E1-2006-178

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EVENT GENERATOR FOR pp INTERACTIONS

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Шубитидзе Н. E1-2006-178
Генератор событий для pp-взаимодействий

Построен феноменологический генератор событий pp-столкновений при энергии $E_{\rm lab} = 70$ ГэВ. Он основан на гауссовой форме матричного элемента взаимодействия. Конечные состояния включают в себя протоны и пионы. Параметры генератора отфитированы на экспериментальных данных по сечениям. Строго выполняются законы сохранения энергии и импульса. Генератор событий обеспечивает малость поперечных импульсов конечных частиц.

Работа выполнена в Лаборатории физики частиц ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна, 2006

Shubitidze N. E1-2006-178
Event Generator for pp Interactions

Phenomenological event generator for pp interactions at energy $E_{\rm lab} = 70$ GeV was created. It is based on the Gaussian form of the matrix element of the interaction. The final states involve two protons and pions. Parameters of the generator are fitted for experimental cross-section data. The energy and momentum conservation laws are strongly satisfied. The event generator provides the smallness of the transverse momentum of the final particles.

The investigation has been performed at the Laboratory of Particle Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna, 2006

INTRODUCTION

The event generator for the process of pp interactions at $E_{\rm lab} = 70$ GeV energy was created in the framework of the project «Thermalization» [1]. The goal of this experiment is the investigation of the collective behavior of particles in the process of multiple hadron production. Recent results of investigation in this direction are reviewed in [2]. The necessity of such a generator arose after testing temporary generators. Some of them (Venus, Nexus, RQMD) were meant for simulation of a process at a higher energy.

PYTHIA [3] is one of a series of more intensively used generators. It covered a wide spectrum of lepton–lepton, lepton–hadron and hadron–hadron processes. It is based on the Lund model of string fragmentation and on the model of parton showers.

HERWIG [4] is a general-purpose Monte-Carlo event generator, which includes the simulation of hard lepton–lepton, lepton–hadron and hadron–hadron scattering and soft hadron–hadron collisions in one package. It uses the parton-shower approach for initial- and final-state QCD radiation, including colour coherence effects, azimuthal correlations both within and between jets, and also the Minimal Supersymmetric Standard Model.

Unfortunately, these generators failed to describe experimental data at energies nearby $E_{\rm lab} = 69$ GeV [5]. The proposed generator is based on a simple model of Gaussian form of matrix element of interaction and has a small number of parameters which are fitted on the experimental data. The work of the generator divides into two phases. The first phase is the sampling of the scattering channel and the second phase is the assignment of the values momenta to particles in a final state.

1. SAMPLING OF A SCATTERING CHANNEL

We restrict our consideration to the reactions of the following kind:

$$p + p \to p + p + \underbrace{\pi^+ + \ldots \pi^-}_{\mu} + \underbrace{\pi^0 + \ldots + \pi^0}_{\nu} \tag{1}$$

because every possible combinations of μ and ν are the numbers of charged and neutral pions in the final state

$$0 \leqslant \mu, \nu \leqslant n_{\rm max} \tag{2}$$

designate a dominant part of scattering channels. For calculation of their probability we use a phase-space integral of the following type:

$$Z_{\mu,\nu} = \int \left\{ \prod_{i=1}^{N} \frac{d^3k_i \, \exp\left[-r_0^2 k_{t,i}^2\right]}{2\sqrt{k_i^2+m_i^2}} \right\} \delta\left(E - \sum_{i=1}^{N}\sqrt{k_i^2+m_i^2}\right) \times \\ \times \delta\left(\sum_{i=1}^{N} k_{x,i}\right) \delta\left(\sum_{i=1}^{N} k_{y,i}\right) \delta\left(\sum_{i=1}^{N} k_{z,i}\right), \quad (3)$$

where E is total energy; r_0 is phenomenological constant which is included into a matrix element of scattering for cutting big transverse momenta of particles; $k_{t,i}$ is transverse momentum of ith particle; $N \equiv 2 + \mu + \nu$ is a complete number of particles in the final state. Values of $Z_{\mu,\nu}$ are proportionate to appropriate cross sections and they are used to build an algorithm of the channel selection.

Integral (3) was calculated by the Monte Carlo method. We use the Dirac delta-function to eliminate the integration on the momentum of the last particle

$$\begin{aligned} k_{N,x} &= -\sum_{i=1}^{N-1} k_{i,x}, \\ k_{N,y} &= -\sum_{i=1}^{N-1} k_{i,y}, \\ k_{N,z} &= -\sum_{i=1}^{N-1} k_{i,z}. \end{aligned} \quad (4)$$

For the residuary $N-1$ momenta we go to the spherical coordinates:

$$\begin{aligned} k_{i,x} &= k_i \, \sin(\pi\tau_i) \, \cos(2\pi\nu_i), \\ k_{i,y} &= k_i \, \sin(\pi\tau_i) \, \sin(2\pi\nu_i), \\ k_{i,z} &= k_i \, \cos(\pi\tau_i). \end{aligned} \quad (5)$$

In new variables integral (3) has got a form:

$$Z_{\mu,\nu} = (2\pi^2)^{N-1} \int \left\{ \prod_{i=1}^{N-1} \frac{k_i^2 dk_i \, \sin(\pi\tau_i) d\tau_i d\nu_i}{2\sqrt{k_i^2+m_i^2}} \, \exp^{-r_0^2 k_i^2 \, \sin(\pi\tau_i)^2} \right\} \times \\ \times \frac{\exp^{-r_0^2 k_{t,N}^2}}{2\sqrt{k_N^2+m_N^2}} \delta\left(E - \sum_{i=1}^{N}\sqrt{k_i^2+m_i^2}\right), \quad (6)$$

where k_N is defined by (4). The next step is to eliminate the Dirac delta-function from (6). We change the total momenta $k_1, \ldots, k_{N-1}$ by the new variables

$\rho, \eta_1, \ldots, \eta_{N-2}$ just like conversion to the hyperspherical coordinates. It is clear if we identify η_i with $\cos^2(\alpha_i)$. The following example for $N = 18$ makes a clear the preceding:

$$\begin{array}{rclllll}
k_1 &=& \rho & \eta_1 & \eta_2 & \eta_4 & \eta_8 & \eta_{16} \\
k_2 &=& \rho & \eta_1 & \eta_2 & \eta_4 & \eta_8 & (1-\eta_{16}) \\
k_3 &=& \rho & \eta_1 & \eta_2 & \eta_4 & (1-\eta_8) & \\
k_4 &=& \rho & \eta_1 & \eta_2 & (1-\eta_4) & \eta_9 & \\
k_5 &=& \rho & \eta_1 & \eta_2 & (1-\eta_4) & (1-\eta_9) & \\
k_6 &=& \rho & \eta_1 & (1-\eta_2) & \eta_5 & \eta_{10} & \\
k_7 &=& \rho & \eta_1 & (1-\eta_2) & \eta_5 & (1-\eta_{10}) & \\
k_8 &=& \rho & \eta_1 & (1-\eta_2) & (1-\eta_5) & \eta_{11} & \\
k_9 &=& \rho & \eta_1 & (1-\eta_2) & (1-\eta_5) & (1-\eta_{11}) & \\
k_{10} &=& \rho & (1-\eta_1) & \eta_3 & \eta_6 & \eta_{12} & \\
k_{11} &=& \rho & (1-\eta_1) & \eta_3 & \eta_6 & (1-\eta_{12}) & \\
k_{12} &=& \rho & (1-\eta_1) & \eta_3 & (1-\eta_6) & \eta_{13} & \\
k_{13} &=& \rho & (1-\eta_1) & \eta_3 & (1-\eta_6) & (1-\eta_{13}) & \\
k_{14} &=& \rho & (1-\eta_1) & (1-\eta_3) & \eta_7 & \eta_{14} & \\
k_{15} &=& \rho & (1-\eta_1) & (1-\eta_3) & \eta_7 & (1-\eta_{14}) & \\
k_{16} &=& \rho & (1-\eta_1) & (1-\eta_3) & (1-\eta_7) & \eta_{15} & \\
k_{17} &=& \rho & (1-\eta_1) & (1-\eta_3) & (1-\eta_7) & (1-\eta_{15}). &
\end{array} \tag{7}$$

Now we have a possibility to change the Dirac delta-function by the following expression:

$$\delta\left(E - \sum_{i=1}^{N} \sqrt{k_i^2 + m_i^2}\right) \to |f'(\rho_0)|^{-1}, \tag{8}$$

where ρ_0 is the root of equation:

$$f(\rho) \equiv E - \sum_{i=1}^{N} \sqrt{k_i^2 + m_i^2} = 0. \tag{9}$$

Nota bene, the subprogram to calculate Jacobians of transformations (7) consists of 3032 rows in the FORTRAN source code. It has been formed by the package «Mathematica».

Thus, all δ-functions are eliminated from the subintegral expression (3) and $3N - 4$ integration variables $\eta_1, \ldots, \eta_{N-2}, \tau_1, \ldots, \tau_{N-1}, \nu_1, \ldots, \nu_{n-1}$ are in the limits (0,1) so integral (3) is completely prepared for integration by the Monte

Carlo method. We have used our realization on the FORTRAN 64-bit pseudo-random number generator LFSR258 with period 2^{258} [6].

We have used the described-above scheme for fitting experimental data and have found an optimal value of the cutting parameter $r_0 = 2.614$ GeV$^{-1} \cdot$ c. Figure 1 presents the results of calculation.

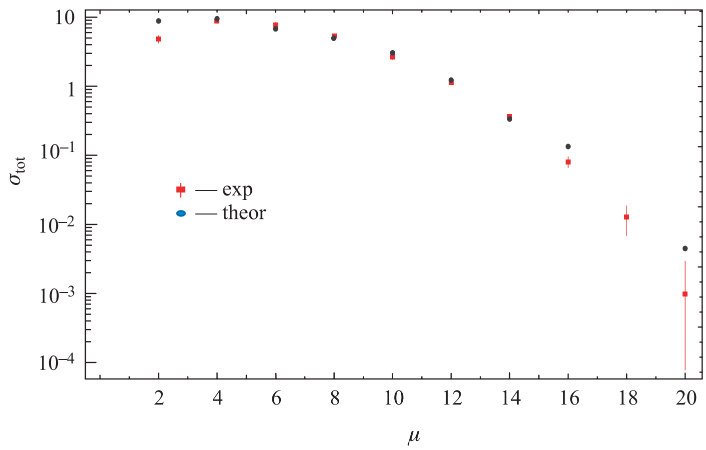

Fig. 1. Experimental (■) and calculated (●) data are compared. The horizontal axis marked the number of charged pions and the vertical axis is the cross sections in mb. The points are the sums of cross sections for all possible numbers of neutral pions when the number of charged pions is fixed

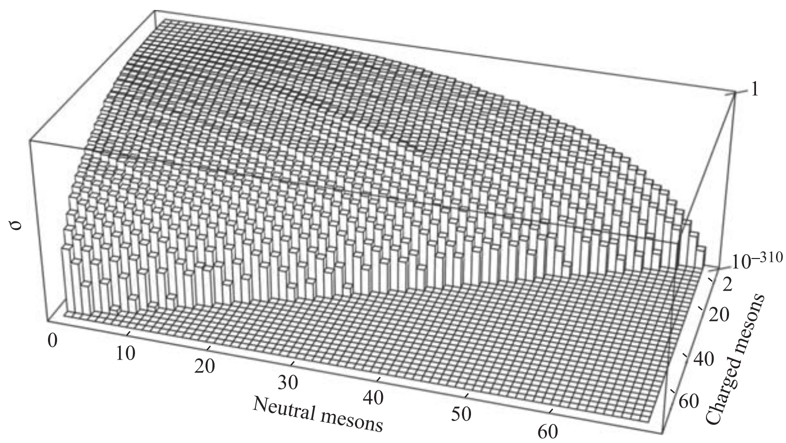

Fig. 2. Cross-section behavior for the various numbers of neutral and charged pions in the final state

After determination of the cutting parameter we calculate $Z_{\mu,\nu}$ for concerned channels. With constant factor of accuracy they are the cross sections of interaction. Figure 2 shows their qualitative behavior.

To determine ν-number of charged pions we need to divide the line segment (0,1) into fragments which are proportional to the inclusive (for neutral pions) cross sections (see Fig. 3), where

$$\widetilde{Z}_i = \widetilde{\sigma}_i / \sum_{\mu=0,2}^{\mu_{\max}} \widetilde{\sigma}_\mu, \tag{10}$$

and $\widetilde{\sigma}_\mu$ is a sum of cross sections with μth number of charged pions in the final state

$$\widetilde{\sigma}_\mu = \sum_{\nu=0}^{\nu_{\max}} \sigma_{\mu,\nu}, \tag{11}$$

where $\nu_{\max}$ is a maximally possible number of π^0 in the final state on the fixed number of $\pi^\pm$.

We define the number of charged pions by sampling a random number in the interval (0,1) and compare it with the line segment in Fig. 3. Having fixed the number of charged pions we analogously define the number of neutral pions in the final state.

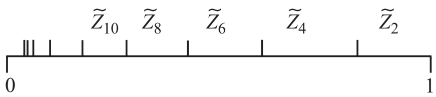

Fig. 3. Scale for determination of the number of charged pions

2. DISTRIBUTION OF MOMENTA

When the channel of interaction is fixed, the next important step of the generator is to confer concrete values for momenta. This means, that we confer to each ith particle values of p_i — total moment, ϑ_i — radial angle and ϕ_i — azimuthal angle. Of course, the laws of conservation energy and momentum must be strongly satisfied.

Obviously, that the distribution on azimuthal angles ϕ must be uniform. To take into account the form of the matrix element of interaction we may claim that the distribution on radial angles must have two maxima in neighbourhoods 0 and π and must be symmetrical under the transformation $\vartheta \to \pi - \vartheta$.

For numerical determination of statistical characteristics of particle distributions on total momenta and radial angles the appropriate histograms have been built for each channels. As the weight function the subintegral function (3) has been used. Received histograms were fitted by the following kind of function:

$$P(p, \vartheta) = \frac{p \sin(\vartheta) \exp\left[-\kappa^2 p^2 \sin(\vartheta)^2\right]}{2p_{\max} F(\kappa p)}, \tag{12}$$

where $\kappa = 3.79$ GeV$^{-1} \cdot$ c is a fitted value of the parameter, $F(x)$ is the Dawson integral [7]

$$F(x) = \mathrm{e}^{-x^2} \int\limits_0^x \mathrm{e}^{-t^2} dt. \tag{13}$$

The graph of functions (12) is shown in Fig. 4.

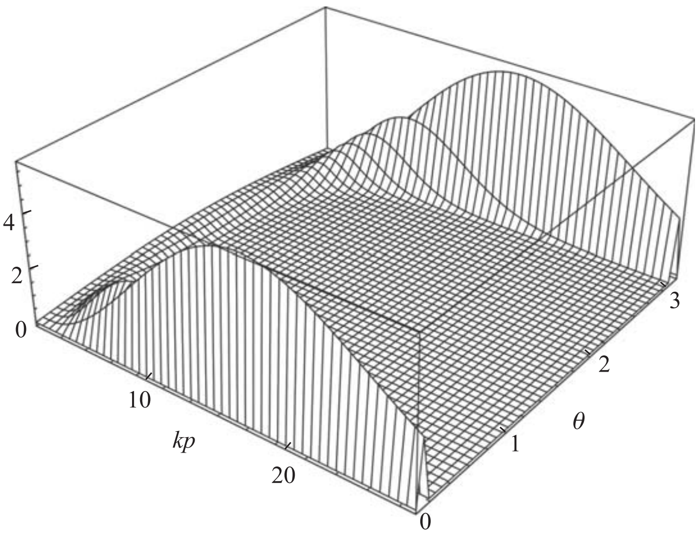

Fig. 4. The behavior of function of statistical distribution of particles on the total momentum p and radial angle ϑ

The direct sampling of momenta and radial angles of appropriate distribution (12) and azimuthal angles in the interval $(0, 2\pi)$ cannot guarantee the accomplishment of laws of conservation energy and momentum.

One can sample momenta of first $N-1$ particles appropriate distribution (12) and for the last particle calculate them from the equations of conservation laws. In this case the statistical distribution on the total momentum and a radial angle of the last particle is totally different from the distribution of other particles. We have also tested the scheme of sampling of momenta of N particles with their further correction by the averaged sum of their values. This method has led to a strong distortion of statistical distribution, especially for cases of small number of particles in the final state.

To eliminate such defects we have selected the following algorithm. The total momenta p and radial angles ϑ on appropriate distribution (12) for $N-1$ particles are being sampled by the method of inverse functions [8]. The scheme of the method is the following. Let $P(x,y)$ be the probability density function of two random variables x and y, which are defined in a rectangle

$$\int\limits_{y_{\min}}^{y_{\max}}\int\limits_{x_{\min}}^{x_{\max}} P(x,y)dxdy = 1. \tag{14}$$

We are sampling two random numbers u and v uniformly distributed in the interval (0,1) and then one after another the system of two equations for x and y is solved

$$u = \int\limits_{y_{\min}}^{y} dy' \int\limits_{x_{\min}}^{x_{\max}} P(x',y')dx' \Rightarrow y = y_0, \tag{15}$$

$$v = \frac{\int\limits_{x_{\min}}^{x} P(x',y_0)dx'}{\int\limits_{x_{\min}}^{x_{\max}} P(x',y_0)dx'} \Rightarrow x = x_0. \tag{16}$$

The sampling for the first $N-4$ particles is going in the alternating order of longitudial components of momenta p_z and random in the interval $(0, p_{\max})$ for the others. This kind of physical limitations is included in the generator because mathematically, states when particles move along Z axis and all the others in the opposite direction are not forbidden. In reality, this kind of states is suppressed. Following a parton model when the partons of interacting hadrons move one off the other, strings stretch between them. The break-off of the last forms a pair of mesons which move in opposite directions.

It is clear, that at the sampling process the state with breaking moment of conservation law may occur. In this case the state is rejected and the process of sampling restarts.

As a result, we have the possibility to receive $p_{z,N}$:

$$p_{z,N} = -\sum_{i=1}^{N-1} p_i \cos(\vartheta_i). \tag{17}$$

The sampling of p_N and ϑ_N doing along the level lines $p_{z,N} = \text{const}$ (see Fig. 5). In accordance with the distribution (12) the probabilities along the level lines have been calculated (see Fig. 6).

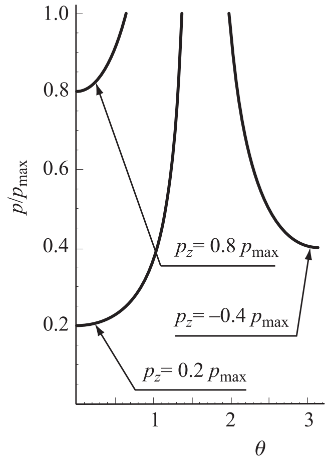

Fig. 5. Level lines for different values of p_z

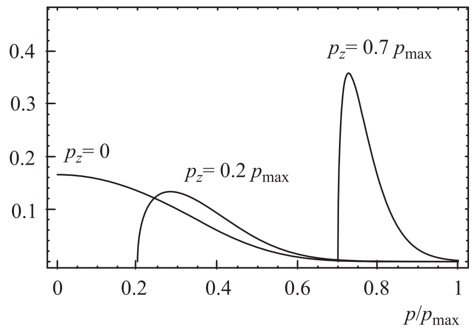

Fig. 6. Probability density along the level lines for different values of p_z

Cumulative probabilities for every channel were fitted by the functions

$$P(y) = \begin{cases} 0, & \text{if } y \leqslant b, \\ \ln\left\{\cosh\left[\dfrac{a_1(y-b)}{1+b_1(y-b)}\right]\right\}, & \text{if } y > b, \end{cases} \tag{18}$$

where

$$y = p_N/p_{\max}, \tag{19}$$

$$b = abs(p_{z,N})/p_{\max}. \tag{20}$$

Figure 7 shows parameters a_1 and b_1 as functions of the parameter b.

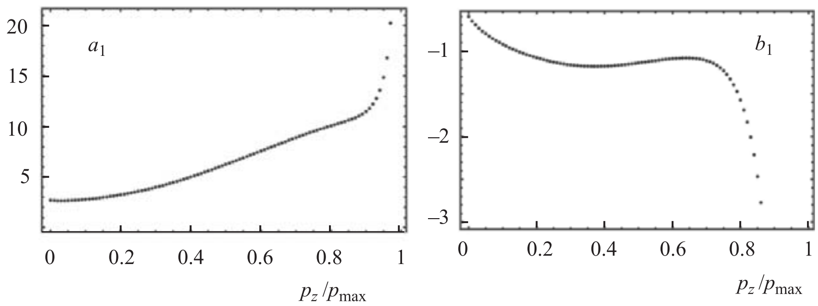

Fig. 7. Behaviors of coefficients a_1 and b_1 as functions of the parameter b

We receive the following equations for p_N:

$$y = b + \frac{\widetilde{A}}{a_1 - b_1\widetilde{A}}, \tag{21}$$

where

$$\widetilde{A} = \operatorname{arccosh}\left\{\left[\cosh\left(\frac{a_1(y-b)}{1+b_1(y-b)}\right)\right]^{\lambda}\right\} \tag{22}$$

and λ is a random number in the interval (0,1).

The sampling of azimuthal angles ϕ_i is a nontrivial problem. The task is: with the given modules of N vectors on the plane (transverse momenta of particles) it is necessary to find N azimuthal angles ϕ_i on condition that the total sum of the vectors is equal to zero (law of conservation of total momentum in c.m. system). At that the algorithm of random finding must include all possible solutions.

In our program we have used the fact that the chain being built by the vectors must be locked, if their sum equal to zero. Therefore, we have the possibility to regulate process of finding the solution. From the preliminary ordered modules of vectors $\xi_1, \ldots, \xi_N$ we build the array of partial sums:

$$R_i = \sum_{k=i+1}^{N} \xi_k \quad (i = 1, \ldots, N-1). \tag{23}$$

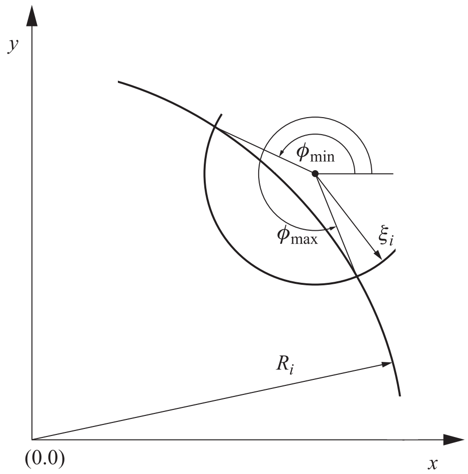

Fig. 8. The calculation of the interval for the sampling of the azimuthal angle for the ith vector is illustrated. The chain of vectors starts from the point (0.0)

At every step the permissible limits for the angle ϕ_i are calculated as is shown in Fig. 8 and then it is sampled. At the first step the allowable interval is equal $(0, 2\pi)$ and at the last step it consists of two points.

The final step for the sampling of particle momenta is the correction of total momenta to satisfy the energy conservation law. An appropriate factor comes out from the following equation:

$$E - \sum_{i=1}^{N} \sqrt{k^2 p_i^2 + m_i^2} = 0. \tag{24}$$

Then, the replacement of the total particle momenta is the following:

$$p_i \longrightarrow k p_i \qquad (i = 1, \dots N). \tag{25}$$

3. RESULTS OF CALCULATIONS

In conclusion the results of calculation of various characteristics received from the generator are presented. The sufficiently high speed of work (million events per 260 seconds) allows one to compose statistics quickly. We have the possibility to change the code of generator and investigate in detail the channels of reaction with less probability.

Figure 9 shows the behavior of cross sections of reaction

$$p + p \to p + p + \pi^+ + \pi^- + \pi^+ + \pi^- + + \underbrace{\pi^0 + \ldots + \pi^0}_{\nu} \tag{26}$$

as a function of neutral pion number ν in the final state.

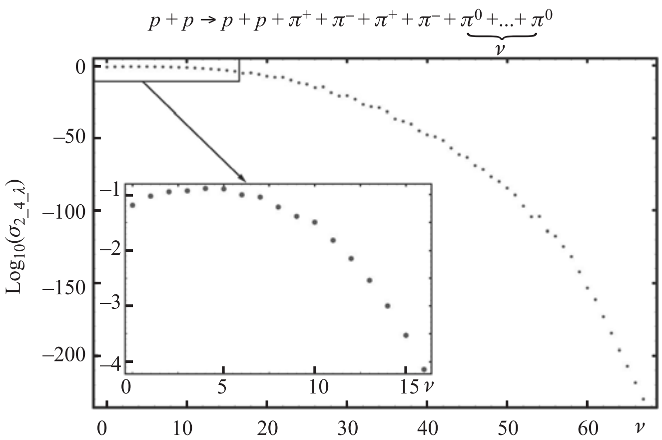

Fig. 9. The typical behavior of cross sections as a function of neutral pion number

Figure 10 shows the distribution of particles in the laboratory system as a function of the radial angle.

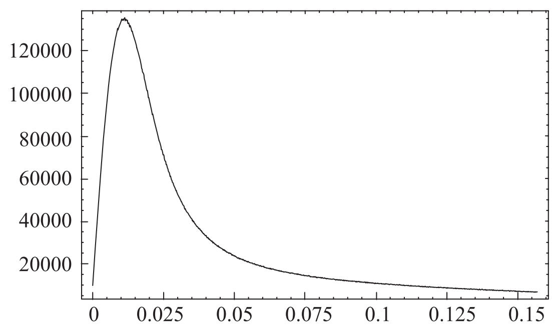

Fig. 10. Distribution of particles in the laboratory system. The values ϑ_{lab} in rad are put in the horizontal axis

Figure 11 shows histograms of particle distributions on the total momenta and radial angles for the case of 6 charged and 6 neutral pions. Figure 12 shows histograms of particle distributions on the total momenta and radial angles for the case of 12 charged and 12 neutral pions.

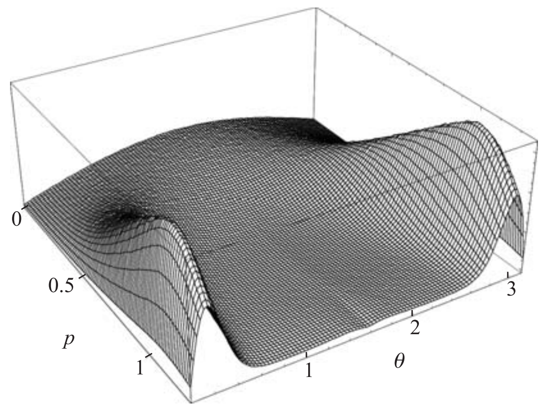

Fig. 11. p–ϑ distribution of particles for the case $pp\ 3(\pi^+\pi^-)\ 6\pi^0$

Figure 13 shows the behavior of transverse and longitudinal momenta as a function of pion number in the final state.

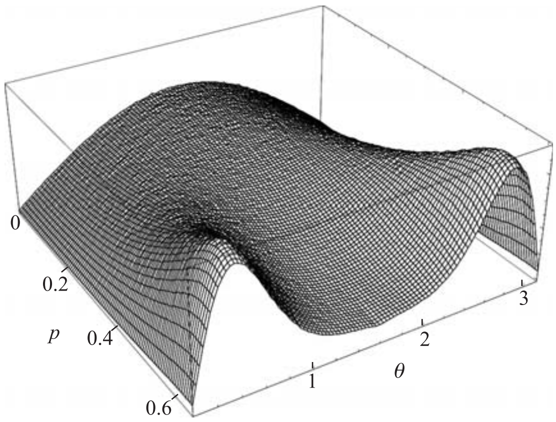

Fig. 12. p-ϑ distribution of particles for the case pp $6(\pi^+\pi^-)$ $12\pi^0$

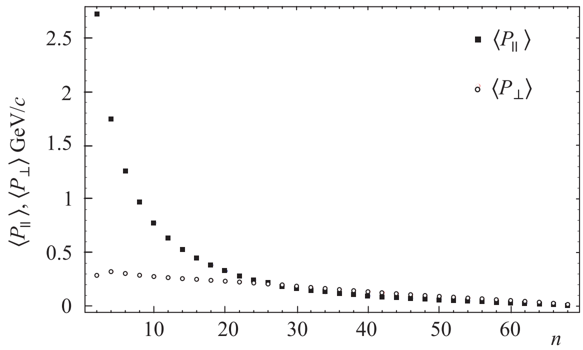

Fig. 13. Behavior of transverse and longitudinal momenta as a function of pion number

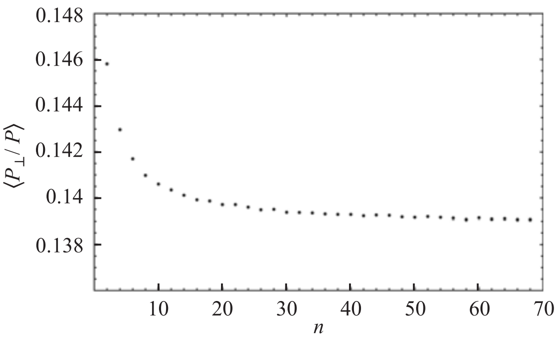

Fig. 14. Behavior of ratios of transverse and total momenta as a function of pions number

Figure 14 shows the behavior of ratios of transverse and total momenta as a function of pions number in the final state.

Acknowledgements. The author wishes to express his gratitude to the J. Manjavidze, V. V. Nikitin, T. Topuria, V. V. Voronyuk and V. V. Uzhinsky for useful discussions. This work was supported by the Russian Foundation for Basic Research under Contract No. 05-02-17693 .

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Received on December 25, 2006.

Корректор *Т. Е. Попеко*

Подписано в печать 15.03.2007.
Формат 60 × 90/16. Бумага офсетная. Печать офсетная.
Усл. печ. л. 1,18. Уч.-изд. л. 1,67. Тираж 365 экз. Заказ № 55708.

Издательский отдел Объединенного института ядерных исследований
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