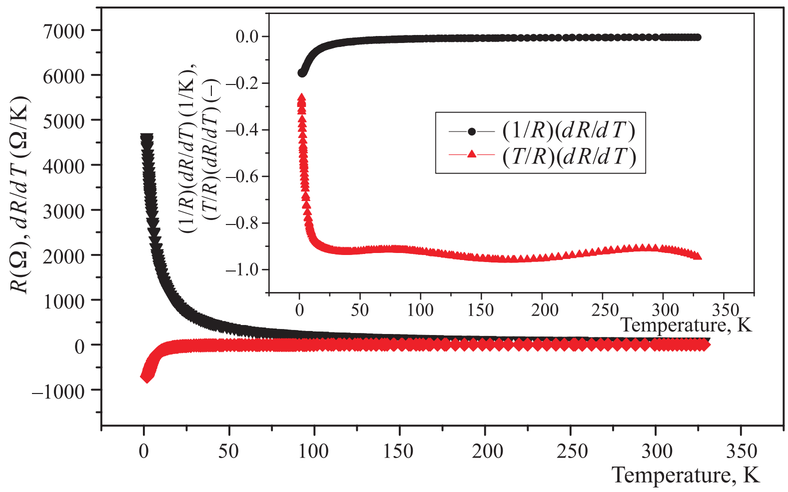

Fig. 1. Temperature dependences of the resistance R, the absolute dR/dT, relative $1/R(dR/dT)$ and specific $(T/R)(dR/dT)$ sensitivities for the whole interval approximation of CERNOX–RuO_2 composition thermometer

Temperature dependences of the deviations ΔT, RMS^T and $\pm\Delta T_{\mathrm{atr}}$ for the whole interval approximation are presented in Fig. 2.

For the whole interval approximation the temperature dependences of the deviations ΔR and ΔR_{te} (Eq. 6) between experimental and approximating values, $\mathrm{RMS}^R_{\mathrm{te}}$, the error corridors $\pm\Delta R_{\mathrm{arr}}$ (Eq. 3) and $\pm\Delta T_{\mathrm{atr}}$ (Eq. 4) are presented in Figs. 3 and 4. The temperature dependences of ΔR_{te} (Eq. 6) and ΔT deviations (Eq. 7) together with $\pm\Delta T_{\mathrm{atr}}$, $\mathrm{RMS}^R_{\mathrm{te}}$ and RMS^T for three-interval approximation are shown in Figs. 5 and 6.

When we follow the cited criteria the deviation results are in the error corridor. The point out of interval — it is a case where we have chosen the appropriated lower degree. RMS for the second and third intervals are undistinguished. The temperature dependences of the absolute (dR/dT), relative $1/R(dR/dT)$ and specific $(T/R)(dR/dT)$ sensitivities are reported at the ISCMP [8].

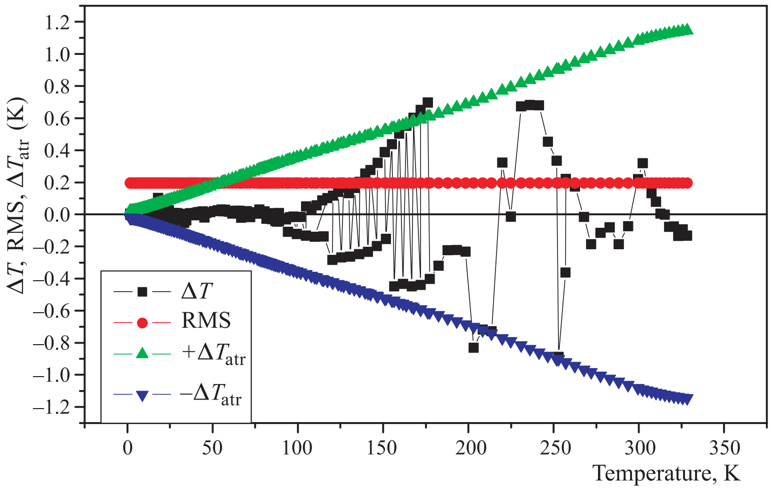

Fig. 2. Temperature dependences of the deviations ΔT, RMS^T and $\pm\Delta T_{\mathrm{atr}}$ for the whole interval approximation of CERNOX–RuO_2 composition thermometer

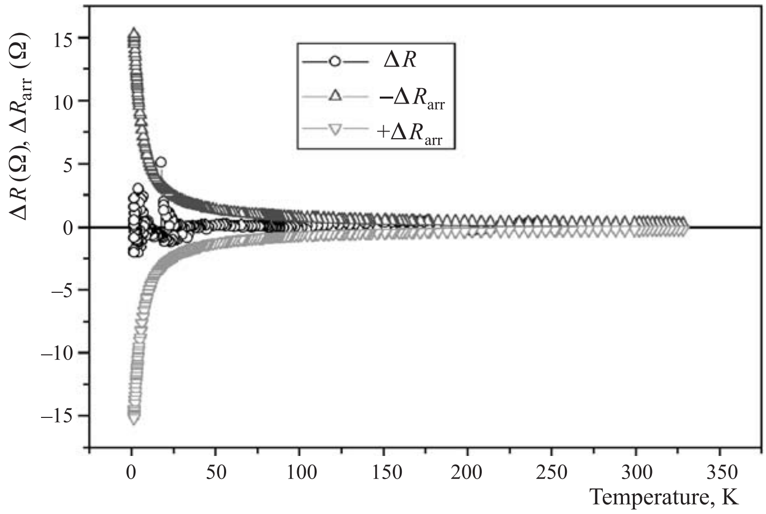

Fig. 3. Temperature dependences of the resistance deviations ΔR and $\pm\Delta R_{\mathrm{arr}}$ for the whole interval approximation of CERNOX–RuO_2 composition thermometer

Note 2. In our previous papers [2, 3] we have analyzed in details the comparison with other appropriated mathematical methods. We have not found the whole interval approximation by other descriptions: the improved interpolation

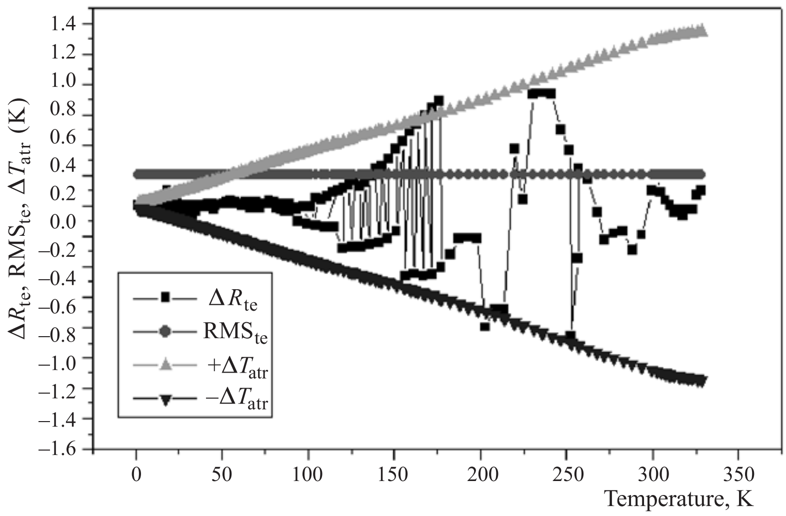

Fig. 4. Temperature dependences of the deviations $(\Delta R)_{\mathrm{te}}$, $\mathrm{RMS}_{\mathrm{te}}^{R}$ and $\pm\Delta T_{\mathrm{atr}}$ for the whole interval approximation of CERNOX–RuO_2 composition thermometer

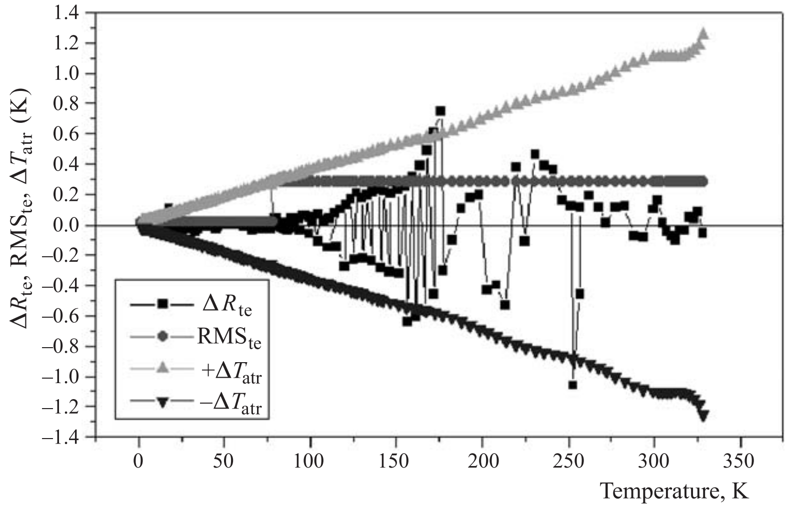

Fig. 5. Temperature dependences of the deviations $(\Delta R)_{\mathrm{te}}$, $\mathrm{RMS}_{\mathrm{te}}^{R}$ and $\pm\Delta T_{\mathrm{atr}}$ for three-interval approximation of CERNOX–RuO_2 composition thermometer

method and the piecewise least squares interpolation [9]. Our method permits the fitting errors to be bounded by specially constructed in a priori defined weighting function ΔT corridor, without excluding bad points. We have also compared

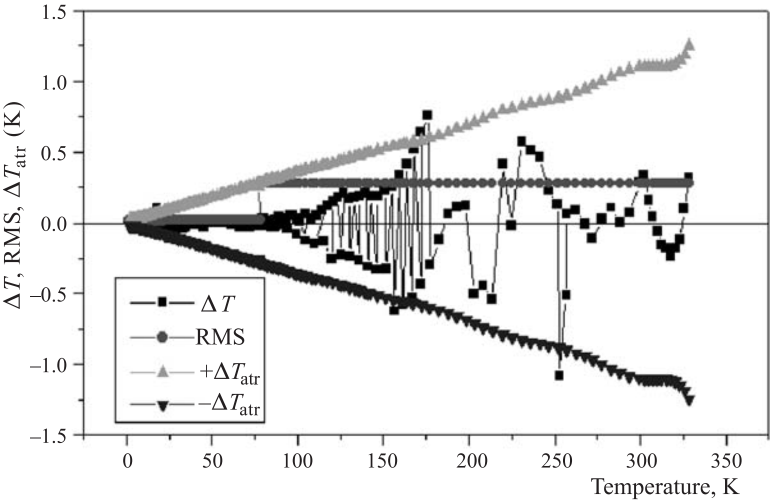

Fig. 6. Temperature dependences of the deviations ΔT, RMS^T and $\pm\Delta T_{\mathrm{atr}}$ for three-interval approximation of CERNOX–RuO_2 composition thermometer

our results [3] with Chebyshev or ordinary polynomials description [10]. They demonstrate also the advantages of our method giving straightforward description.

CONCLUSIONS

The proposed mathematical approach is useful for description of calibrated test data of studied thermometers. The calculated thermometric characteristics are also necessary for automating the low-temperature physics experiment carried out in high magnetic fields. The obtained numerical results demonstrate the perspectives of the applied method in cryogenic thermometry.

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