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THE FAST NEUTRON INDUCED (n, p) REACTION CROSS
SECTIONS. Compound Reaction Mechanism

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Сечения (n, p) -реакции, вызываемой быстрыми нейтронами.
Компаунд-механизм

В рамках компаунд-механизма с использованием модели испарения, приближения постоянной температуры, полуклассического приближения для сечения инверсной реакции и формулы Вайцекера для энергии связи получена общая формула для сечения реакции с вылетом частиц, индуцированной быстрыми нейтронами. В энергетическом интервале от 6 до 16 МэВ полученная формула используется для систематического анализа известных экспериментальных сечений (n, p) -реакции. Найдено, что разница между теоретическими и экспериментальными сечениями (n, p) -реакции увеличивается с возрастанием параметра относительного избытка нейтронов в ядрах мишени $(N - Z + 1)/A$. Для широкой области энергии нейтронов от 6 до 16 МэВ пересмотрены выводы, полученные Левковским.

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The Fast Neutron Induced (n, p) Reaction Cross Sections.
Compound Reaction Mechanism

In the framework of the compound mechanism the general formula for fast neutron induced and particle emission reaction cross section was deduced. The evaporation model, constant nuclear temperature approximation, semi-classical approach to an inverse reaction cross section and Weizsäcker's formula for nuclear binding energy were used. For the systematic analysis of known experimental (n, p) cross sections the obtained formula was used in the energy range from 6 to 16 MeV. It was found that discrepancy between the theoretical and experimental (n, p) cross sections increases with growth of the neutron relative excess parameter $(N - Z + 1)/A$. Levkovsky's conclusions were considered and revised for a wide energy range from 6 to 16 MeV.

The investigation has been performed at the Frank Laboratory of Neutron Physics, JINR.

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INTRODUCTION

Investigation of charged particle emission reactions induced by fast neutrons is of interest to both nuclear reactor technology and the understanding of nuclear reaction mechanisms. In particular, the study of (n, p) reaction cross sections is necessary to estimate radiation damage due to hydrogen production, nuclear heating and transmutations in the structural materials of fission and fusion reactors. On the other hand, it is often necessary in practice to evaluate the neutron cross sections of the nuclides, for which no experimental data are available.

Because of this, in last decade we carried out the systematic analysis of the known (n, p) cross sections and observed the so-called isotopic effect in the wide energy interval of neutrons ($E_n = 6 - 16$ MeV) and for the broad mass range ($A = 24 - 209$) of target nuclei [1–3]. However, consistent theoretical substantiation on existence of the systematical regularity for the (n, p) cross sections up to now is no apparently available. Several formulae have been suggested to describe the isotopic dependence of the (n, p) cross sections around the neutron energy of 14–15 MeV only [4–13].

In this paper, to explain the isotopic effect of known experimental (n, p) cross sections in the framework of the compound reaction mechanism using the evaporation model and the constant nuclear temperature approximation some formulae are deduced. From the point of view of the statistical model the fast neutron induced (n, p) reaction experimental cross sections are analyzed. Also, three main conclusions proposed in 1973 by Levkovsky for the 14.5 MeV neutron induced (n, p) and (n, α) cross sections are considered and revised in a wide energy range.

1. STATISTICAL MODEL FORMULAE

1.1. The (n, x) Reaction Cross Section. For several MeV neutrons in order to deduce general expressions for the (n, x) reaction cross sections we can use the statistical model based upon Bohr's postulate of compound mechanism in which nuclear reactions proceed in two stages as

$$\sigma(n, x) = \sigma_c(n) G(x). \quad (1)$$

Here

$$\sigma_c(n) = \pi (R + \lambda)^2 \quad (2)$$

is the compound nucleus formation cross section, where R is the target nucleus radius and λ is the wavelength of the incident neutrons divided by 2π .

The probability of the compound nucleus decay into channel x ($x = p, n, \alpha \dots$) is expressed as

$$G(x) = \frac{\Gamma_x}{\Gamma} = \frac{\Gamma_x}{\sum_i \Gamma_i}, \quad (3)$$

where Γ_x and Γ are the partial and total level widths.

Using the evaporation model [14–16] we can determine the partial level width Γ_x as following:

$$\Gamma_x = \frac{2S_x + 1}{\pi^2 h^2 \rho_c(E_c)} M_x \int_{V_x}^{\varepsilon_x^{\max}} \varepsilon_x \sigma_c(\varepsilon_c) \rho_y(U_x) d\varepsilon_x. \quad (4)$$

Here S_x , M_x , ε_x and V_x are the spin, mass, energy and the Coulomb potential for the outgoing particle x , respectively; $\rho_y(U_x)$ is the level density of the residual nuclei; U_x is the excitation energy of the residual nuclei; $\rho_c(E_c)$ is the level density of compound nuclei; $\sigma_c(\varepsilon_x)$ is the inverse reaction cross section.

The maximum energy of the outgoing particle x is determined as

$$\varepsilon_x^{\max} = E_c - B_x + \delta_x \approx E_n + Q_{nx}. \quad (5)$$

Here E_c is the compound nucleus excitation energy; B_x is the binding energy of the outgoing x particle; δ_x is the even-odd effect parameter; E_n is the neutron energy and Q_{nx} is the (n, x) reaction energy.

In the semi-classical approximation we can determine the inverse reaction cross section as follows:

$$\sigma_c(\varepsilon_x) = \begin{cases} \pi R^2 \left(1 - \frac{V_x}{\varepsilon_x}\right) & \varepsilon_x > V_x, \\ 0 & \varepsilon_x < V_x. \end{cases} \quad (6)$$

The nuclear entropy is determined as

$$S = \ln N, \quad (7)$$

where N is the number of states.

Then, using the relation for the entropy

$$\frac{dS}{dE} = \frac{1}{T} \quad (8)$$

we can write the following expression for level densities ratio of the residual and compound nuclei:

$$\frac{\rho_y(U_x)}{\rho_c(E_c)} \approx e^{S_y - S_c} = e^{-\frac{1}{\Theta}(B_x + \delta_x + \varepsilon_x)}. \quad (9)$$

Using (6) and (9) for the formula (4) we can get the following formula for the partial level width:

$$\Gamma_x = \frac{2S_x + 1}{\pi h^2} M_x R^2 \int_{V_x}^{\varepsilon_x^{\max}} \varepsilon_x \left(1 - \frac{V_x}{\varepsilon_x}\right) e^{-\frac{B_x + \delta_x + \varepsilon_x}{\Theta}} d\varepsilon_x. \quad (10)$$

Here Θ is the nuclear thermodynamical temperature.

Then, neglecting γ emission and after integration of (10), from (1), (3) and (10) we finally get [17] for the (n, x) cross section the following general formula:

$$\sigma(n, x) = \sigma_c(n) \frac{(2S_x + 1) M_x e^{-\frac{B_x + \delta_x + V_x}{\Theta}} \left\{ 1 - \frac{W_{nx}}{\Theta} e^{-\frac{W_{nx}}{\Theta}} - e^{-\frac{W_{nx}}{\Theta}} \right\}}{\sum_i (2S_i + 1) M_i e^{-\frac{B_i + \delta_i + V_i}{\Theta}} \left\{ 1 - \frac{W_{ni}}{\Theta} e^{-\frac{W_{ni}}{\Theta}} - e^{-\frac{W_{ni}}{\Theta}} \right\}}, \quad (11)$$

where $W_{nx} = E_n + Q_{nx} - V_x$ and $W_{ni} = E_n + Q_{ni} - V_i$.

For fast neutrons total level width can be approximately taken as $\Gamma \approx \Gamma_n$.

Then from (11) we get

$$\sigma(n, x) = \sigma_c(n) \frac{2S_x + 1}{2S_n + 1} \frac{M_x}{M_n} e^{\frac{Q_{nx} - V_x}{\Theta}} \left\{ \frac{1 - \frac{W_{nx}}{\Theta} e^{-\frac{W_{nx}}{\Theta}} - e^{-\frac{W_{nx}}{\Theta}}}{1 - \frac{E_n}{\Theta} e^{-\frac{E_n}{\Theta}} - e^{-\frac{E_n}{\Theta}}} \right\}. \quad (12)$$

1.2. Analysis of the Formula for the (n, x) Cross Sections. For fast neutrons using the following assumption:

$$(E_n + Q_{ni} - V_i) \gg \Theta \quad (13)$$

Ericson's formula [18] can be obtained from (11):

$$\sigma(n, x) = \sigma_c(n) \frac{(2S_x + 1) M_x e^{\frac{Q_{nx} - V_x}{\Theta}}}{\sum_i (2S_i + 1) M_i e^{\frac{Q_{ni} - V_i}{\Theta}}}. \quad (14)$$

If we use the approximation $\Gamma \approx \Gamma_n$ and (13), we can write from (11) the convenient formula of Cuzzocrea et al. [19]

$$\sigma(n, x) = \sigma_c(n) \frac{(2S_x + 1) M_x}{(2S_n + 1) M_n} e^{\frac{Q_{nx} - V_x}{\Theta}}. \quad (15)$$

1.3. The (n, p) Cross Section. The (n, p) cross section can be obtained from (15) as follows:

$$\sigma(n, p) = \sigma_c(n) \frac{(2S_p + 1) M_p}{(2S_n + 1) M_n} e^{\frac{Q_{np} - V_p}{\Theta}}. \quad (16)$$

Since $S_p = S_n = \frac{1}{2}$ and $M_p \approx M_n$, using (2) and (16) the (n, p) cross section can be rewritten in the simple form:

$$\sigma(n, p) = \pi (R + \lambda)^2 e^{\frac{Q_{np} - V_p}{\Theta}}. \quad (17)$$

Reaction energy can be found from Weizsäcker's formula for the target and residual nuclei. Then from (17) it can be written as

$$\sigma(n, p) = \pi (R + \lambda)^2 \exp \frac{1}{\Theta} \left\{ \gamma \frac{2Z - 1}{A^{1/3}} - 4\xi \frac{N - Z + 1}{A} + \frac{\Delta}{A^{3/4}} - V_p \right\}. \quad (18)$$

Here A , N and Z are the mass number, number of neutrons and number of protons for the target nucleus, respectively, and $\Delta = \delta_i - \delta_f$, where $|\delta| = 34$ MeV or 0, $\gamma = 0.71$ MeV, and $\xi = 23.7$ MeV.

Thus, for systematical analysis of the (n, p) cross sections the following formula from (18) can be obtained:

$$\sigma(n, p) = C \pi (R + \lambda)^2 e^{-K \frac{N - Z + 1}{A}}, \quad (19)$$

where

$$C = \exp \frac{1}{\Theta} \left\{ \gamma \frac{2Z - 1}{A^{1/3}} + \frac{\Delta}{A^{3/4}} - V_p \right\} \quad (20)$$

and

$$K = \frac{4\xi}{\Theta}. \quad (21)$$

2. SYSTEMATICAL ANALYSIS OF THE (n, p) REACTION CROSS SECTIONS

2.1. Analysis of the Experimental (n, p) Cross Sections. Using formula (19) the systematic analyses of the experimental (n, p) cross sections for neutrons of $E_n = 6$ and 10 MeV, are shown in Fig. 1 as examples.

It can be seen from Fig. 1 that for $N = Z$ isotopes of ${}^{24}_{12}\text{Mg}$ ($Q_{np} = -4.732$ MeV), ${}^{28}_{14}\text{Si}$ ($Q_{np} = -3.860$ MeV), ${}^{32}_{16}\text{S}$ ($Q_{np} = -0.528$ MeV) and ${}^{40}_{20}\text{Ca}$ ($Q_{np} = -0.529$ MeV) the reduced cross sections have lower values in comparison with other isotopes.

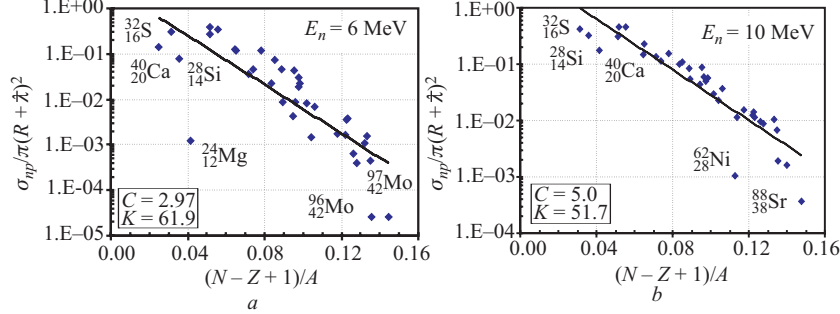


Fig. 1. The dependence of reduced (n, p) cross sections on the relative neutron excess parameter $(N - Z + 1)/A$ for neutron energy 6 MeV (a) and 10 MeV (b)

Also, the reduced cross sections for isotopes ^{96}Mo ($Q_{np} = -2.406$ MeV), ^{97}Mo ($Q_{np} = -1.153$ MeV), ^{62}Ni ($Q_{np} = -4.44$ MeV) and ^{88}Sr ($Q_{np} = -4.522$ MeV), which have high threshold energy, are lower than averaged fitted line. So, excepting these 8 isotopes the systematic regularity of the (n, p) cross sections for $E_n = 6 - 16$ MeV is shown in Fig. 2.

It is seen that in these cases the fitted lines are in agreement with experimental points. The parameters K and C in formula (19) are fitted to experimental data as constants for all isotopes at each neutron energy and also are given in Fig. 2.

2.2. The Parameters K and C . The parameters K and C in formula (19) can be determined by two methods. First, they can be fitted as the constant parameters at each energy point for all isotopes as shown in Fig. 2 (see the Table).

E_n , MeV	K	C
6	75.2	17.5
8	62.8	11.9
10	52.1	6.80
13	38.8	2.74
14.5	37.3	2.43
16	33.5	1.42

Second, the C and K parameters can be immediately obtained from the statistical model formulae (18) and (19).

If we use a formula for nuclear thermodynamic temperature as in [14] and the Fermi gas model formula for the level density parameter [20] we can get from

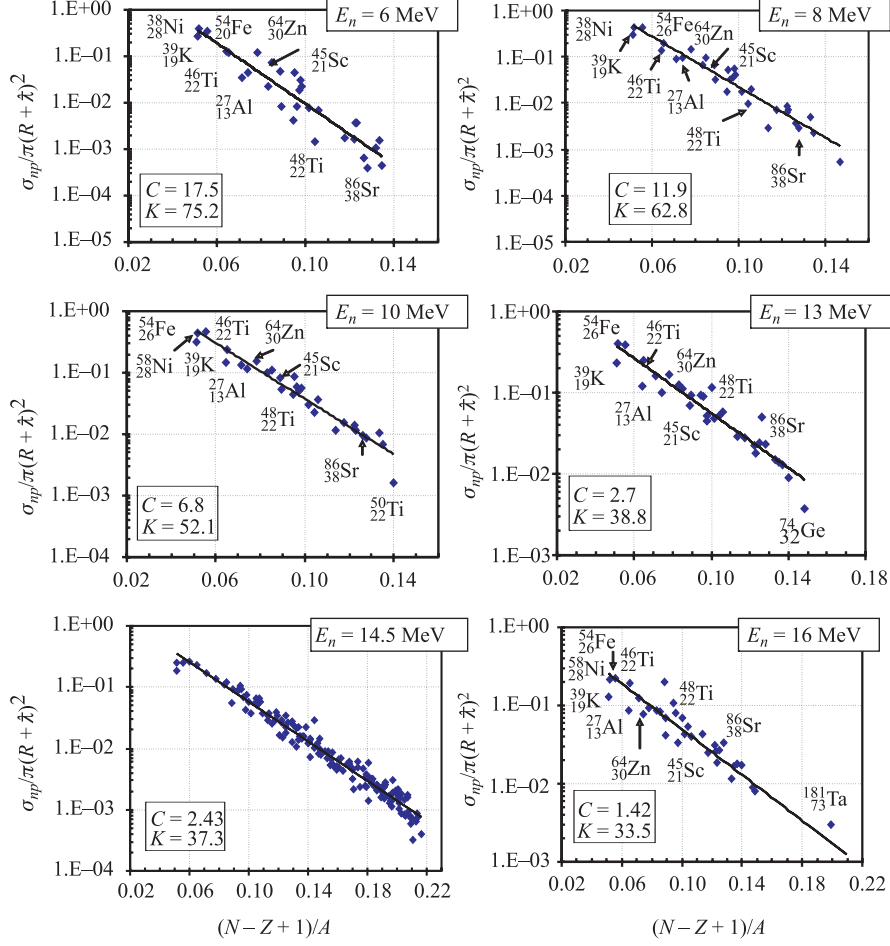


Fig. 2. The dependence of reduced (n, p) cross sections on the relative neutron excess parameter $(N - Z + 1)/A$ for $E_n = 6, 8, 10, 13, 14.5$ and 16 MeV

(21) the following expression for the parameter K :

$$K = 4\xi \sqrt{\frac{A}{13.5(E_n + Q_{np})}}. \quad (22)$$

We can use the following approximation for the Coulomb potential of protons [17, 21]:

$$V_p \approx \frac{Z}{A^{1/3}} \text{ (MeV)}. \quad (23)$$

Thus, if we assume $\Delta = 0$ for (n, p) reaction, the parameter C is determined from (20) as follows:

$$C = \exp \left\{ Z A^{1/6} \frac{2\gamma - 1}{\sqrt{13.5 (E_n + Q_{np})}} \right\} . \quad (24)$$

So, from (22) and (24) for parameters C and K the following relation can be written:

$$\ln C = \frac{Z}{A^{1/3}} \left(\frac{2\gamma - 1}{4\xi} \right) K. \quad (25)$$

It is seen from (22) and (24) that the parameters C and K depend on charge and mass numbers of target nuclei. However, for the number Z and A we can consider effective average value, which are constants for all nuclei and are determined by fitting to experimental data. Then, from (22) and (24) we can verify energy dependence of the parameters C and K , which is shown in Fig. 3. Also, in Fig. 3 the fitted parameters A , Q_{np} and $Z A^{1/6}$ are given. It is seen that the fitted parameter Q_{np} has the same value in the cases of K and C . It can be rewritten the relation (25) in the following form:

$$C = e^{BK}, \quad (26)$$

where

$$B = \frac{Z}{A^{1/3}} \frac{(2\gamma - 1)}{4\xi} = 0.037. \quad (27)$$

Here Z and A were taken from Fig. 3.

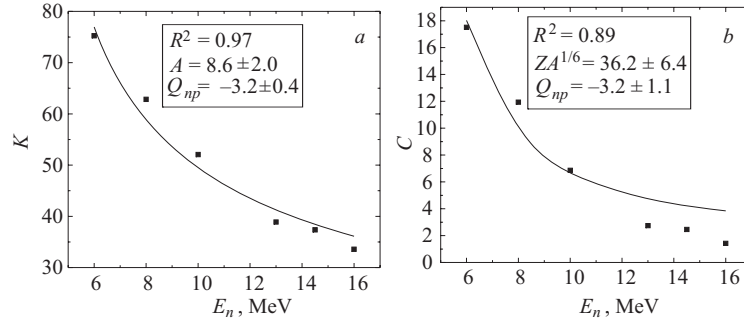


Fig. 3. Energy dependences for parameters K (a) and C (b)

Moreover, the relationship between the parameters C and K shown in Fig. 4 was established by formulae (25) and (26). It is seen that the fitted parameter B is in agreement with calculated value (27).

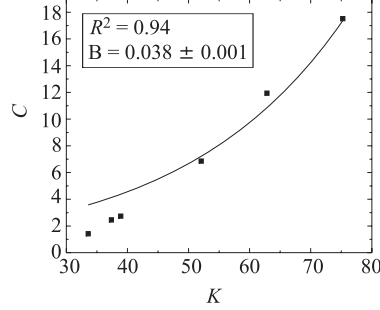


Fig. 4. The relationship between the parameters C and K

In Figs. 3 and 4 the black points for C and K parameters were taken from the Table and the solid curves were obtained by formulae (22), (24) and (25), respectively.

We can conclude from Figs. 3 and 4 that the theoretical parameters C and K are in satisfactory agreement with values fitted to experimental data.

3. ANALYSIS OF LEVKOVSKY'S CONCLUSIONS

3.1. Levkovsky's Conclusions. Analyzing the known experimental values of the 14.5 MeV neutron induced (n, p) and (n, α) reaction cross sections in 1973 Levkovsky drew the three following conclusions [5]:

1. The (n, p) reaction cross section is expressed by the following formula:

$$\sigma(n, p) = C\pi r_0^2(1 + A^{1/3})^2 e^{-K \frac{N-Z}{A}}. \quad (28)$$

Here the fitting parameters are determined as $C = 0.73$ and $K = 33$.

2. The relation between the (n, p) and (n, α) cross sections is obtained as follows:

$$\sigma(n, \alpha) = 0.4\sigma(n, p) \text{ or } \frac{\sigma(n, p)}{\sigma(n, \alpha)} = 2.5. \quad (29)$$

3. Systematic regularity of (n, p) and (n, α) cross sections expressed by formulae (28) and (29) takes place in the energy region of 14.5 MeV only, because in this case the excitation function has maximum value for the most of nuclei.

Now from the point of view of the statistical model we will reconsider these conclusions for a wide energy range of neutrons using the revised data for neutron cross sections.

3.2. Fast Neutron Induced (n, p) Reaction Cross Section Formula. It can be seen that the empirical formula of Levkovsky (28) is similar to expression (19), which was deduced using the statistical model. In addition, it should be noted that the parameters C and K in expression (19) can be obtained not only by fitting to experimental data, but directly by formulae (22) and (24). If we use fitted values of the parameters C and K , as in case of Levkovsky's analysis, experimental (n, p) cross sections for wide energy range from 6 to 16 MeV (not only for 14.5 MeV) which are shown in Fig.2 are satisfactorily described by formula (19).

3.3. Relation between the (n, p) and (n, α) Reaction Cross Sections. The ratio of the (n, p) cross section to the (n, α) one in the energy range from 6 to 16 MeV is shown in Fig.5. It is seen from Fig.5 that at energies of 8, 10, 13

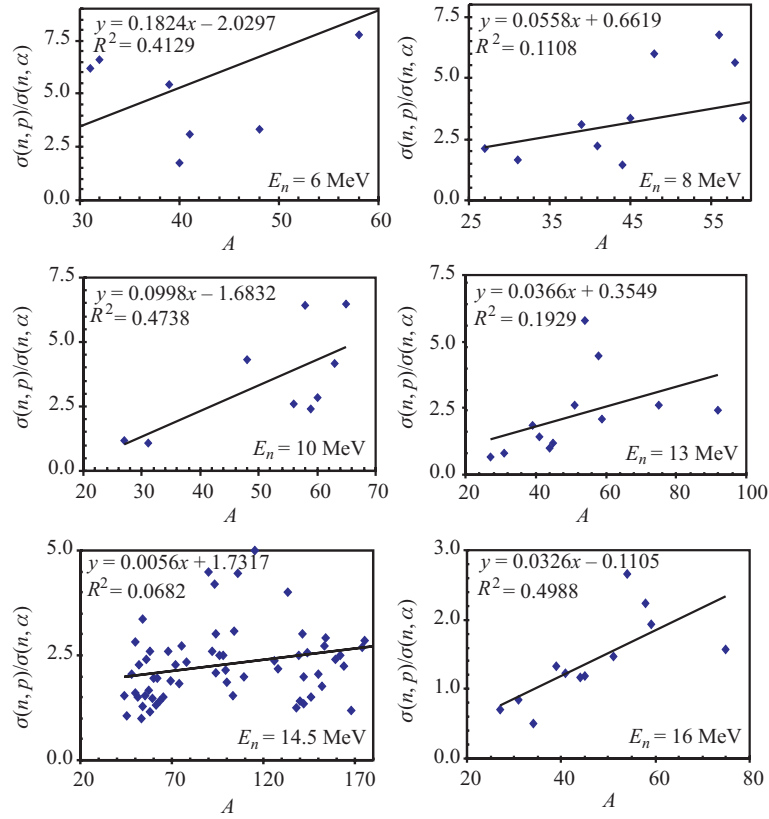


Fig. 5. The relation between the (n, p) and (n, α) cross sections for energy range of $E_n = 6 - 16$ MeV

and 14.5 MeV this ratio is approximately equal to 2.5, as Levkovsky concluded, although there is the essential deviation from average value. At that time, this ratio is equal to ~ 5 at 6 MeV and ~ 1.5 at 16 MeV. This fact and certain increase of this ratio with growth of the number A for the target nuclei, perhaps, are qualitatively explained by the difference of the Coulomb barriers for proton and α particle.

4. THE STATISTICAL MODEL CALCULATIONS OF (n, p) REACTION CROSS SECTION

Calculated by statistical model formula (19) values and known experimental data for (n, p) reaction cross sections at energies of 6, 8, 10, 13, 14.5 and 16 MeV are shown in Fig. 6. It is seen from Fig. 6 that discrepancy between

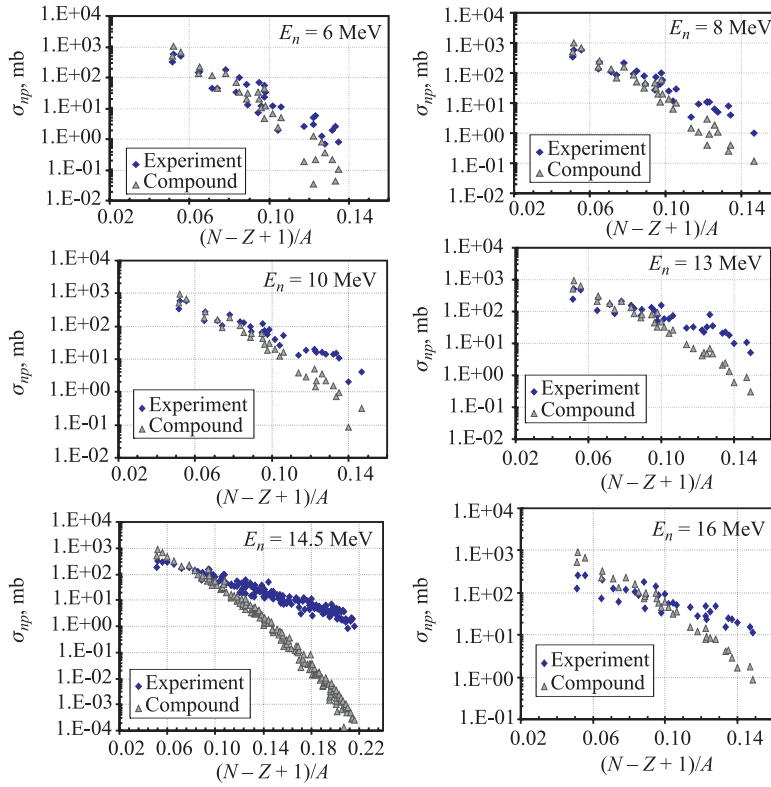


Fig. 6. The values and experimental data of (n, p) cross section calculated by statistical model

the experimental and theoretical cross sections rises with increase of the parameter $(N - Z + 1)/A$ and neutron energy. Such results, on the one hand, show that the proton and neutron numbers asymmetry parameter $(N - Z + 1)/A$ is very important in the fast neutron induced (n, p) reaction cross sections. On the other hand, it should be taken into account besides compound mechanism also the pre-equilibrium and direct reaction mechanisms in the theoretical model calculations for the fast neutron induced (n, p) reaction cross sections.

CONCLUSIONS

1. In the framework of the compound mechanism the general formula for fast neutron induced and particle emission reaction cross section was deduced. The evaporation model, constant nuclear temperature approximation, semi-classical approach to an inverse reaction cross section and Weizsäcker's formula for nuclear binding energy were used.

2. From the general formula a formula for the fast neutron induced (n, p) reaction cross section was obtained.

3. For the systematic analysis of known experimental (n, p) cross sections the obtained formula was used in the energy range from 6 to 16 MeV. It was found that discrepancy between the theoretical and experimental (n, p) cross sections increases with growth of the neutron relative excess parameter $(N - Z + 1)/A$.

4. The parameters C and K in the formula for the (n, p) reaction cross section were determined by fitting to experimental data. Also, theoretical relations for the C and K parameters were established. The energy dependence of the calculated and fitted values for the parameters K and C were obtained.

5. In the wide energy range from 6 to 16 MeV Levkovsky's conclusions were considered and revised.

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