E2-2006-186

E. A. Kuraev, V. V. Bytev

CHARGE ASYMMETRY FOR ELECTRON(POSITRON)–PROTON ELASTIC SCATTERING

Submitted to «Particles and Nuclei, Letters»

Кураев Э. А., Бытьев В. В. E2-2006-186
Зарядовая асимметрия в электрон(позитрон)-протонных
упругих столкновениях

Зарядовая асимметрия в электрон(позитрон)-протонных столкновениях возникает из-за интерференции вклада от борновской амплитуды и амплитуды обмена двумя фотонами. Ее можно получить из результатов экспериментов по электрон-протонному и позитрон-протонному рассеянию в одинаковой кинематике. Для виртуального комптоновского рассеяния на протоне, который входит в диаграмму обмена двумя фотонами, мы выделили вклады от упругого промежуточного состояния протона, неупругого рассеяния и параметризовали формфакторы протона как сумму двух частей — вкладов от сильного и электромагнитного взаимодействий. Аргументы, основанные на аналитичности, приводят к сокращению неупругих вкладов и вкладов сильных взаимодействий в упругий формфактор протона. В рамках этой модели сделаны численные оценки для величины асимметрии. Для упрощения расчета численной части была взята модель для формфакторов, которая близка, но не совпадает с общепринятой дипольной параметризацией упругих формфакторов протона.

Работа выполнена в Лаборатории теоретической физики им. Н. Н. Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 2006

Kuraev E. A., Bytev V. V. E2-2006-186
Charge Asymmetry for Electron(Positron)–Proton Elastic Scattering

Charge asymmetry in electron(positron)–proton scattering arises from the interference of the Born amplitude and the box-type amplitude corresponding to two virtual photons exchange. It can be extracted from electron–proton and positron–proton scattering experiments in the same kinematical conditions. Considering the virtual photon-Compton scattering tensor, which contributes to the box-type amplitude, we separate proton and inelastic contributions in the intermediate state and parametrize the proton form factors as the sum of a pure QED term and a strong interaction term. Arguments, based on analyticity, are given in favor of cancellation of contributions from proton strong interaction form factors and inelastic intermediate states in the box-type amplitudes. In the framework of this model, and assuming a dipole character of form factors, numerical estimations are given for moderately high energies.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 2006

INTRODUCTION

Recently, a lot of attention was devoted to 2γ-exchange amplitude both in scattering and annihilation channels [1–3], in connection with the experimental data on electromagnetic proton form factors (FFs) [4].

Extraction of box-type (two-photon exchange amplitude (TPE)) contribution to elastic electron–proton scattering amplitude is one of long-standing problems of experimental physics. It can be obtained from electron–proton and positron–proton scattering at the same kinematical conditions. A similar information about TPE amplitude in the annihilation channel can be obtained from the measurement of the forward-backward asymmetry in proton–antiproton production in electron–positron annihilation (and the reversal process).

The theoretical description of TPE amplitude is strongly model-dependent. Two reasons should be mentioned: the experimental knowledge of nucleon FFs is restricted in a small kinematical region and the precision of the data is often insufficient, and the contribution of the intermediate hadronic states can be only calculated with large uncertainty.

A general approximation for proton electromagnetic form factors follows the dipole approximation:

$$G_E(q^2) = \frac{G_M(q^2)}{\mu} = G_D(Q^2) = (1+Q^2/0.71\ \mathrm{GeV}^2)^{-2}, Q^2 = -q^2 = -t, \quad (1)$$

where μ is the anomalous magnetic moment of proton. However, recent experiments [4] showed a deviation of the proton electric FF from this prescription, when measured following the recoil polarization method, which is more precise than the traditional Rosenbluth separation. Such a deviation was tentatively explained, advocating the presence of a two-photon contribution.

The motivation of this paper is to perform the calculation of charge-odd correlation

$$A^{\mathrm{odd}} = \frac{\mathrm{d}\sigma^{e^-p} - \mathrm{d}\sigma^{e^+p}}{2d\sigma_B^{ep}} \quad (2)$$

in the process of electron–proton scattering in the framework of an analytical model (AM), free from uncertainties connected with inelastic hadronic state in intermediate state of the TPE amplitude. In the framework of this model it is possible to show that the effects due to strong-interaction FFs and those due to the inelastic intermediate states almost completely compensate each other, within an accuracy discussed below.

Our paper is organized as follows. In Secs. 1 and 2 we consider the charge-odd contribution of triangle and box-type diagram. In Sec. 3 we describe the procedure of numerical integration. In Sec. 4 we present the results of numerical integration for asymmetries and in Conclusions we estimate the accuracy of the obtained results. The appendices contain the tables of four-fold integrals and some details of calculations.

1. ANALYTICAL MODEL FORMULATION

In the analysis of the TPE amplitude we consider the electromagnetic interactions in the lowest order of perturbation theory. Hadron electromagnetic FFs are functions of one kinematical variable, Q^2 and the static value of the Dirac FF of the proton (for $Q^2 = 0$) is unity due to QED origin. Therefore we parametrize the proton FFs in the form

$$F_1(q^2) = 1 + F_{1s}(q^2), \quad F_2(q^2) = F_{2s}(q^2), \quad F_{1s}(0) = 0, \quad F_{2s}(0) = \mu. \quad (3)$$

Let us discuss now the arguments in favor of a cancellation of the terms of order of F_s^2 with the contribution of the inelastic hadronic intermediate states, in TPE amplitude.

The TPE amplitude contains the virtual photon-Compton scattering tensor. It can be splitted in two terms, when only strong-interaction contributions to the Compton amplitude are taken into account. One term (the elastic term) is the generalization of the Born term with the strong-interaction FFs at the vertices of the interaction of the virtual photons with the hadron. We suppose that the hadron before and after the interaction with the photons remains unchanged. The second term (inelastic) corresponds to inelastic channels formed by pions and nucleons or similar hadronic states which can be excited in the intermediate state, between the vertices of the virtual photon interaction with the hadron.

For this aim let us present the loop-momentum integration element in the form

$$\mathrm{d}^4 k = \frac{1}{2s}\mathrm{d}^2 k_\perp \mathrm{d}s_1 \mathrm{d}s_2, \quad (4)$$

where $s = 2pp_1$ is the total energy, $s_1 = (k - p_1)^2$, and $s_2 = (k + p)^2$ are the invariant mass squares of the upper (electronic) part of the TPE Feynman diagram and the lower (hadronic) ones; $\mathrm{d}^2 k_\perp$ represents the integration on the components of the loop momentum $k_\perp p_1 = k_\perp p = 0$ which are transversal to the initial electron p_1 and proton p momenta.

We can consider the upper and lower block tensors to be both gauge invariant (the factor $1/2$ is introduced to avoid double counting and two Feynman diagrams are included in each block). Therefore, the TPE amplitude can be written in the form (we omit the factors corresponding to fermion spinors):

$$A = \frac{1}{2!}\frac{(4\pi\alpha)^2(2\pi i)^2}{(2\pi)^4}\int\frac{\mathrm{d}^2k_\perp ds_2}{(k)(\bar{k})}L_{\mu\nu}H^{\mu\nu}, \tag{5}$$

where $L_{\mu\nu} = \gamma_\nu(\hat{p}_1 - \hat{k})\gamma_\mu$, and $H_{\mu\nu}$ is the Compton tensor of proton. The integration contour is drawn in Fig. 1, a: it starts from $-\infty - i0$ and follows to $+\infty + i0$, so that it belongs to the physical sheets of s and u channels [5].

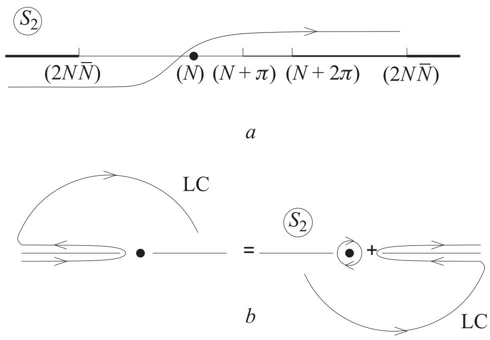

Fig. 1. Integration contour

In the physical sheet, the Compton amplitude has a pole, corresponding to a single-proton state in the intermediate state and two cuts: the right one, corresponding to the inelastic states in s_2 channel, and the left one, starting at $s_2 < -9M^2$ (M is the proton mass). Closing the integration contour to the left and to the right side (see Fig. 1, a) it was shown (see [5]) that the following relation holds:

$$A_{\text{left}} = A_{\text{elastic}} + A_{\text{inelastic}}. \tag{6}$$

Comparing with the sum rule obtained in [5], here the Born amplitude contribution is omitted, as well as only strong-interaction effects are considered here. Omitting the left cut contribution A_{left} (our estimate shows that it can be included in 10% error bar [5]), the effects of strong-interaction contributions to the hadron FFs compensate the strong-interaction contributions arising from the inelastic channels.

As an example, the Feynman amplitude at the origin of the left cut in the s_2 plane of virtual Compton scattering which contributes to the TPE blocks, is drawn in Fig. 2, a, underlying the proton propagators which correspond to real 3-proton u-channel intermediate state.

Its physical meaning is the interference of amplitudes of proton–antiproton pair production in virtual photon–proton collisions due to the Fermi statistics (see Fig. 2, b). A rough estimate of the ratio of contributions of typical right-hand cut $R_{\rm right}$ to the left-hand cut $R_{\rm left}$ is the ratio of cross sections of pion photoproduction to nucleon–antinucleon photoproduction cross section on proton: $R_{\rm left}/R_{\rm right} \sim (2Mm_\pi)/(10M^2) \leqslant 10\%$.

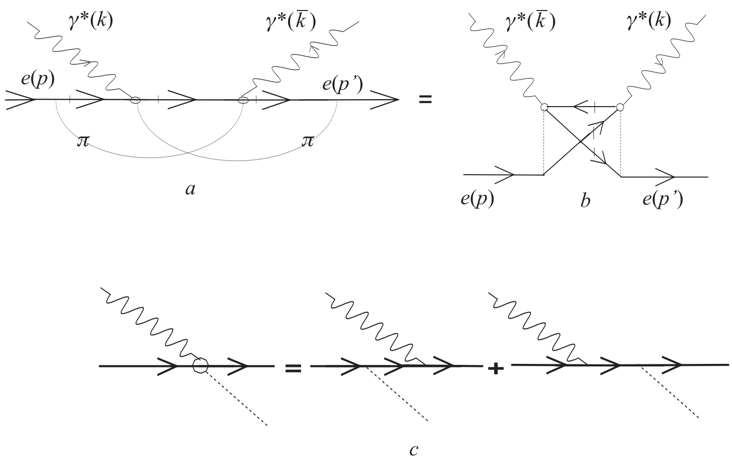

Fig. 2. Compton scattering Feynman diagram illustrating the left cut (a) which is equivalent to the u-channel discontinuity of the Feynman amplitude (b). The gauge invariant set of Compton subdiagrams is shown in (c)

This argument was shown to be exact in the framework of QED [5], where a rather specific kinematics was considered: forward scattering amplitude at high energies. The application to the case of nonforward TPE amplitude requires a more rigorous proof, which is outside of the purpose of this paper. Here we suggest to consider our approach as a model, the validity of which should be experimentally verified.

For example, experiments measuring charge-odd observables in ep scattering will be critical for verification of the validity of our model.

Proton FFs enter in the box amplitude in a form which can be schematically written as

$$\int \frac{d^4k}{i\pi^2(e)(p)} \frac{1+F_s(k^2)}{(k)} \frac{1+F_s(\bar{k}^2)}{(\bar{k})}, \qquad (7)$$

$(k) = k^2-\lambda^2$, $(\bar{k}) = (k-q)^2-\lambda^2$, $(e) = (k-p_1)^2-m_e^2$, $(p) = (k+p)^2-M^2$, where we extract the QED part and do not distinguish the Dirac and Pauli form factors. This expression can be rearranged as

$$\frac{1+F_s(k^2)}{(k)}\,\frac{1+F_s(\bar{k}^2)}{(\bar{k})}=\frac{F_s(k^2)F_s(\bar{k}^2)}{(k)(\bar{k})}+\frac{1}{(\bar{k})}\left[\frac{F_s(k^2)}{(k)}-\frac{F_s(q^2)}{q^2}\right]$$
$$+\frac{1}{(k)}\left[\frac{F_s(\bar{k}^2)}{(\bar{k})}-\frac{F_s(q^2)}{q^2}\right]+\frac{F_s(q^2)}{q^2}\left[\frac{1}{(k)}+\frac{1}{(\bar{k})}\right]+\frac{1}{(k)(\bar{k})} \tag{8}$$
$$=A_{\rm int}+A_{\rm FR}+A_{\rm TR}+A_{\rm Box}. \tag{9}$$

According to our model assumption the first term $A_{\rm int}$ in the right-hand side (r.h.s.) of (9) is compensated by the inelastic intermediate hadron state contribution and so it will be omitted below. The next two terms in (8) do not contain infrared (IR) singularities but contain the ultraviolet (UV) ones. The fourth term in (8) suffers from both IR and UV divergences, the last one suffers only from the IR divergences. Contributions of the last two terms can be calculated analytically as well as they do not contain FFs uncertainties at the loop-momentum integration. The explicit results for them are given below.

Keeping in mind the UV convergence of the initial amplitude, we extract the UV cut-off Λ depending contribution containing $\ln\Lambda^2/M^2$ from the fourth term in (8) and add it to the contribution of the 2nd and 3rd terms providing their UV convergence. This procedure denoted in equation (9). Part of calculations concerning pure QED contribution (the last term in this equation) was performed in our paper [2].

The cross section of elastic ep scattering

$$e(p_1)+p(p)\to e(p_1')+p(p') \tag{10}$$

in the Born approximation in the laboratory frame ($p=(M,0,0,0)$) has the form

$$\frac{{\rm d}\sigma_B}{{\rm d}\Omega}=\frac{\sigma_M\sigma_{\rm red}}{\varepsilon(1+\tau)},\ \sigma_M=\frac{\alpha^2\cos^2\frac{\theta}{2}}{4E^2\sin^4\frac{\theta}{2}}\frac{1}{\rho},\ \rho=1+\frac{2E}{M}\sin^2\frac{\theta}{2},\ \tau=\frac{Q^2}{4M^2},$$
$$t=\frac{s(1-\rho)}{\rho},\ Q^2=-q^2=-t=2p_1p_1',\ s=2ME,\ u=-\frac{s}{\rho}=-2pp_1',\tag{11}$$
$$s+t+u=0,\ \varepsilon^{-1}=1+2(1+\tau)\tan^2\frac{\theta}{2}$$

with

$$\sigma_{\rm red}=\tau G_M^2+\varepsilon G_E^2,\quad G_M=F_1+F_2,\quad G_E=F_1-\tau F_2. \tag{12}$$

Here θ — electron scattering angle and $F_1=1+F_{1s}$, $F_2=F_{2s}$ — the Dirac and Pauli proton's FFs.

Keeping in mind the representation in the form of Eq. (9), it will be convenient to use another (equivalent) form of the Born cross section:

$$\frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega} = \frac{\mathrm{d}\sigma_{Bt}}{\mathrm{d}\Omega} + \frac{\mathrm{d}\sigma_{B\mathrm{box}}}{\mathrm{d}\Omega} = \frac{\alpha^2}{M^2\rho^2 t^2}(B_t + B_{\mathrm{box}}), \tag{13}$$

$$B_t = \frac{1}{2}(F_1^2 - F_1)(2tM^2 + s^2 + u^2) + t^2 F_1 F_2 - \tau F_2^2(tM^2 + su) - \frac{1}{2}F_2 t^2,$$
$$B_{\mathrm{box}} = \frac{1}{2}F_1(2tM^2 + s^2 + u^2) + \frac{1}{2}t^2 F_2. \tag{14}$$

The IR divergence from virtual photon emission contribution is, as usually, canceled when summing with contribution from emission of soft real photons

$$\frac{\mathrm{d}\sigma^{\mathrm{soft}}}{\mathrm{d}\Omega} = \left[\frac{\mathrm{d}\sigma_{Bt}}{\mathrm{d}\Omega} + \frac{\mathrm{d}\sigma_{B\mathrm{box}}}{\mathrm{d}\Omega}\right]\delta^{\mathrm{odd}}_{\mathrm{soft}} = \frac{\mathrm{d}\sigma^{\mathrm{soft}}_{Bt}}{\mathrm{d}\Omega} + \frac{\mathrm{d}\sigma^{\mathrm{soft}}_{B\mathrm{box}}}{\mathrm{d}\Omega}. \tag{15}$$

The quantity $\delta^{\mathrm{odd}}_{\mathrm{soft}}$ was considered in [2, 7]:

$$\delta^{\mathrm{odd}}_{\mathrm{soft}} = -2\,\frac{4\pi\alpha}{16\pi^3}\int \frac{d^3k}{\omega}\left(\frac{p_1'}{p_1'k} - \frac{p_1}{p_1k}\right)\left(\frac{p'}{p'k} - \frac{p}{pk}\right)\Big|_{S_0,\omega\leqslant\Delta E}$$
$$= \frac{2\alpha}{\pi}\left[2\ln\frac{1}{\rho}\ln\frac{2\rho\Delta E}{\lambda} + \ln x\ln\rho + \mathrm{Li}_2\left(1 - \frac{1}{\rho x}\right) - \mathrm{Li}_2\left(1 - \frac{\rho}{x}\right)\right],$$
$$x = \frac{\sqrt{1+\tau} + \sqrt{\tau}}{\sqrt{1+\tau} - \sqrt{\tau}}, \tag{16}$$

with $\lambda, \Delta E$ — soft photon mass and its maximal energy in the laboratory frame.

Note that $\delta^{\mathrm{odd}}_{\mathrm{soft}}$ does not have a definite symmetry under substitution $p \leftrightarrow -p'; s \leftrightarrow u$ due to the specific definition of soft photon in the laboratory frame.

The virtual contribution to the cross section can be splitted in three terms (according to Eq. (9)):

$$\frac{\mathrm{d}\sigma_v}{\mathrm{d}\Omega} = \frac{\mathrm{d}\sigma_{vt}}{\mathrm{d}\Omega} + \frac{\mathrm{d}\sigma_{vb}}{\mathrm{d}\Omega} + \frac{\mathrm{d}\sigma_F}{\mathrm{d}\Omega} = \frac{\alpha^3}{2\pi t^2 M^2\rho^2}(a_t + a_b + a_f). \tag{17}$$

The first term appears from the contribution of triangle-type diagram and can be put in the form

$$a_t = (1 - P(s \leftrightarrow u))\int \frac{\mathrm{d}^4k}{i\pi^2}\frac{1}{(k)}\left[\frac{S_e S_T}{(e)(p)} + \frac{S_{\bar{e}} S_{\bar{T}}}{(\bar{e})(\bar{p})}\right] \tag{18}$$

with

$$(\bar{e}) = (k + p_1')^2 - m_e^2,\ (\bar{p}) = (p' - k)^2 - M^2, \tag{19}$$

where $P(s \leftrightarrow u) f(s,u) = f(u,s)$ is the substitution operation and

$$
\begin{aligned}
S_e &= \frac{1}{4} Tr \hat{p}_1' \gamma_\mu (\hat{p}_1 - \hat{k}) \gamma_\nu \hat{p}_1 \gamma_\eta, \\
S_{\bar{e}} &= \frac{1}{4} Tr \hat{p}_1' \gamma_\mu (\hat{p}_1' + \hat{k}) \gamma_\nu \hat{p}_1 \gamma_\eta, \\
S_T &= \frac{1}{4} Tr R ((\Gamma_\mu(q) - \gamma_\mu)(\hat{p} + \hat{k} + M) \gamma_\nu, \\
S_{\bar{T}} &= \frac{1}{4} Tr R \gamma_\mu (\hat{p}' - \hat{k} + M)(\Gamma_\nu(q) - \gamma_\nu), \\
\Gamma_\mu(q) &= \gamma_\mu F_1(q^2) + \frac{1}{4M} F_2(q^2)(\hat{q}\gamma_\mu - \gamma_\mu \hat{q}); \ R = (\hat{p} + M)\Gamma_\eta(-q)(\hat{p}' + M).
\end{aligned} \tag{20}
$$

The box-type contribution is parametrized as

$$
\begin{aligned}
a_b &= (1 - P(s \leftrightarrow u)) \int \frac{\mathrm{d}^4 k}{i\pi^2} \frac{t S_e Z_p}{(k)(\bar{k})(p)(e)}, \\
Z_p &= \frac{1}{4} Tr R \gamma_\mu (\hat{p} + \hat{k} + M) \gamma_\nu.
\end{aligned} \tag{21}
$$

Some of the necessary integrals have been previously calculated in [6]. However, for completeness, they are all given in Appendix A.

The finite part contribution (2nd and 3rd terms in r.h.s. of Eq. (9)) are parametrized as

$$
a_f = (1 - P(s \leftrightarrow u)) \int \frac{\mathrm{d}^4 k}{i\pi^2} \frac{t}{(\bar{k})} \left[\frac{S_e S_F}{(e)(p)} + \frac{S_{\bar{e}} S_{\bar{F}}}{(\bar{e})(\bar{p})} \right] = A_f(s,u) - A_f(u,s) \tag{22}
$$

and

$$
\begin{aligned}
S_F &= \frac{1}{4} Tr R \gamma_\mu (\hat{p} + \hat{k} + M) \Phi_\nu, \\
S_{\bar{F}} &= \frac{1}{4} Tr R \Phi_\mu (\hat{p}' - \hat{k} + M) \gamma_\nu, \\
\Phi_\mu &= \left[\frac{1}{(k)} (F_1(k^2) - 1) - \frac{1}{q^2} (F_1(q^2) - 1) \right] \gamma_\mu - \\
&\quad - \frac{1}{4M} \left[\frac{[\hat{q}\gamma_\mu]}{q^2} F_2(q^2) - - \frac{[\hat{k}, \gamma_\mu]}{(k)} F_2(k^2) \right].
\end{aligned} \tag{23}
$$

In the integration over the four-vector k in a_f and a_t enters the UV cut-off parameter $|k|^2 < \Lambda^2$. To obtain A_{FR} and A_{TR} in Eq. (9) the terms containing $\ln(\Lambda^2/M^2)$ are singled out from the contribution of the fourth term of r.h.s. (8) and added to the contribution of the 2nd and 3rd terms in (8). So we can put in (17) $a_t + a_b + a_f = a_{tr} + a_b + a_{fr}$, explicitly eliminating the cut-off parameter dependence.

2. VIRTUAL AND SOFT-PHOTON EMISSION CONTRIBUTIONS OF TRIANGLE-TYPE DIAGRAM

The interference of the Born amplitude with the part of TPE arising from the fourth term from the r.h.s. of Eq. (9) with the corresponding part of soft photon emission leads to

$$\begin{aligned}\frac{\mathrm{d}\sigma_T}{\mathrm{d}\Omega} &= \frac{\mathrm{d}\sigma_{vt}}{\mathrm{d}\Omega} + \frac{\mathrm{d}\sigma_{Bt}^{\mathrm{soft}}}{\mathrm{d}\Omega} \\ &= \frac{2\alpha^3}{M^2\rho^2 t^2 \pi}\left\{2B_t\left[\ln\frac{1}{\rho}\ln\frac{2\Delta E}{M} - \ln^2\rho + \frac{1}{2}\ln\rho\ln x + \frac{1}{2}\mathrm{Li}_2\left(1-\frac{1}{\rho x}\right)\right.\right. \\ &\left.\left.-\frac{1}{2}\mathrm{Li}_2\left(1-\frac{\rho}{x}\right)\right] + \frac{1}{2}D_{tsv}\right\} + \frac{3\alpha^3(u-s)}{2M^2\rho^2 t\pi}(F_2+F_1)(1-F_1)\ln\frac{\Lambda^2}{M^2} \qquad (24)\end{aligned}$$

with

$$\begin{aligned}D_{tsv} = B_t&\left[\ln^2\frac{s}{M^2} - \ln^2\frac{-u}{M^2} - 2\mathrm{Li}_2\left(1+\frac{M^2}{s}\right) + 2\mathrm{Li}_2\left(1+\frac{M^2}{u}\right) - \pi^2\right] \\ &+ [1-P(s\leftrightarrow u)]\left(A(s,t)\ln\frac{s}{M^2} + B(s,t)\right), \qquad (25)\end{aligned}$$

and

$$\begin{aligned}A(s,t) &= a_1F_1(1-F_1) + a_2F_2 + a_3F_2^2 + a_4F_1F_2, \\ B(s,t) &= b_1F_1(1-F_1) + b_2F_2 + b_3F_2^2 + b_4F_1F_2\end{aligned}$$

with

$$\begin{aligned}a_1 &= \frac{M^4(M^2+t)}{2(M^2+s)^2}(3M^2+4s) + \frac{1}{4}[-6M^4+4sM^2+2tM^2+6u^2], \\ a_2 &= \frac{tM^4(7M^2+9s)}{4(M^2+s)^2} - \frac{t}{4}(5u+7M^2), \\ a_3 &= -\frac{M^2t}{16(M^2+s)^2}(2M^4+5st+3sM^2+5tM^2)+ \\ &+ \frac{t}{16M^2}(2M^4+M^2(u-2t)-8su), \\ a_4 &= -\frac{M^2t}{8(M^2+s)^2}(8M^4+2st+23sM^2+2tM^2) - \frac{t}{8}(-18M^2+18t+13s), \\ b_1 &= -\frac{s}{4}(7t+2M^2) - \frac{M^4(M^2+t)}{2(M^2+s)}, \quad b_2 = -\frac{7}{4}st - \frac{tM^4}{2(M^2+s)}, \\ b_3 &= \frac{st}{16} + \frac{tM^4}{16(M^2+s)}, \quad b_4 = \frac{15}{8}st + \frac{5tM^4}{8(M^2+s)}. \qquad (26)\end{aligned}$$

Note that function D_{tsv} in the r.h.s. of Eq. (24) changes the sign at substitution $s \leftrightarrow u$, in particular,

$$\rho = \frac{s}{-u} \to \frac{1}{\rho}, \quad Re\left[\ln^2 \frac{-s-i0}{M^2}\right] = \ln^2 \frac{s}{M^2} - \pi^2 \to \ln^2 \frac{-u}{M^2}, \qquad \ln \frac{s}{M^2} \to \ln \frac{-u}{M^2}. \tag{27}$$

The last term in (24), which contains the cut-off parameter is necessary in order to remove the Λ-dependence of a_f.

3. VIRTUAL AND SOFT-PHOTON EMISSION CONTRIBUTIONS OF QED BOX-TYPE DIAGRAM

The box-type contribution (last term in (9)) with corresponding part of soft-photon emission is given by (the list of necessary loop-momentum integrals and details of the calculation are given in Appendix A):

$$\begin{aligned}\frac{d\sigma_B}{d\Omega} &= \frac{d\sigma_{vb}}{d\Omega} + \frac{d\sigma^{\rm soft}_{B\rm box}}{d\Omega_e} = \\ &= \frac{2\alpha^3}{\pi M^2\rho^2 t^2}\left\{B_{\rm box}\left[\ln\frac{1}{\rho}\ln\frac{(2\Delta E)^2}{4\tau M^2} - 2\ln^2\rho + \ln\rho\ln x + \right.\right. \\ &\left.\left. + \ {\rm Li}_2\left(1-\frac{1}{\rho x}\right) - {\rm Li}_2\left(1-\frac{\rho}{x}\right)\right] - \frac{t^2}{2}[1-P(s\leftrightarrow u)](d_1F_1 - d_2F_2)\right\}\end{aligned} \tag{28}$$

with

$$\begin{aligned} d_1 &= \frac{s}{t}\left[\frac{1}{2}\ln^2(4\tau) - \frac{\tau}{1+\tau}\ln(4\tau) + M^2F_Q\left(6\tau + 2 - \frac{2\tau^2}{1+\tau}\right)\right. \\ &\left. - \ \ln^2\frac{s}{-t} + \pi^2 + 2{\rm Li}_2\left(1+\frac{M^2}{s}\right)\right] - \\ &- \frac{(1-2\tau)}{4\tau}\left[2\ln\rho\ln(4\tau) - \ln^2\frac{s}{M^2} + \pi^2 + 2{\rm Li}_2\left(1+\frac{M^2}{s}\right)\right] + \\ &+ \left(2M^2 - \frac{su}{t}\right)\frac{\ln\frac{s}{M^2}}{s+M^2}, \\ d_2 &= \frac{s}{2(1+\tau)}\left[F_Q - \frac{1}{2M^2}\ln(4\tau)\right] - \frac{M^2}{M^2+s}\ln\frac{s}{M^2} - \ln\rho\ln(4\tau) + \\ &+ \frac{1}{2}\left(\ln^2\frac{s}{M^2} - \pi^2\right) - {\rm Li}_2\left(1+\frac{M^2}{s}\right),\end{aligned} \tag{29}$$

and F_Q is given in (47).

4. INFRARED SINGULARITIES FREE CONTRIBUTIONS OF BOX AMPLITUDES

In this section we use the following ansatz for nucleon FFs:

$$F_1(q^2) = F_2(q^2)/\mu = \left(\frac{Q_0^2}{q^2 - Q_0^2}\right)^2, \tag{30}$$

setting the parameter Q_0^2 to 0.71 GeV^2. This form permits us to carry on analytical calculations. Note that this presctiption differs from those ones given above, Eq. (1). Let us rewrite the expressions of Eq. (23) for Φ_μ as follows:

$$\Phi_\mu = \Phi_1\gamma_\mu - \frac{[\hat{q}\gamma_\mu]}{4M}F_2(t)\Phi_2 - \mu\frac{[\hat{q}-\hat{k},\gamma_\mu]}{4M}\Phi_3 \tag{31}$$

with (here we use dipole approximation for FFs, Eq. 30)

$$\begin{aligned}
\Phi_1 &= \frac{(q^2-k^2)}{(Q_k)^2}[A(Q_k)+B],\ A = \frac{2Q_0^2-q^2}{(Q_q)^2} = \frac{1-F_1(t)}{t},\ B = \frac{Q_0^2}{(Q_q)},\\
\Phi_2 &= \frac{(k^2-q^2)}{(k)t(Q_k)^2}\left[(Q_k)^2 + q^2(Q_k) + q^2(Q_q)\right],\\
\Phi_3 &= \frac{(Q_0^2)^2}{(k)(Q_k)^2},\ (Q_q) = q^2 - Q_0^2,\ (Q_k) = k^2 - Q_0^2.
\end{aligned} \tag{32}$$

When integrating on the loop-momenta, the Φ_1 and Φ_2 terms give origin to UV divergences of contributions containing $L_\Lambda = \ln\frac{\Lambda^2}{M^2}$ in the form:

$$\begin{aligned}
A_f^\Lambda(s,u) \simeq \int dxydy\Bigg[&\frac{1-F_1(t)}{t}\left(\ln\frac{\Lambda^2}{d_0} - \frac{3}{2}\right)G(s,t)+\\
&+\frac{F_2(t)}{4Mt}\left(\ln\frac{\Lambda^2}{\mathcal{D}_0} - \frac{3}{2}\right)F(s,t)\Bigg],
\end{aligned} \tag{33}$$

with D_0, d_0 given in Appendix B, and

$$\begin{aligned}
G(s,t) &= -\frac{1}{4}Tr\hat{p}_1'\gamma_\mu\gamma_\sigma\gamma_\nu\hat{p}_1\gamma_\eta\cdot\frac{1}{4}TrR\gamma_\mu\gamma_\sigma\gamma_\nu =\\
&= -F_2(q^2)(2t^2-6st) - F_1(q^2)(2u^2+8s^2+10tM^2),\\
F(s,t) &= -\frac{1}{4}Tr\hat{p}_1'\gamma_\mu\gamma_\sigma\gamma_\nu\hat{p}_1\gamma_\eta\cdot\frac{1}{4}TrR\gamma_\mu\gamma_\sigma[\hat{q},\gamma_\nu] =\\
&= -F_2(q^2)\left(8Mt^2 + \frac{8stu}{M}\right) - 16t^2MF_1(q^2).
\end{aligned} \tag{34}$$

The Φ_2 and Φ_3 terms contain IR divergences. However, in both regions where IR divergences are present, i.e., $k \to 0$, $k \to q$, the sum of the contributions $\Phi_2 + \Phi_3$ converges. As for UV contributions, we note that the quantity

$$a_{fr} \simeq A_f^\Lambda(s,u) - A_f^\Lambda(u,s) - 3(u-s)(F_2+F_1)(F_1-1)L_\Lambda,\ L_\Lambda = \ln\frac{\Lambda^2}{M^2} \quad (35)$$

is finite at the limit $\Lambda \to \infty$. It results in the replacement $\Lambda^2 = M^2$, $L_\Lambda = 0$. The explicit expression for a_{fr} in terms of three-fold integrals is given in Appendix B. We note that UV divergences associated with Φ_2 are canceled due to the symmetry $F(s,t) = F(u,t)$.

The relevant contribution to the differential cross section can be written as

$$\frac{d\sigma_F}{d\Omega} = \frac{d\sigma_B}{d\Omega}\frac{\alpha}{\pi}D_f,\ D_f = \frac{a_{fr}}{2(B_t + B_{\rm box})}. \quad (36)$$

5. RESULTS AND DISCUSSION

The differential cross section with two-photon exchange and the relevant soft-photon emission is given by

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{\rm Born}}{d\Omega} + \frac{d\sigma_T}{d\Omega} + \frac{d\sigma_B}{d\Omega} + \frac{d\sigma_F}{d\Omega}. \quad (37)$$

Our result for charge asymmetry (2) has the form

$$A^{\rm odd} = \frac{\alpha}{\pi}\left[2\ln\frac{1}{\rho}\ln\frac{2\Delta E}{M} + \mathcal{D}(\frac{E}{M},\theta)\right],\ \mathcal{D}(\frac{E}{M},\theta) = D_{tbs} + D_f, \quad (38)$$

where D_{tbs} denotes the analytical part

$$D_{tbs} = \ln x\ln\rho - 2\ln^2\rho + \mathrm{Li}_2\left(1-\frac{1}{\rho x}\right) - \mathrm{Li}_2\left(1-\frac{\rho}{x}\right) + \frac{1}{B_t+B_{\rm box}}\times$$
$$\times\left\{B_t\left[\ln^2\frac{s}{M^2} - \ln^2\frac{-u}{M^2} - \pi^2 - 2\mathrm{Li}_2\left(1+\frac{M^2}{s}\right) + 2\mathrm{Li}_2\left(1+\frac{M^2}{u}\right)\right] + \right.$$
$$+ B_{\rm box}\ln\rho\ln(4\tau) + [1-P(s\leftrightarrow u)]\times$$
$$\left.\times\left[A(s,t)\ln\frac{s}{M^2} + B(s,t) - t^2(d_1F_1 - d_2F_2)\right]\right\}. \quad (39)$$

We note that charge asymmetry $A^{\rm odd}$ is finite in zero electron mass limit, but contains the soft-photon emission parameter $\Delta E/M$.

The function D_{tbs}, which can be calculated analytically and the function D_f, Eq. (39), for which a numerical integration has been performed, have been calculated for several values of angles and energies. Both contributions are larger (in absolute value) at large E/M.

Ansatz (3) which reflects the possibility to separate the QED and the strong-interaction contributions to FFs and that assumes $F_{\rm QED} = 1$ is useful as it allows one to perform, at least partly, analytical calculations. However, it cannot be considered exactly. A better parametrization of QED and strong interaction contributions to FFs is

$$F_1(Q^2) = F_{1Q}(x) + F_{1s}(x),\ F_{1Q}(x) = \frac{1+x^2}{(1+x)^4},$$
$$F_{1s}(x) = \frac{2x}{(1+x)^4},\quad x = \frac{-q^2}{Q_0^2} = \frac{Q^2(\text{ GeV})}{0.71}. \tag{40}$$

Therefore, in order to find the global correction due to two-photon exchange, one should weight the individual contributions D_{tbs} and D_f, by the ratio of strong and EM FFs. This can be done by multiplying D_{tbs} by the factor $F_{1Q}(x)$. The total contribution is therefore

$$D \to \tilde{\mathcal{D}} = F_{1Q}(x)(D_{tbs} + D_f).$$

Tabulated values of the numerical results are presnted in the Table. One can see that the two-photon contribution to the asymmetry is of the order of percent, keeping in mind the multiplicative factor α/π. The behavior is smooth in the considered kinematical ranges. Singularities are expected for $\theta = 0$ $\epsilon = 1$, due to symmetry properties of the 2γ exchange [1].

Numerical values of $\tilde{\mathcal{D}}$ as a function of E/M and θ, with dipole parametrization of the form factors

E/M–θ	30	60	90	120	150
1	1.16	0.58	–0.72	–1.29	–1.81
2	3.71	1.19	–0.22	–0.96	–1.61
3	4.23	1.54	–0.15	–0.97	–1.73
4	4.46	1.86	0.32	–0.62	–1.35
5	4.52	2.43	1.12	–0.13	–0.99

Taking into account the factor α/π, the corrections do not exceed 1%, in the considered kinematical range.

CONCLUSIONS

Charge asymmetry in electron–proton elastic scattering contains essential information on the contribution of 2γ exchange to the reaction amplitude. This amplitude can shed light on Compton scattering of virtual photons on proton. It contains a part corresponding to proton intermediate state, which carries the information on proton FFs. Another term corresponds to excited nucleon states and inelastic states such as $N\pi$, $N2\pi$, $N\bar{N}N$. Their theoretical investigation is strongly model-dependent.

Our main assumption about the compensation of pure strong-interaction induced contributions to FFs and inelastic channels allows us to avoid additional uncertainty connected with inelastic channels.

Our choice of photon form factors (see Ref. (30)) is nonphysical one. The physical case corresponds to

$$G_E = \frac{1}{\mu} G_M = \frac{Q_0^4}{(Q_0^2 - q^2)^2}. \tag{41}$$

The aim of the choice is the simplification of the analytical calculation.

The assumption about the possibility to omit $A_{\rm int}$ in (9) was proved to hold in the framework of QED, and for ep elastic scattering for the kinematics of almost forward scattering. The application to large angle scattering requires, in principle, a rigorous proof.

The parameterization of the $NN^*\gamma^*$ vertex is also approximated, since one of the nucleons is off mass shell. Nevertheless, they can be estimated to be small for $Q^2 < 5$ GeV^2 and included in the sources of theoretical uncertainties. The reliability of our assumption can be estimated from the ratio of cross sections of pion and nucleon–antinucleon pair photoproduction. The uncertainty of our results does not exceed 10%.

Another interesting question is if such a compensation is present for the annihilation channel. The measurement of charge asymmetry is, in this case, associated with polar angle odd contribution to the differential cross section.

Similar effects of charge and angular asymmetries can also be due to Z-boson exchange, but such a contribution is small for moderate-high energy colliders. The ratio of corresponding contributions can be evaluated as: $\sim (\pi g_V g_A s)/(\alpha M_Z^2) < 5 \cdot 10^{-3}$ for $s < 10$ GeV^2 (g_V (g_A) is the vector(axial)-coupling constant of the Z boson to fermion).

The analytical calculation of 2γ amplitude with FFs encounters mathematical difficulties. In Ref. [8] the results of the box amplitude with arbitrary FFs were investigated. Similar attempt was done by A. Ilichev [9]. These works use different approaches to include FFs, however the numerical results are close to ours, and show that the two-photon contribution cannot be responsible for the discrepancy in the recent FFs measurements.

The other works [10] devote much attention to the excited intermediate states as Δ and N^* resonances, introducing additional uncertainties. In our approach, excited states should not be included, as they correspond to poles in the second physical sheet.

Our numerical results show that charge-odd correlations are of the order of percent, in the kinematical region considered here. Such a value is expected to be larger at larger q^2 values and could be measured in very precise experiments, at present facilities.

Acknowledgments. One of us (E. A. K.) acknowledges the kind hospitality of Saclay, where part of this work was done, and especially to Prof. Egle Tomasi-Gustafsson for critical remarks and help. This work was partly supported by grant MK-2952.2006.2. The authors are grateful to C. C. Adamuščín for technical help.

APPENDIX A
LIST OF NECESSARY INTEGRALS

We give here a list of scalar, vector and tensor-type loop-momentum integrals with three denominators $(k),(e),(p)$

$$\int \frac{\mathrm{d}^4 k}{i\pi^2} \frac{1; k_\mu; k_\mu k_\nu}{(k)(e)(p)} = Z_1;\ Z_2 p_{1\mu} + Z_3 p_\mu,$$

$$Z_4 g_{\mu\nu} + Z_5 p_{1\mu} p_{1\nu} + Z_6 p_\mu p_\nu + Z_7 (p_{1\mu} p_\nu + p_{1\nu} p_\mu). \tag{42}$$

Standard procedure of joining the denominators leads to integrals of the form

$$\int\limits_0^1 \mathrm{d}x \int\limits_0^1 2y\mathrm{d}y \int \frac{1; k_\mu; k_\mu k_\nu}{[(k - y p_x)^2 - y^2 p_x^2 - \lambda^2 (1-y)]^3}, \tag{43}$$

with $p_x = x p_1 - (1-x) p, p_x^2 = m^2 x^2 + M^2 (1-x)^2 + s_1 x (1-x)$, $s_1 = -s - i0$. Further integration on loop momentum (see (37)) with Λ is the UV cut-off parameter, leads to

$$\begin{aligned}
Z_1 &= \frac{1}{2s}\left[L_s^2 - \frac{1}{2}L^2 - 2Li_s + \ln\frac{M^2}{\lambda^2}(2L_s + L)\right], \\
L &= \ln\frac{M^2}{m^2}, \quad L_s = \ln\frac{s}{M^2} - i\pi, \\
Li_s &= Li_2(1 + \frac{M^2}{s}) - i\pi\ln(1 + \frac{M^2}{s}), \\
Z_2 &= \frac{1}{s}\left[L + (1 + \frac{M^2}{s + M^2})L_s\right],
\end{aligned}$$

$$
\begin{aligned}
Z_3 &= -\frac{1}{s+M^2}L_s, \\
Z_4 &= \frac{1}{4}L_\Lambda + \frac{3}{8} - \frac{s}{4(s+M^2)}L_s, \\
Z_5 &= \frac{1}{2s}\left[-1 - \frac{M^2}{s+M^2} + L_s(1 - \frac{M^4}{(s+M^2)^2}) + L\right], \\
Z_6 &= \frac{1}{2}\frac{1}{s+M^2}\left[\frac{s}{s+M^2}L_s - 1\right], \\
Z_7 &= -\frac{1}{2}\frac{1}{s+M^2}\left[1 + \frac{M^2}{s+M^2}L_s\right].
\end{aligned}
$$

Integrals with denominators $(k)(\bar{e})(\bar{p})$ can be obtained from those given above by replacements $p_1 \to -p_1'$, $p \to -p'$ with the same coefficients. Integrals with denominators $(k)(\bar{e})(p)$ can be obtained from those given above by replacements $p_1 \to -p_1'$, $p \to p$, and coefficients, which can be obtained from those mentioned above by replacement $s \to u$.

Box-type integrals defined as

$$
Y_1; I_\mu; I_{\mu\nu} = \int \frac{d^4k}{i\pi^2}\frac{1; k_\mu; k_\mu k_\nu}{(0)(q)(e)(p)}; \quad I_\mu = Y_2\Delta_\mu + Y_3 P_\mu,
$$

$$
I_{\mu\nu} = Y_4 g_{\mu\nu} + Y_5\Delta_\mu\Delta_\nu + Y_6 P_\mu P_\nu + Y_7(P_\mu\Delta_\nu + P_\nu\Delta_\mu) + Y_8 Q_\mu Q_\nu \tag{44}
$$

with

$$
Q = \frac{p_1 - p_1'}{2}, \quad P = \frac{p_1 + p_1'}{2}, \quad \Delta = \frac{p + p'}{2}. \tag{45}
$$

Explicit expressions for Y_k are

$$
\begin{aligned}
Y_1 &= \frac{2}{st}\ln\frac{-s-i0}{Mm}\ln\frac{-t}{\lambda^2}, \\
Y_2 &= -\frac{1}{2d}\left[-\frac{\tau}{2}(F + F_Q) - P^2(F + F_\Delta)\right], \\
Y_3 &= \frac{1}{2d}\left[-\frac{\tau}{2}(F + F_\Delta) - \Delta^2(F + F_Q)\right], \\
Y_4 &= \frac{1}{\tau}\Big[-\frac{\tau}{2}(F - G + H_p + H_\Delta + H_Q) + H_\Delta\left(-\frac{\tau}{2} - P^2\right) - \\
&\quad - H_Q\left(-\frac{\tau}{2} - \Delta^2\right) + 2Q^2 Y_3(P^2 + \tau) + P^2 G_\Delta - \Delta^2 G_Q + 2Q^2\Delta^2 Y_2\Big],
\end{aligned}
$$

$$
\begin{aligned}
Y_5 &= \frac{1}{\tau d}\Bigg[P^2\frac{\tau}{2}H + 2(P^2)^2(H_\Delta - 2Q^2Y_3 - G_\Delta) + \\
&+ \left(\frac{\tau^2}{4} - 2P^2\Delta^2\right)(H_Q + 2Q^2Y_2 - G_Q)\Bigg], \\
Y_6 &= \frac{1}{\tau d}\Bigg[-\frac{\tau}{2}\Delta^2(G - F - H_p - 3H_\Delta + 6Q^2Y_3) + \\
&+ \left(P^2\Delta^2 + \frac{\tau^2}{4}\right)(H_\Delta - 2Q^2Y_3 - G_\Delta) - \Delta^2(H_Q + 2Q^2Y_2 - G_Q)\Bigg], \\
Y_7 &= -\frac{1}{\tau d}\left[\frac{\tau^2}{4}H - \Delta^2\frac{\tau}{2}(H_Q + 2Q^2Y_2 - G_Q) + 2P^2\frac{\tau}{2}(H_\Delta - 2Q^2Y_3 - G_\Delta)\right], \\
Y_8 &= -\frac{1}{Q^2\tau}\Bigg[-\tau\left(H_\Delta - 2Q^2Y_3 + H_P + \frac{1}{2}F - \frac{1}{2}G\right) +) \\
&+ \Delta^2(H_Q - 2Q^2Y_3 - G_Q) - P^2(H_\Delta - 2Q^2Y_3 - G_\Delta)\Bigg], \\
\tau &= 2P\Delta, \quad d = P^2\Delta^2 - \frac{\tau^2}{4}, \quad H = F - G + H_P + 3H_\Delta + 6P^2Y_3. \qquad (46)
\end{aligned}
$$

The quantities entering here are

$$
\begin{aligned}
M^2F_Q &= -\frac{1}{4\sqrt{\tau(1+\tau)}}\left[\pi^2 + \ln(4\tau)\ln x + \mathrm{Li}_2\left(-2\sqrt{\tau x}\right) - \mathrm{Li}_2\left(\frac{2\sqrt{\tau}}{\sqrt{x}}\right)\right], \\
F_\Delta &= -\frac{1}{t}\left[\frac{1}{2}\ln^2\frac{-t}{m^2} + \frac{\pi^2}{6}\right], \quad G_Q = -\frac{1}{4M^2(1+\tau)}\left[-tF_Q - 2\ln\frac{-t}{M^2}\right], \\
F &= \frac{1}{2s}\Bigg[2\ln\frac{-s-i0}{Mm}\ln\frac{-t}{M^2} - \ln^2\left(\frac{-s-i0}{M^2}\right) + \\
&+ 2\ln^2\frac{M}{m} + 2\mathrm{Li}_2\left(1 + \frac{M^2}{s}\right)\Bigg], \\
H_Q &= \frac{1}{s+M^2}\ln\frac{-s-i0}{M^2}. \qquad (47)
\end{aligned}
$$

The explicit expression for the QED box-type Born amplitude (see Eq. (21)) is $a_b(s,u) = A_b(s,u) - A_b(u,s)$ with

$$
\begin{aligned}
A_b(s,u) &= (F_1P_q - F_2Q_q)F_Q + (F_1P_\Delta - F_2Q_\Delta)F_\Delta + (F_1P_G - F_2Q_G)G_Q + \\
&+ (F_1P_H - F_2Q_H)H_Q + (F_1P_F - F_2Q_F)F + (F_1P_Y - F_2Q_Y)Y_1,
\end{aligned}
$$

with

$$
\begin{aligned}
P_q &= \frac{1}{8}\left[M^2(s-u)+s(3s-u)\right]; \; Q_q=\frac{st}{8}; \; P_\Delta=\frac{M^2t}{4}+\frac{3s^2}{8}; \\
Q_\Delta &= -\frac{t^2}{8}; \; P_G=-\frac{M^2t}{4}-\frac{tu}{8}; \; Q_G=\frac{M^2t}{4}-\frac{st}{8}; \\
P_H &= \frac{M^2t}{2}-\frac{su}{4}; \; Q_H=-\frac{tM^2}{4}; \; P_F=\frac{sM^2}{2}+\frac{s(s-u)}{4}; \\
Q_F &= -\frac{st}{4}; \; P_Y=-\frac{M^2ts}{2}-\frac{s(s^2+u^2)}{4}; \; Q_Y=\frac{st^2}{4}.
\end{aligned} \tag{48}
$$

APPENDIX B

The contributions from the uncrossed box-type Feynman amplitude, A_f, can be written as the sum of terms associated to Φ_1, Φ_2, Φ_3

$$
A_f = A_{f1} - \frac{1}{4M}\left(F_2(t)A_{f2}+(Q_0^2)^2 t\mu A_{f3}\right), \; a_{fr}=(1-P(s\leftrightarrow u))A_f.
$$

We follow the procedure described in Appendix C.

The Φ_2 and Φ_1 terms are calculated by using the relation

$$
\frac{k^2-q^2}{(k-q)^2} = 1-2q_\alpha\frac{(q-k)_\alpha}{(\bar{k})}, \tag{49}
$$

in order to eliminate explicitly the divergence at $k\to q$. For A_{f1} we have

$$
\begin{aligned}
A_{f1} = t\int_0^1 dxydy\Bigg\{ & G(s,t)\left[A\left(\ln\frac{M^2}{d_0}-\frac{3}{2}\right)+\frac{B\bar{y}}{d_0}\right]- \\
& -2N(yp_x)\left(\frac{A}{d_0}+\frac{B\bar{y}}{d_0^2}\right)\Bigg\} + t\int_0^1 dxydyz^2dz\Bigg\{\mathcal{S}\left(\frac{A}{D_1}+\frac{B\bar{z}}{D_1^2}\right)- \\
& -2N(zP_0)\left(\frac{A}{D_1^2}+\frac{2B\bar{z}}{D_1^3}\right)t(2\bar{z}+yz)\Bigg\}
\end{aligned} \tag{50}
$$

with

$$
\mathcal{S} = tG(s,t)(2\bar{z}+yz)+2(z\bar{y}-\bar{z})[2t(s^2+tM^2)F_1-2st^2F_2],
$$

and

$$
N(b) = \frac{1}{4}Tr\hat{p}_1'\gamma_\mu(\hat{p}_1-\hat{b})\gamma_\nu\hat{p}_1\gamma_\eta\cdot\frac{1}{4}TrR\gamma_\mu(\hat{p}+\hat{b}+M)\gamma_\nu,
$$

and A, B given above (Eq. (32)). The expression for A_{f2} is

$$A_{f2} = \int dxdyyz^2dz \left\{ F(s,t) \left[\left(\ln \frac{\Lambda^2}{D_0} - \frac{3}{2} \right) - q^2\bar{y}L_1 + q^2(Q_q) \cdot \bar{y}^2 L_2 \right] + \right.$$

$$\left. + \left[-\frac{1}{D_0} + q^2\bar{y}L_2 - 2q^2(Q_q)\bar{y}^2 L_3 \right] [M(yp_x) + \bar{M}(yp_x')] \right\} -$$

$$-2 \int dxdyyz^2dz \left\{ \left[-\frac{1}{2a} + \frac{q^2\bar{z}}{2}\mathcal{J}_2 - q^2(Q_q)\bar{z}^2\mathcal{J}_3 \right] q^\alpha \left[F_\alpha(zP_0) + \bar{F}_\alpha(zP_0') \right] + \right.$$

$$\left. + \frac{t[2\bar{z} + zy]}{2} \left[\frac{1}{a^2} - 2\bar{z}q^2\mathcal{J}_3 + 6q^2(Q_q)\bar{z}^2\mathcal{J}_4 \right] [M(zP_0) + \bar{M}(zP_0')] \right\}, \quad (51)$$

with $\mathcal{J}_i, L_i$ given in Appendix C and

$$M(b) = \frac{1}{4} Tr\hat{p}_1' \gamma_\mu (\hat{p}_1 - \hat{b}) \gamma_\nu \hat{p}_1 \gamma_\eta \cdot \frac{1}{4} TrR\gamma_\mu (\hat{p} + \hat{b} + M)[\hat{q}, \gamma_\nu],$$

$$\bar{M}(b) = \frac{1}{4} Tr\hat{p}_1' \gamma_\mu (\hat{p}_1' + \hat{b}) \gamma_\nu \hat{p}_1 \gamma_\eta \cdot \frac{1}{4} TrR[\hat{q}, \gamma_\mu](\hat{p}' - \hat{b} + M)\gamma_\nu.$$

$$F_\alpha(b) = \frac{1}{4} Tr\hat{p}_1' \gamma_\mu \gamma_\alpha \gamma_\nu \hat{p}_1 \gamma_\eta \cdot \frac{1}{4} TrR\gamma_\mu (\hat{p} + \hat{b} + M)[\hat{q}, \gamma_\nu] -$$

$$- \frac{1}{4} Tr\hat{p}_1' \gamma_\mu (\hat{p}_1 - \hat{b}) \gamma_\nu \hat{p}_1 \gamma_\eta \cdot \frac{1}{4} TrR\gamma_\mu \gamma_\alpha [\hat{q}, \gamma_\nu] + (q - b)^\alpha F(s,t), \quad (52)$$

and similar expression for $\bar{F}_\alpha$.

The form of A_{f3} is

$$A_{f3} = \int_0^1 dxydy(z\bar{z})^2 dz \times$$

$$\times \left\{ -[H(zP_0) + \bar{H}(zP_0')]\mathcal{J}_3 + 6[G(zP_0) + \bar{G}(zP_0')]\mathcal{J}_4 \right\} \quad (53)$$

with

$$G(k) = \frac{1}{4} Tr\hat{p}_1' \gamma_\mu (\hat{p}_1 - \hat{k}) \gamma_\nu \hat{p}_1 \gamma_\eta \cdot \frac{1}{4} TrR\gamma_\mu (\hat{p} + \hat{k} + M)[\hat{q} - \hat{k}, \gamma_\nu],$$

$$\bar{G}(k) = \frac{1}{4} Tr\hat{p}_1' \gamma_\mu (\hat{p}_1' + \hat{k}) \gamma_\nu \hat{p}_1 \gamma_\eta \cdot \frac{1}{4} TrR[\hat{q} - \hat{k}, \gamma_\mu](\hat{p}' - \hat{k} + M)\gamma_\nu, \quad (54)$$

$$
\begin{aligned}
H(b) = {} & \frac{1}{4} Tr \hat{p}_1' \gamma_\mu \gamma_\sigma \gamma_\nu \hat{p}_1 \gamma_\eta \times \\
& \times \left\{ -\frac{1}{4} TrR \gamma_\mu \gamma_\sigma [\hat{q} - \hat{b}, \gamma_\nu] + \frac{1}{4} TrR \gamma_\mu (\hat{p} + \hat{b} + M) [\gamma_\sigma, \gamma_\nu] \right\} \\
& - \frac{1}{4} Tr \hat{p}_1' \gamma_\mu (\hat{p_1} - \hat{b}) \gamma_\nu \hat{p}_1 \gamma_\eta \cdot \frac{1}{4} TrR \gamma_\mu \gamma_\sigma [\gamma_\sigma, \gamma_\nu]. \\
\bar{H}(b) = {} & \frac{1}{4} Tr \hat{p}_1' \gamma_\mu \gamma_\sigma \gamma_\nu \hat{p}_1 \gamma_\eta \times \\
& \times \left\{ -\frac{1}{4} TrR [\gamma_\sigma \gamma_\mu] (\hat{p}' - \hat{b} + M) \gamma_\nu - \frac{1}{4} TrR [\hat{q} - \hat{b}, \gamma_\nu] \gamma_\sigma \gamma_\nu \right\} \\
& + \frac{1}{4} Tr \hat{p}_1' \gamma_\mu (\hat{p_1}' + \hat{b}) \gamma_\nu \hat{p}_1 \gamma_\eta \cdot \frac{1}{4} TrR [\gamma_\sigma \gamma_\mu] \gamma_\sigma \gamma_\nu. \qquad (55)
\end{aligned}
$$

Infrared singularities (divergences of A_{f2}, A_{f3} at $y \to 0$, $z \to 1$ are mutually compensated in the sum A_f.

APPENDIX C

Let us describe the procedure employed for compacting the denominators and for the loop-momentum integration. Taking $\mathcal{D}(z) = az + b\bar{z}$, $\bar{z} = 1 - z$, the Feynman prescription for compacting the denominators leads to

$$
\begin{aligned}
\frac{1}{ab} &= \int_0^1 \frac{dz}{\mathcal{D}(z)^2}; \ \frac{1}{a^2 b} = \int_0^1 \frac{2z dz}{\mathcal{D}(z)^3}; \ \frac{1}{a^2 b^2} = \int_0^1 \frac{6 z \bar{z} dz}{\mathcal{D}(z)^4}, \\
\frac{1}{a^3 b} &= \int_0^1 \frac{3z^2 dz}{\mathcal{D}(z)^4}; \ \frac{1}{a^3 b^2} = \int_0^1 \frac{12 z^2 \bar{z} dz}{\mathcal{D}(z)^5}; \ \frac{1}{a^3 b^3} = \int_0^1 \frac{30 (z\bar{z})^2 dz}{\mathcal{D}(z)^6}. \qquad (56)
\end{aligned}
$$

Applying (56) to the set of denominators

$$
\begin{aligned}
& (e) = k^2 - 2kp_1, \ (\bar{e}) = k^2 + 2kp_1', \ (p) = k^2 + 2kp, \\
& (\bar{p}) = k^2 - 2p'k, \ (k) = K^2, (\bar{k}) = (q-k)^2, (Q_k) = k^2 - Q_0^2,
\end{aligned}
$$

one obtains

$$
\begin{aligned}
\frac{1}{(epk)} &= \int_0^1 \frac{2dxydy}{[k - yp_x)^2 - \mathcal{D}_0]^3}, \qquad \frac{1}{(ep\bar{k})} = \int_0^1 \frac{2dxydy}{[(k - P_0)^2 - \mathcal{D}_0]^3}, \\
\frac{1}{(\bar{e}\bar{p}\bar{k})} &= \int_0^1 \frac{2dxydy}{[(k - P_0')^2 - \mathcal{D}_0]^3}; \quad \frac{1}{(epQ_k)} = \int_0^1 \frac{2dxydy}{[(k - yp_x)^2 - d_0]^3}, \qquad (57)
\end{aligned}
$$

with $\mathcal{D}_0 = y^2 p_x^2$; $d_0 = y^2 p_x^2 + \bar{y} Q_0^2$, $P_0 = y p_x + \bar{y} q$, $P_0' = y p_x' + \bar{y} q$, $p_x = x p_1 - \bar{x} p$, $p_x' = \bar{x} p' - x p_1'$. The following relation holds: $P_0^2 = P_0'^2 = y^2 p_x^2 + \bar{y} q^2$, $p_x^2 = p_x'^2 = \bar{x}^2 M^2 - s x \bar{x}$.

We do not develop further (explicitly) the denominators which contain $(\bar{e})(\bar{p})$, because the corresponding results can be obtained by those depending on $(e)(p)$ under replacement $p_x \to p_{x'}$.

In a similar way one obtains

$$\begin{aligned}
\frac{1}{(epQ_k^2)} &= \int_0^1 \frac{6y\bar{y}dxdy}{[(k-yp_x)^2 - d_0]^4}; \frac{1}{(kepQ_k)} = \int_0^1 \frac{6y\bar{y}dxdydt}{[(k-yp_x)^2 - d_t]^4},\\
&\quad d_t = \mathcal{D}_0 + t\bar{y}Q_0^2;\\
\frac{1}{(kepQ_k^2)} &= \int_0^1 \frac{24dxtdty\bar{y}^2dy}{[(k-yp_x)^2 - d_t]^5}; \frac{1}{(ep\bar{k}Q_k)} = \int_0^1 \frac{6ydydxz^2dz}{[(k-zP_0)^2 - \mathcal{D}_1]^4},\\
&\quad \mathcal{D}_1 = a + \bar{z}Q_0^2;\\
\frac{1}{(epk\bar{k})} &= \int_0^1 \frac{6ydydxz^2dz}{[(k-zp_0)^2 - a]^4};\ a = z^2P_0^2 - z\bar{y}q^2,\\
\frac{1}{(epk\bar{k}Q_k)} &= \int_0^1 \frac{24ydyz^2\bar{z}dzdtdx}{[(k-zP_0)^2 - \mathcal{D}_2]^5};\ \mathcal{D}_2 = a + \bar{z}Q_0^2 t,\\
\frac{1}{(epk\bar{k}Q_k^2)} &= \int_0^1 \frac{120ydydxtdt(z\bar{z})^2dz}{[(k-zP_0)^2 - \mathcal{D}_2]^6}.
\end{aligned} \tag{58}$$

After replacing the loop momenta by $k - b \to \kappa$, a symmetrical integration on κ is performed. For polynomials $N(k)$ of order in k less than 3 one finds $N(\kappa + b) = \frac{1}{4}\kappa^2(b) + N(b)$. The application of the Wick rotation leads to

$$\begin{aligned}
\int \frac{\kappa^2 d\kappa}{(\kappa^2 - d)^3} &= \ln\frac{\Lambda^2}{d} - \frac{3}{2}; & \int \frac{d\kappa}{(\kappa^2 - d)^3} &= -\frac{1}{2d},\\
\int \frac{\kappa^2 d\kappa}{(\kappa^2 - d)^4} &= -\frac{1}{3d}; & \int \frac{d\kappa}{(\kappa^2 - d)^4} &= \frac{1}{6d^2},\\
\int \frac{\kappa^2 d\kappa}{(\kappa^2 - d)^5} &= \frac{1}{12d^2}; & \int \frac{d\kappa}{(\kappa^2 - d)^5} &= -\frac{1}{12d^3},\\
\int \frac{\kappa^2 d\kappa}{(\kappa^2 - d)^6} &= -\frac{1}{30d^3}; & \int \frac{d\kappa}{(\kappa^2 - d)^6} &= \frac{1}{20d^4},
\end{aligned} \tag{59}$$

with

$$d\kappa = \frac{d^4\kappa}{i\pi^2} = \kappa_e^2 d\kappa_e^2,\ \kappa^2 = -\kappa_e^2,\ \kappa_e^2 = \kappa_0^2 + \kappa_1^2 + \kappa_2^2 + \kappa_3^2,$$

and κ_e — the Euclidean four-vector. Note that the integration of the Feynman parameter t can be provided explicitly, as it enters only in phase volume

$$\begin{aligned}
L_1 &= \int\limits_0^1 \frac{dt}{d_t} = \frac{1}{\bar{y}Q_0^2} \ln\frac{d_0}{\mathcal{D}_0}, L_2 = \int\limits_0^1 \frac{dt}{d_t^2} = \frac{1}{\mathcal{D}_0 d_0}, \\
L_3 &= \int\limits_0^1 \frac{tdt}{d_t^3} = \frac{1}{(\bar{y}Q_0^2)^2}\left[\ln\frac{td_0}{\mathcal{D}_0} - 1 + \frac{\mathcal{D}_0}{d_0}\right], \\
\mathcal{J}_2 &= \int\limits_0^1 \frac{dt}{\mathcal{D}_2^2} = \frac{a+\mathcal{D}_1}{2a^2\mathcal{D}_1^2}, \\
\mathcal{J}_3 &= \int\limits_0^1 \frac{tdt}{\mathcal{D}_2^3} = \frac{1}{2a\mathcal{D}_1^2}; \mathcal{J}_4 = \int\limits_0^1 \frac{tdt}{\mathcal{D}_2^4} = \frac{\mathcal{D}_1 + 2a}{6a^2\mathcal{D}_1^3}.
\end{aligned} \tag{60}$$

REFERENCES

1. *Rekalo M. P., Tomasi-Gustafsson E.* // Eur. Phys. J. A. 2004. V. 22. P. 331;
   Rekalo M. P., Tomasi-Gustafsson E. // Nucl. Phys. A. 2004. V. 740. P. 271;
   Rekalo M. P., Tomasi-Gustafsson E. // Nucl. Phys. A. 2004. V. 742. P. 322.

2. *Kuraev E. A., Bytev V. V., Bystritskiy Y. M., Tomasi-Gustafsson E.* // Phys. Rev. D. 2006. V. 74. P. 013003.

3. *Blunden P. G., Melnitchouk W., Tjon J. A.* // Phys. Rev. Lett. 2003. V. 91. P. 142304;
   Chen Y.-C., Afanasev A., Brodsky S. J., Carlson C. E., Vanderhaeghen M. // Phys. Rev. Lett. 2004. V. 93. P. 122301;
   Guichon P. A. M., Vanderhaeghen M. // Phys. Rev. Lett. 2003. V. 91. P. 142303.

4. *Jones M. K. et al.* // Phys. Rev. Lett. 2000. V. 84. P. 1398;
   Gayou O. et al. // Phys. Rev. Lett. 2002. V. 88. P. 092301;
   Punjabi V. et al. // Phys. Rev. C. 2005. V. 71. P. 055202 [Erratum-ibid. C. 2005. V. 71. P. 069902].

5. *Baier V. N. et al.* // Phys. Rep. 1981. V. 78. P. 293, see Appendix «Sum rules»;
   Kuraev E., Secansky M., Tomasi-Gustafsson E. // Phys. Rev. D. 2006. V. 73. P. 125016.

6. *Kuraev E. A., Meledin G.* INP Preprint 76-91. Novosibirsk, 1976 (unpublished).

7. *Maximon L. C., Tjon L. A.* // Phys. Rev. C. 2000. V. 62. P. 054320.

8. *Borisyuk D., Kobushkin A.* arXiv:nucl-th/0606030.

9. *Ilichev A.* Private communication.

10. *Kondratyuk S., Blunden P. G., Melnitchouk W., Tjon J. A.* // Phys. Rev. Lett. 2005. V. 95. P. 172503.

Received on December 28, 2006.

Корректор *Т. Е. Попеко*

Подписано в печать 13.03.2007.
Формат $60 \times 90/16$. Бумага офсетная. Печать офсетная.
Усл. печ. л. 1,68. Уч.-изд. л. 2,37. Тираж 415 экз. Заказ № 55699.

Издательский отдел Объединенного института ядерных исследований
141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.
E-mail: publish@pds.jinr.ru
www.jinr.ru/publish/