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PSEUDOSPIN SYMMETRY AND STRUCTURE  
OF NUCLEI WITH  $Z \geq 100$

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Псевдоспиновая симметрия и структура ядер с  $Z \geq 100$

В рамках релятивистской теории среднего поля рассмотрена зависимость остаточных сил в ядрах от псевдоспина. Показано, что эта зависимость слабая. Как следствие, слабой оказывается и зависимость от псевдоспина взаимодействия частица–остов. Это приводит к малой величине расщепления псевдоспиновых дублетов, вызванной векторной связью псевдоспина нечетной частицы и псевдоорбитального момента остова. Указано на некоторые возможности экспериментального исследования проявлений псевдоспиновой симметрии в спектрах нечетных ядер с  $Z \geq 100$ .

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Pseudospin Symmetry and Structure of Nuclei with  $Z \geq 100$

In the framework of the Relativistic Mean Field approach (RMF) a pseudospin dependence of the residual forces in nuclei is considered. It is shown that this dependence is relatively weak. As a consequence, a pseudospin dependence of the particle–core coupling is weak as well. This leads to a small splitting of the pseudospin doublets produced by a vector coupling of an odd particle pseudospin and a pseudo-orbital momentum of the core. Some possibilities for experimental investigations of the manifestations of the pseudospin symmetry in the spectra of odd nuclei with  $Z \geq 100$  are indicated.

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## INTRODUCTION

The pseudospin symmetry [1–3] is known as approximate symmetry of the nuclear mean field. This symmetry is manifested in the nuclear excitation spectra by the presence of quasi-degenerate doublets. At the same time, the existence of this symmetry is strongly related to the strength of the spin–orbit interaction term of the nuclear mean field, and therefore, to the next proton magic number. The strong spin–orbit interaction in nuclei and the presence of approximate pseudospin symmetry in a nuclear mean field are two sides of the same medal.

It is well known that the mean field in nuclear theory plays a role of a basic theory for several more specific advanced theories. These theories can be built upon introducing the single-particle mean field basis. Therefore, it is very important for the whole field of nuclear structure physics to examine consequences of the pseudospin symmetry, although this symmetry is approximate.

Any dynamical symmetry implies the existence of a characteristic multiplet structure. These multiplets are characterized by a magnitude of the multiplet splitting. The characteristic magnitude of a splitting of the pseudospin doublets in spherical nuclei is of the order of  $0.1\hbar\omega_0$ , where  $\hbar\omega_0$  is a frequency of the single particle oscillator.

However, this splitting demonstrates a dependence on a ratio between the numbers of protons and neutrons and it is very small in some nuclei.

Single-particle pseudospin doublets in deformed nuclei are characterized by a projection of the pseudo-orbital momentum on the symmetry axis. The splittings of these doublets are several times smaller than in spherical nuclei. The doublet structure is also observed in the rotational bands of odd deformed nuclei based on the pseudospin singlets, i. e., on the states with the projection of the pseudo-orbital momentum on the symmetry axis equal to zero. The doublet structure in these bands arises as a result of coupling of an odd particle pseudospin and a total pseudo-orbital momentum. This pseudo-orbital momentum is a sum of a core rotational momentum and a pseudo-orbital momentum of an odd particle. A splitting of these doublets is quite small and equals several tens of keV.

These facts mean that the term describing a particle–core coupling in a phenomenological nuclear Hamiltonian is pseudospin independent with good accuracy. For this reason, the spectra of odd deformed nuclei and especially rotational bands based on the pseudospin singlets are the most interesting objects to look for pseudospin symmetry manifestations.

The calculations performed in [10] have shown that the goodness of the pseudospin symmetry improves when a nucleon binding energy decreases and a pseudo-orbital momentum decreases. Therefore, weakly bound exotic nuclei are the most exciting ones to search for the pseudospin symmetry manifestation.

It is the aim of the present paper to investigate a pseudospin dependence of the particle–core coupling and indicate some experimental possibilities to study the pseudospin symmetry manifestation in the spectra of odd superheavy nuclei.

## 1. PSEUDOSPIN SYMMETRY ORIGIN

A strong spin–orbit interaction introduced into nuclear physics in 1949 [4] was an unusual idea at that moment as the majority of nuclear physicists believed in the  $L$ – $S$  coupling scheme. However, strong spin–orbit interaction was necessary to reproduce the known magic numbers. The simplest Hamiltonian which can describe the nuclear mean field is the Hamiltonian with a harmonic oscillator potential, spin–orbit and orbit–orbit terms

$$h = h_{\text{osc}} + \nu_{ls} \mathbf{l} \cdot \mathbf{s} + \nu_{ll} (\mathbf{l}^2 - \langle \mathbf{l}^2 \rangle_{\text{shell}}). \quad (1)$$

The value of  $\nu_{ls}$  is such that a splitting generated by the  $\mathbf{l} \cdot \mathbf{s}$  term in (1) is large.

Twenty years later [1, 2] quasidegeneracy in the single particle level scheme was observed. Namely, single-particle states with  $j_1 = l_1 + 1/2$  and  $j_2 = l_2 (= l_1 + 2) - 1/2 = j_1 + 1$  lie very close in energy. They are labeled as pseudospin doublets with the following quantum numbers:

$$\begin{aligned} \tilde{N} &= N - 1, \\ \tilde{l} &= \begin{cases} l_1 + 1, j_1 = l_1 + 1/2 \\ l_2 - 1, j_2 = l_2 - 1/2, \end{cases} \\ \tilde{s} &= 1/2, \end{aligned} \quad (2)$$

where tilde marks the pseudo-oscillator quantum numbers. Examples of pseudo-spin doublets are:  $3s_{1/2}$  and  $2d_{3/2}$  ( $\tilde{l} = 1, \tilde{N} = 3$ ),  $1g_{7/2}$  and  $2d_{5/2}$  ( $\tilde{l} = 3, \tilde{N} = 3$ ),  $1h_{9/2}$  and  $2f_{7/2}$  ( $\tilde{l} = 4, \tilde{N} = 4$ ). An example of a pseudospin singlet is  $3p_{1/2}$  ( $\tilde{l} = 0, \tilde{N} = 4$ ).

In terms of the pseudospin–orbit operators the Hamiltonian (1) takes the form [5]

$$h = \tilde{h}_{\text{osc}} + (4\nu_{ll} - \nu_{ls}) \tilde{\mathbf{l}} \cdot \tilde{\mathbf{s}} + \nu_{ll} (\tilde{\mathbf{l}}^2 - \langle \tilde{\mathbf{l}}^2 \rangle_{\text{shell}}) + \text{const.} \quad (3)$$

It is known empirically that

$$4\nu_{ll} - \nu_{ls} \approx 0. \quad (4)$$

As a result, pseudospin–orbit interaction is several times weaker than usual spin–orbit interaction.

The physical grounds for appearance of the pseudospin symmetry in nuclei was clarified in the works of J. N. Ginocchio [3]. It was shown that this problem should be considered in the framework of the Relativistic Mean Field Theory. The Lorentz-covariant Dirac equation for a single particle with mass  $M$  is

$$(\gamma^\mu(cp_\mu + g_v A_\mu) + Mc^2 + V_s) \Psi = 0, \quad (5)$$

where  $V_s$  is a scalar potential, attractive in the case of nucleons, and  $A_\mu(A_0, \mathbf{A})$  is a vector potential. Assuming that these potentials are time-independent we obtain a Dirac Hamiltonian

$$H = \boldsymbol{\alpha} (c\mathbf{p} + g_v \mathbf{A}) + V_v + \beta (Mc^2 + V_s), \quad (6)$$

where  $V_v = g_v A_0$  is repulsive [6]. Neglecting  $\mathbf{A}$  which is not presented in a mean field [6] of an even–even nucleus we obtain the following equation for the large ( $g$ ) and small ( $f$ ) components of the Dirac spinor [6]:

$$\begin{pmatrix} M + V_v + V_s & \boldsymbol{\sigma} \cdot \mathbf{p} \\ \boldsymbol{\sigma} \cdot \mathbf{p} & -M + V_v - V_s \end{pmatrix} \begin{pmatrix} g \\ f \end{pmatrix} = E \begin{pmatrix} g \\ f \end{pmatrix}. \quad (7)$$

Representing  $E$  as  $E = M + \epsilon$  and using the fact that  $|\epsilon| \ll 2\tilde{M}$ , where  $\tilde{M} = M - 1/2(V_v - V_s)/c^2$ , we derive from (7) the Schrödinger equation for the large component  $g$  [6]

$$\left( \mathbf{p} \frac{1}{2\tilde{M}} \mathbf{p} + \frac{\hbar^2}{4\tilde{M}^2 c^2} \frac{1}{r} \frac{\partial(V_v - V_s)}{\partial r} \mathbf{1} \cdot \mathbf{s} + (V_v + V_s) \right) g = \epsilon g. \quad (8)$$

It is seen from (8) that different combinations of  $V_v$  and  $V_s$  contribute to the spin–orbit term and the radial potential well. The depth of the radial potential ( $V_v + V_s$ ) is equal approximately to 50 MeV. The value of ( $V_v - V_s$ ) is equal to 700–800 MeV inside the nucleus.

As was shown in [3], pseudospin symmetry takes place if  $V_s/V_v = -1$ . For this reason, this symmetry is not exact because in this case  $(V_v + V_s) = 0$ , i. e., there is no binding potential for nucleons. However, as it follows from the QCD sum rule, the ratio  $V_s/V_v \approx -1$  with an estimated accuracy of 20%. Indeed the detailed QCD sum rule gives

$$\begin{aligned} V_s &= -4\pi^2 \sigma_N \rho_N / M^2 m_q, \\ V_v &= 32\pi^2 \rho_N / M^2, \end{aligned} \quad (9)$$

where  $\rho_N$  is the nuclear matter density,  $\sigma_N$  is the so-called sigma term ( $\sigma_N \approx 45 \pm 8$  MeV) and  $m_q$  is the mass of a light quark. Thus,

$$\frac{V_s}{V_v} = -\frac{\sigma_N}{8m_q} \approx -1.1. \quad (10)$$

## 2. PSEUDOSPIN DEPENDENCE OF THE PARTICLE–CORE COUPLING

To describe the properties of the low-lying collective states and of a coupling of a single particle and a collective motion, it is useful to replace a realistic residual interaction by a schematic interaction. A useful application of this concept is the RPA [7]. The RPA is equivalent to the time-dependent Hartree–Fock. It means that in the framework of the RPA all interactions generating the same time-dependent mean field are equivalent. For the time-dependent mean field  $U(\mathbf{r}, t)$  we have a relation

$$U(\mathbf{r}, t) = \int d^3r' V_{\text{res}}(\mathbf{r}, \mathbf{r}') \rho(\mathbf{r}', t), \quad (11)$$

where  $\rho$  is a nuclear density and  $V_{\text{res}}$  is a residual interaction. Having in mind a description of nuclear shape oscillations and their coupling to a single particle motion let us parameterize a time dependence of the mean field and the nuclear density by the following expressions:

$$U(\mathbf{r}, t) = U_0 \left( \frac{r}{1 + \sum_{\lambda, \mu} \alpha_{\lambda, \mu}(t) Y_{\lambda, \mu}(\mathbf{r})} \right), \quad (12)$$

$$\rho(\mathbf{r}, t) = \rho_0 \left( \frac{r}{1 + \sum_{\lambda, \mu} \alpha_{\lambda, \mu}(t) Y_{\lambda, \mu}(\mathbf{r})} \right), \quad (13)$$

where  $\rho_0$  and  $U_0$  are the static mean field and density. Expanding in powers of  $\alpha_{\lambda, \mu}$  in Eqs. (12) and (13), restricting ourselves to the first orders in  $\alpha_{\lambda, \mu}$

$$U(\mathbf{r}, t) = U_0(r) - r \frac{d\rho_0(r)}{dr} \sum_{\lambda, \mu} \alpha_{\lambda, \mu}(t) Y_{\lambda, \mu}, \quad (14)$$

$$\rho(\mathbf{r}, t) = \rho_0 - r \frac{dU_0(r)}{dr} \sum_{\lambda, \mu} \alpha_{\lambda, \mu}(t) Y_{\lambda, \mu} \quad (15)$$

and substituting the result into (11) we obtain  $V_{\text{res}}$  in a separable form

$$V_{\text{res}}(\mathbf{r}, \mathbf{r}') = \chi r \frac{dU_0(r)}{dr} \cdot r' \frac{dU_0(r')}{dr'} \sum_{\lambda, \mu} Y_{\lambda, \mu}(\mathbf{r}) Y_{\lambda, \mu}^*(\mathbf{r}') \quad (16)$$

with the condition for  $\chi$

$$1 = \chi \int r^2 dr r \frac{dU_0}{dr} \cdot r \frac{d\rho_0}{dr}. \quad (17)$$

For the nuclear mean field  $U_0$  the following expression results from Eq. (8):

$$U_0(r) = (V_s(r) + V_v(r)) + \frac{\hbar^2}{4\tilde{M}^2 c^2} \frac{1}{r} \frac{d}{dr} (V_s(r) - V_v(r)) (\mathbf{1} \cdot \mathbf{s}). \quad (18)$$

Let us approximate  $V_s(r)$  and  $V_v(r)$  by the terms linear in  $\rho(r)$  [8]

$$\begin{aligned} V_s(r) &= -V_{0s} \frac{\rho_0(r)}{\rho_{av}}, \\ V_v(r) &= V_{0v} \frac{\rho_0(r)}{\rho_{av}}, \\ V_{0s} - V_{0v} &\approx 50 \text{ MeV}, \\ V_{0s} + V_{0v} &= 700 \div 800 \text{ MeV}, \\ \tilde{M} &= M - (V_v - V_s)/2c^2, \end{aligned} \quad (19)$$

where  $\rho_{av}$  is the nuclear density inside the nucleus. Then, assuming a Saxon–Woods form of  $\rho_0$  we obtain the following expression for the form factor  $r \frac{dU_0(r)}{dr}$ :

$$\begin{aligned} r \frac{dU_0(r)}{dr} &= \frac{\rho_0(r)}{\rho_{av}} \left(1 - \frac{\rho_0(r)}{\rho_{av}}\right) \left\{ (V_{0s} - V_{0v}) \frac{r}{a} + \frac{\hbar^2}{2Ma^2} \frac{\mathbf{1} \cdot \mathbf{s}}{\left(1 - \frac{V_{0v} + V_{0s}}{2Mc^2} \frac{\rho_0(r)}{\rho_{av}}\right)^3} \right. \\ &\times \left[ \frac{(V_{0v} + V_{0s})}{2Mc^2} \left(1 - \frac{V_{0v} + V_{0s}}{2Mc^2} \frac{\rho_0(r)}{\rho_{av}}\right) \frac{a}{r} + \frac{(V_{0v} + V_{0s})}{2Mc^2} \left(1 - 2 \frac{\rho_0(r)}{\rho_{av}}\right) \right. \\ &\left. \left. + \left(\frac{V_{0v} + V_{0s}}{2Mc^2}\right)^2 \frac{\rho_0(r)}{\rho_{av}} \right] \right\}, \end{aligned} \quad (20)$$

where  $a$  is a diffusion parameter of the nuclear density  $\rho_0$ . The function  $\frac{\rho_0(r)}{\rho_{av}} \left(1 - \frac{\rho_0(r)}{\rho_{av}}\right)$  is localized at the nuclear surface. So we can put in the figure brackets in (20)  $r = R$ , where  $R$  is a nuclear radius. Therefore, we can approximate the form factor  $r \frac{dU_0}{dr}$  by the expression

$$r \frac{dU_0}{dr} = \frac{\rho_0(r)}{\rho_{av}} \left(1 - \frac{\rho_0(r)}{\rho_{av}}\right) (c + b + b(\mathbf{1} \cdot \mathbf{s})) \quad (21)$$

or in terms of the pseudospin and pseudo-orbital momentum operators as

$$r \frac{dU_0}{dr} = \frac{\rho_0(r)}{\rho_{av}} \left(1 - \frac{\rho_0(r)}{\rho_{av}}\right) (c - b(\tilde{\mathbf{l}} \cdot \tilde{\mathbf{s}})). \quad (22)$$

Using the values of  $V_s$  and  $V_v$  given above and putting  $R = 7$  fm,  $a = 0.6$  fm we obtain  $c \approx 550$  MeV and  $b \approx 45$  MeV. With the form factor (22) substituted into (16) the residual forces obtained look like the surface delta interaction [9].

From a comparison of the values of the parameters  $c$  and  $b$  we can see that the main part of the residual interaction is pseudospin-independent. Therefore, the Hamiltonian with the pseudospin symmetric mean field term and the pseudospin independent part of the residual forces derived above can be considered as an approximate model for description of low-lying nuclear excitations. In the framework of this model the excited states of both even–even and odd nuclei will be characterized by the total pseudo-orbital momentum. The pseudo-orbital momentum is finally coupled to the pseudospin forming pseudospin multiplets. The eigenstates of this Hamiltonian with one sort of particles belong to the basis of irreducible representations of  $U(\Omega) \otimes U(2)$ . Here  $\Omega$  is the total number of the pseudo-orbital  $\tilde{m}$  states. The spatial parts of the nucleon wave functions form the basis of irreducible representations of  $U(\Omega)$  characterized by their symmetry type. A more detailed characterization of these states could be provided by a subgroup of  $U(\Omega)$  containing  $O(3)$ . In the case of well-deformed nuclei with their rotational bands as basic elements of the excitation spectra the intermediate group is  $SU(3)$ . In the case of even–even nuclei the lowest bands correspond to the most symmetric representation characterized by the following sequence of values of the pseudo-orbital momenta:  $\tilde{L} = 0, 2, 4, \dots$ . In the case of odd nuclei, the value of the pseudo-orbital momentum of the lowest state can be different from zero. This value depends on the pseudo-orbital momenta of the single particle state near the Fermi surface.

A splitting of the pseudospin multiplets is determined by the matrix element of the pseudospin-dependent part of the residual forces  $\delta V_{\text{res}}$ . In the first approximation

$$\begin{aligned} \delta V_{\text{res}}(\mathbf{r}_1, \mathbf{r}_2) = & -2cb\chi \left( \tilde{\mathbf{I}}_1 \cdot \tilde{\mathbf{s}}_1 \right) \frac{\rho_0(r_1)}{\rho_{av}} \left( 1 - \frac{\rho_0(r_1)}{\rho_{av}} \right) \\ & \times \frac{\rho_0(r_2)}{\rho_{av}} \left( 1 - \frac{\rho_0(r_2)}{\rho_{av}} \right) \sum_{\lambda, \mu} Y_{\lambda, \mu}(\mathbf{r}_1) Y_{\lambda, \mu}^*(\mathbf{r}_2). \end{aligned} \quad (23)$$

In the lowest states pseudospin takes the minimum possible value since in this case the coordinate depending part of the wave function is the most symmetric and the pseudospin of the nucleon pairs is zero. Thus, the total pseudospin is equal to that of an odd particle, i. e., to  $1/2$ . In this case index «1» in (23) belongs to an odd particle, but index «2» describes all other particles forming the core. Then the interaction term (23) takes the form of the particle–core coupling term considered, for example, in the Bohr–Mottelson model. The pseudospin independent analog of the Bohr–Mottelson particle–core coupling term with the

radial form factor derived above is

$$\begin{aligned} \delta V_{\text{res}}(\mathbf{r}_1, \mathbf{r}_2) = & -c^2 \chi \frac{\rho_0(r_1)}{\rho_{av}} \left(1 - \frac{\rho_0(r_1)}{\rho_{av}}\right) \\ & \times \frac{\rho_0(r_2)}{\rho_{av}} \left(1 - \frac{\rho_0(r_2)}{\rho_{av}}\right) \sum_{\lambda, \mu} Y_{\lambda, \mu}(\mathbf{r}_1) Y_{\lambda, \mu}^*(\mathbf{r}_2). \end{aligned} \quad (24)$$

Comparing (23) and (24) we can see that the strength of the interaction term (23) is  $2\frac{b}{c}\langle \tilde{\mathbf{l}}_i \cdot \tilde{\mathbf{s}}_i \rangle$  times smaller than that of the pseudospin independent particle–core coupling term. Let us estimate at first the average  $\langle \tilde{\mathbf{l}}_i \cdot \tilde{\mathbf{s}}_i \rangle$  where index «i» denotes an odd particle. The pseudo-orbital momentum of an odd particle contributes to the total pseudo-orbital momentum  $\tilde{L}$  of the state. It can be taken to be equal to  $\frac{1}{N}\tilde{L}$ , where  $N$  is approximately the number of particles in the open shell if the number of nucleons contributing to the total pseudo-orbital momentum does not depend on  $\tilde{L}$ . If this number increases proportionally to  $\tilde{L}$  the contribution of the odd particle to  $\tilde{L}$  can be estimated as  $\frac{\tilde{l}_0}{N}$ , where  $\tilde{l}_0$  is a constant of the order of unity. For well-deformed nuclei we can take  $N \approx 30$ . Since  $\tilde{s} = 1/2$ , we obtain

$$2\frac{b}{a}\langle \tilde{\mathbf{l}}_i \cdot \tilde{\mathbf{s}}_i \rangle \approx \frac{\tilde{L}}{360} \quad \text{or} \quad \frac{\tilde{l}_0}{360}. \quad (25)$$

The matrix element of the particle–core interaction term in the Bohr–Mottelson model in the case of deformed nuclei can be estimated as  $\sim 2.3$  MeV. Thus, a splitting of the pseudospin doublets in the rotational bands of odd nuclei is  $\sim 7\tilde{L}$  or  $\sim 7\tilde{l}_0$  keV. This estimate is in a correspondence with an experimentally observed splitting which is equal to  $10 \div 30$  keV; though, without proportionality to  $\tilde{L}$ . Thus, we should use an estimate with  $\tilde{l}_0$ .

### 3. WHAT IS INTERESTING TO OBSERVE

As was mentioned above, experimental data on the single particle spectra of the stable well-investigated nuclei show that pseudospin symmetry is fulfilled only approximately. It is known from the consideration in the framework of the Relativistic Mean Field approach that pseudospin symmetry improves as the binding energy of nucleons decreases. The calculations [10] have also shown that pseudospin symmetry improves as the pseudo-orbital momentum  $\tilde{l}$  decreases. Therefore, it is interesting to look for the manifestations of pseudospin symmetry in nuclei far removed from the valley of stability. Thus, the low-lying states in nuclei with  $Z \geq 100$  are interesting objects for investigations. In these nuclei

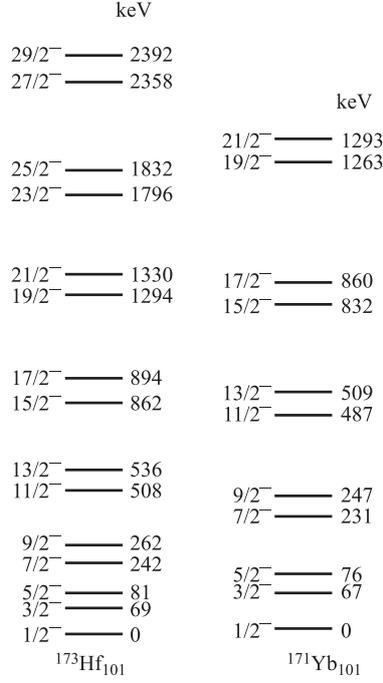


Fig. 1. Ground state rotational bands of  $^{173}\text{Hf}$  and  $^{171}\text{Yb}$  based on the pseudospin singlet states. Experimental data are taken from [14]

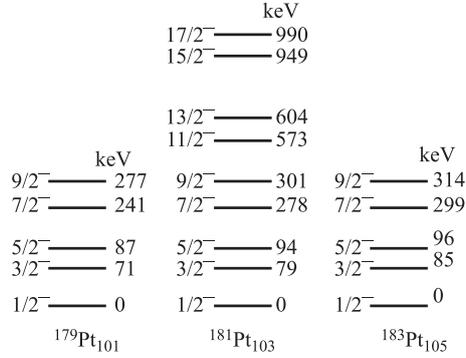


Fig. 2. Ground state rotational bands of  $^{179,181,183}\text{Pt}$  based on the pseudospin singlet states. Experimental data are taken from [14]

more interesting are single particle states with a small pseudo-orbital momentum, or in the case of deformed nuclei with a small projection of the pseudo-orbital momentum on the axial symmetry axis.

	keV	keV	L
$9/2^-$	508	$11/2^-$	5
$7/2^-$	333	$9/2^-$	4
$5/2^-$	187	$7/2^-$	3
$3/2^-$	74	$5/2^-$	2
$1/2^-$	0	$3/2^-$	1
$1/2[510]$		$3/2[512]$	

$^{187}\text{Os}_{111}$

Fig. 3. The lowest-lying rotational band of  $^{187}\text{Os}$  based on the single particle states belonging to the pseudospin doublet with  $\tilde{\Lambda} = 1$ . Experimental data are taken from [14]

At the same time, calculations of the single particle spectra of superheavy nuclei performed up to now demonstrate different results: from small to large splitting of the pseudospin doublets. This is an additional argument to carry out experimental investigations in order to clarify the problem.

The experimental data on well-investigated nuclei [11] and the consideration in the previous section have shown that the most clear manifestation of the pseudospin symmetry is expected in the spectra of the low-lying rotational bands of odd nuclei based on the pseudospin singlets or on the pseudospin doublets with a projection of the pseudo-orbital momentum on the axial symmetry axis  $\Lambda$  equal to 1. Below we consider this suggestion in detail.

The calculations of A. Parchomenko and A. Sobiczewski [12] show that with a large probability in odd Md and Lr isotopes the ground state or one of the low-lying states is  $[521]1/2^-$ . This state is the pseudospin singlet state having the following pseudo-oscillator quantum numbers  $[\widetilde{420}]1/2^-$ . The rotational band based on this state consists of a singlet and a sequence of doublets:  $1/2^-$ ;  $(3/2^-, 5/2^-)$ ;  $(7/2^-, 9/2^-)$ ... For illustration of possible observations let us consider nuclei with odd numbers of neutrons  $N$  equal to 101 and 103 and even numbers of protons. Their excitation spectra are shown in Figs. 1 and 2 for  $^{173}\text{Hf}_{101}$ ,  $^{171}\text{Yb}_{101}$  and  $^{179-185}\text{Pt}_{101-107}$ . A small splitting of doublets equal to several tens of keV is the main signature of the pseudospin symmetry. As is seen from Figs. 1 and 2, the doublet structure can be seen in several isotopes of the same element and for several values of  $Z$ . Thus, it is not necessary to search for one very special nucleus in which this effect is pronounced. If this effect exists, it should be seen in several neighbouring nuclei.

The other interesting possibility for observation of the pseudospin symmetry effects is related to the spectra of odd isotopes of the element  $Z = 111$ . As is shown by the calculations of A. Parchomenko and A. Sobiczewski, the pseudospin doublet with  $\Lambda = 1$ :  $[512]3/2^-$  and  $[510]1/2^-$  can exist in these nuclei. The expected spectra of the low-lying states in this case can be similar to that observed in  $^{187}\text{Os}_{111}$  having the number of neutrons equal to 111. This spectrum is shown in Fig. 3. It is seen that a splitting of the states in doublets is very small and does not exceed 10 keV.

The last example considered is the low-lying spectra of the spherical nuclei with  $Z = 115$  and 117. Single particle states with pseudo-orbital momenta  $\tilde{l} = 0$  and 2 can be located in these nuclei near the ground state. The low-lying spectra should be similar to the spectrum of  $^{195}\text{Pt}_{117}$  which was very well investigated in [13].

## CONCLUSION

The existence of the approximate pseudospin symmetry is supported by the experimental data for nuclei belonging to the traditionally investigated region of the nuclide chart.

The pseudospin symmetry is justified theoretically and has its grounds in an approximate equality of the scalar and vector potentials in the Dirac equation describing a motion of nucleons in a relativistic mean field.

It is interesting, whether the pseudospin symmetry will be confirmed by experimental data for exotic nuclei, for instance, for superheavy nuclei.

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