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TRANSVERSE EMITTANCE BLOW-UP FROM BEAM INJECTION ERRORS IN SYNCHROTRONS WITH NONLINEAR FEEDBACK SYSTEMS

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О росте эмиттанса пучка вследствие ошибок инжекции в синхротронах с нелинейными системами подавления когерентных поперечных колебаний

Рассматривается проблема роста эмиттанса пучка, возникающего вследствие ошибок инжекции, в синхротронах с системами подавления когерентных поперечных колебаний, в которых передаточная функция в цепи обратной связи является нелинейной. Рассчитан относительный рост эмиттанса пучка для линейной и нелинейной передаточных функций цепи обратной связи системы подавления. Обсуждаются эффекты увеличения постоянной затухания когерентных колебаний пучка и уменьшения разброса амплитуд когерентных поперечных колебаний сгустков при использовании нелинейного режима системы подавления колебаний с положительной кубической добавкой в передаточной функции цепи обратной связи.

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Transverse Emittance Blow-Up from Beam Injection Errors in Synchrotrons with Nonlinear Feedback Systems

The problem of transverse emittance blow-up from beam injection errors in synchrotrons with nonlinear feedback systems is considered. The relative emittance growth is calculated for linear and nonlinear feedback transfer functions. Effects of an increase of the damping decrement of the beam coherent oscillations and of a decrease of the coherent transverse amplitude spread of different bunches in case of the damper with a positive cubic term in the feedback transfer function are discussed.

The investigation has been performed at the Laboratory of Particle Physics, JINR.

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INTRODUCTION

Emittance preservation is an important issue during injection of a beam into a circular accelerator. An initial position or angular error can lead to an increase in the transverse beam size due to decoherence or filamentation. It is well known [1] that the emittance blow-up due to the decoherence in presence of the injection error is

$$\varepsilon = \left(1 + \frac{\bar{a}_{\varepsilon}^2}{2\sigma_{x0}^2}\right)\varepsilon_0, \tag{1}$$

where $\varepsilon_0 = \sigma_{x0}^2/\beta$ is an initial transverse emittance with an initial RMS beam size σ_{x0}. The amplitude $\bar{a}_{\varepsilon}$ of the beam deviation due to the injection error is

$$\bar{a}_{\varepsilon} = \Delta\bar{r}_0 = \sqrt{(\Delta\bar{x}_0)^2 + (\beta\Delta\bar{x}_0' + \alpha\Delta\bar{x}_0)^2},$$

where $\Delta\bar{x}_0$ is an initial displacement injection error, $\Delta\bar{x}_0'$ is an initial angular injection error, β and α are the optic Twiss parameters at the injection point. It is assumed in (1) that all particles of the injected beam with the emittance ε_0 are being redistributed on the phase space and fill out after a long time the larger phase space, which corresponds to emittance ε, due to the decoherence only (Fig. 1). Other effects such as an active damping of coherent oscillations or a transverse instability of a beam are not taken into account in Eq. (1).

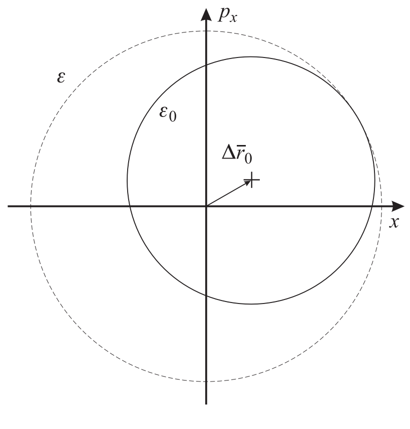

Fig. 1. Injected emittance ε_0 dilution to ε due to error $\Delta\bar{r}_0$

The damper kicker (DK) of a transverse feedback system (TFS) corrects the beam transverse momentum in accordance with the beam displacement from the closed orbit at the location of the beam position monitor (BPM). Hence, the feedback system leads to a steady decrease of the coherent amplitude, and the emittance blow-up does not happen without the decoherence. However, with the presence of the decoherence, the coherent amplitude decreases in time, and the displacement of the centre of gravity which is measured by the BPM at every turn has a smaller magnitude than without the decoherence. Therefore, the effect of the decoherence can produce the emittance blow-up despite the active damping of the coherent oscillations by the transverse damper.

The emittance blow-up in case of a classical linear transverse damping system was discussed in [2]. A more general approach that includes effects of transverse coherent instabilities and nonlinear damping of coherent transverse oscillations is described below.

1. BASIC EQUATIONS AND DEFINITIONS

The amplitude $\bar{a}(t)$ of transverse coherent oscillations of a beam decreases in time due to decoherence with the time constant τ_{dec} and satisfies the differential equation

$$\frac{d\bar{a}(t)}{dt} = -\frac{\bar{a}(t)}{\tau_{\mathrm{dec}}} \tag{2}$$

with the starting condition $\bar{a}(t = 0) = \Delta r_0$. The term $\bar{a}(t)$ describes the dependence of the amplitude of the oscillations of the beam centre of gravity on time due to the filamentation that leads to redistribution of particles on phase space. At BPM it looks like a damped coherent oscillation. Hence, in presence of the decoherence effect only, the impact of the injection error $\Delta\bar{r}_0$ to the emittance growth in time can be described by the function $\bar{a}_\varepsilon(t) = \Delta\bar{r}_0 - \bar{a}(t)$. Therefore, the part $\bar{a}_\varepsilon(t)$ of the amplitude of transverse coherent oscillations $\bar{a}(t)$ that goes to the emittance blow-up due to decoherence satisfies the differential equation

$$\frac{d\bar{a}_\varepsilon(t)}{dt} = \frac{\bar{a}(t)}{\tau_{\mathrm{dec}}} \tag{3}$$

with the starting condition $\bar{a}_\varepsilon(t = 0) = 0$. The differential Eq. (3) can be used for obtaining a new dependence of $\bar{a}_\varepsilon(t)$ on time after including the active damping and instability effects in dependence of $\bar{a}(t)$ on time in the differential Eq. (2).

An action of a transverse feedback system can be taken into account in (2) by including an additional term $d\bar{a}_{\mathrm{d}}(t)/dt$, which corresponds to the decrease in the amplitude of oscillation of the beam centre of gravity. A transverse instability with the time constant of growth τ_{inst} leads to an additional positive term $\bar{a}(t)/\tau_{\text{inst}}$. Therefore, the differential equation for the amplitude $\bar{a}(t)$ is given by

$$\frac{d\bar{a}(t)}{dt} = -\frac{\bar{a}(t)}{\tau_{\text{dec}}} + \frac{d\bar{a}_{\text{d}}(t)}{dt} + \frac{\bar{a}(t)}{\tau_{\text{inst}}}. \tag{4}$$

Let us assume that the dependences $\bar{a}(t)$ and $\bar{a}_{\varepsilon}(t)$ have been obtained from Eqs. (4) and (3). The total amplitude not corrected by the active feedback in presence of the transverse instability is the following:

$$\lim_{t\to\infty} \bar{a}_{\varepsilon}(t) = F_{\varepsilon} \cdot \Delta\bar{r}_0, \tag{5}$$

where F_{ε} is the form factor. Its value determines the part of the initial error Δr_0 that leads to the emittance blow-up. So, $F_{\varepsilon} = 1$ in presence of decoherence effect only and $F_{\varepsilon} < 1$ in case of an active damping. Therefore, the relative emittance blow-up can be expressed by the formula:

$$\frac{\Delta\varepsilon}{\varepsilon_0} = \frac{\varepsilon - \varepsilon_0}{\varepsilon_0} = \frac{(\Delta\bar{r}_0)^2}{2\sigma_{x0}^2} F_{\varepsilon}^2. \tag{6}$$

2. TRANSVERSE EMITTANCE BLOW-UP IN PRESENCE OF FEEDBACK SYSTEMS

The term $d\bar{a}_{\text{d}}(t)/dt$ in (4) depends on the type of a feedback transfer function $f(\bar{x})$:

$$g \cdot f(\bar{x}[n, s_{\text{P}}]) = \sqrt{\beta_{\text{P}}\beta_{\text{K}}}\Delta\bar{x}'[n, s_{\text{K}}],$$

where g is the gain of the feedback loop, β_{P} and β_{K} are the transverse betatron amplitude functions at the BPM and DK locations, $\bar{x}[n, s_{\text{P}}]$ is the displacement of the beam centre of gravity at the BPM location s_{P}, and $\Delta\bar{x}'[n, s_{\text{K}}]$ is the correction kick at the DK location s_{K} at the n-th turn. So, the transfer function for the linear feedback system is given by

$$g \cdot f(\bar{x}) = g\bar{x},$$

and the derivative $d\bar{a}_{\text{d}}(t)/dt$ is expressed by the formula

$$\frac{d\bar{a}_{\text{d}}(t)}{dt} = -\frac{\bar{a}(t)}{\tau_{\text{d}}}. \tag{7}$$

Here the time constant of damping is given by

$$\tau_{\text{d}} = 2T_{\text{rev}}/g, \tag{8}$$

where T_{rev} is the revolution period of a particle in a synchrotron. The formula (8) corresponds to the classical ideal transverse feedback system if the phase advance from BPM to DK is equal to an odd number of $\pi/2$ radians. In that case, the best damping can be ensured by the TFS, and the coherent transverse oscillations as well as the injection errors are damped if the decrement of the oscillations exceeds the increment of the instability [3].

Several analytical expressions for $d\bar{a}_{\rm d}(t)/dt$ in case of nonlinear feedback systems were presented in [4]. So, for the feedback transfer function with a cubic term

$$g \cdot f(\bar{x}) = g\bar{x} + gg_3\bar{x}^3$$

the derivative $d\bar{a}_{\rm d}(t)/dt$ in accordance with [4] for $|g_3|\bar{a}^2 < 1$ and $g \ll 1$ is

$$\frac{d\bar{a}_{\rm d}(t)}{dt} = -\frac{\bar{a}(t)}{\tau_{\rm d}} - \frac{3g_3}{4}\frac{\bar{a}^3(t)}{\tau_{\rm d}}, \tag{9}$$

where $\tau_{\rm d}$ coincides with its definition in (8).

2.1. Linear Feedback Systems. By substituting (7) in (4), the differential equation for the amplitude $\bar{a}(t)$ can be written as follows:

$$\frac{d\bar{a}(t)}{dt} = -\frac{\bar{a}(t)}{\tau_{\rm dec}} - \frac{\bar{a}(t)}{\tau_{\rm d}} + \frac{\bar{a}(t)}{\tau_{\rm inst}} = -\frac{\bar{a}(t)}{\tau}, \tag{10}$$

where the time constant of decay τ

$$\frac{1}{\tau} = \frac{1}{\tau_{\rm dec}} + \frac{1}{\tau_{\rm d}} - \frac{1}{\tau_{\rm inst}} \tag{11}$$

corresponds to the damped oscillation if $\tau_{\rm d} < \tau_{\rm inst} \ll \tau_{\rm dec}$. The solution of (10) is given by

$$\bar{a}(t) = \Delta\bar{r}_0 \exp\left(-t/\tau\right), \tag{12}$$

and the solution of Eq. (3) with $\bar{a}(t)$ from (12) is:

$$\bar{a}_\varepsilon(t) = \frac{\tau}{\tau_{\rm dec}}\left(1 - \exp\left(-t/\tau\right)\right)\Delta r_0. \tag{13}$$

Therefore, in accordance with (5), the form factor F_ε for the total amplitude not corrected by the active linear feedback in presence of the transverse instability is:

$$F_\varepsilon = \frac{1}{\Delta r_0}\lim_{t\to\infty}\bar{a}_\varepsilon(t) = \frac{\tau}{\tau_{\rm dec}} = \left(1 + \frac{\tau_{\rm dec}}{\tau_{\rm d}} - \frac{\tau_{\rm dec}}{\tau_{\rm inst}}\right)^{-1}. \tag{14}$$

The emittance blow-up is given by

$$\varepsilon = \left(1 + \frac{(\Delta\bar{r}_0)^2}{2\sigma_{x0}^2}\left(1 + \frac{\tau_{\rm dec}}{\tau_{\rm d}} - \frac{\tau_{\rm dec}}{\tau_{\rm inst}}\right)^{-2}\right)\varepsilon_0. \tag{15}$$

If $\tau_{\rm inst} = \tau_{\rm d}$ or $\tau_{\rm inst} > \tau_{\rm d} \to \infty$ then (15) coincides with (1). If $\tau_{\rm inst} \to \infty$ then (15) coincides with the formula for the emittance blow-up presented in [2]:

$$\varepsilon = \left(1 + \frac{(\Delta \bar{r}_0)^2}{2\sigma_{x0}^2}\left(1 + \frac{\tau_{\rm dec}}{\tau_{\rm d}}\right)^{-2}\right)\varepsilon_0. \tag{16}$$

It is clear from (15) that a faster decoherence (a smaller magnitude of $\tau_{\rm dec}$) for the fixed parameters $\tau_{\rm d}$ and $\tau_{\rm inst}$ leads to a larger emittance blow-up.

2.2. Nonlinear Feedback Systems. The differential equation for $\bar{a}(t)$ in case of nonlinear feedback systems with a cubic term, after substituting (9) in (4), takes the form:

$$\frac{d\bar{a}(t)}{dt} = -\frac{\bar{a}(t)}{\tau_{\rm dec}} - \frac{\bar{a}(t)}{\tau_{\rm d}} - \frac{3g_3}{4}\frac{\bar{a}^3(t)}{\tau_{\rm d}} + \frac{\bar{a}(t)}{\tau_{\rm inst}} = -\frac{\bar{a}(t)}{\tau} - \frac{3g_3}{4}\frac{\bar{a}^3(t)}{\tau_{\rm d}}, \tag{17}$$

where τ was already defined in (11). The solution of (17) is given by

$$\bar{a}(t) = \frac{\Delta r_0 \cdot \exp\left(-t/\tau\right)}{\sqrt{1 + \xi \cdot \left(1 - \exp\left(-2t/\tau\right)\right)}}, \tag{18}$$

where

$$\xi = \frac{3g_3}{4}\frac{\tau}{\tau_{\rm d}}(\Delta r_0)^2. \tag{19}$$

After solving Eq. (3) for $\bar{a}_\varepsilon(t)$ with $\bar{a}(t)$ from (18) and substituting $\bar{a}_\varepsilon(t)$ in (5), the form factor F_ε for the total amplitude not corrected by the active nonlinear feedback with cubic term in presence of the transverse instability can be expressed by the formula

$$F_\varepsilon = \frac{1}{\Delta r_0}\lim_{t\to\infty}\bar{a}_\varepsilon(t) =$$
$$= \frac{\tau}{\tau_{\rm dec}\sqrt{|\xi|}} \cdot \begin{cases} \arcsin\left(\sqrt{\frac{|\xi|}{1+\xi}}\right), & \text{if } g_3 > 0\,; \\ \ln\left|\sqrt{\frac{|\xi|}{1+\xi}} + \sqrt{1 + \frac{|\xi|}{1+\xi}}\right|, & \text{if } g_3 < 0. \end{cases} \tag{20}$$

Dependences of the form factor F_ε on instability increments for linear and nonlinear feedback systems are shown in Fig. 2 in case of the LHC specifications [5]. So, the emittance blow-up is smaller for the nonlinear feedback system with a positive magnitude of g_3 in the cubic term than for the linear feedback system. It should be emphasised that the form factor F_ε depends strongly on the time damping constant $\tau_{\rm d}$. So, if excess of $\tau_{\rm d}$ above $\tau_{\rm inst}$ is a small value ($\tau_{\rm d} \to \tau_{\rm inst}$), then $F_\varepsilon \to 1$ and the initial injection error Δr_0 leads to the emmitance magnitude as in presence of the decoherence process only with small influence of the transverse damper. Therefore, the magnitude of F_ε can be used as criteria for choosing the time damping constant $\tau_{\mathrm{d}} > \tau_{\mathrm{inst}}$ and the correction force in the damper kicker.

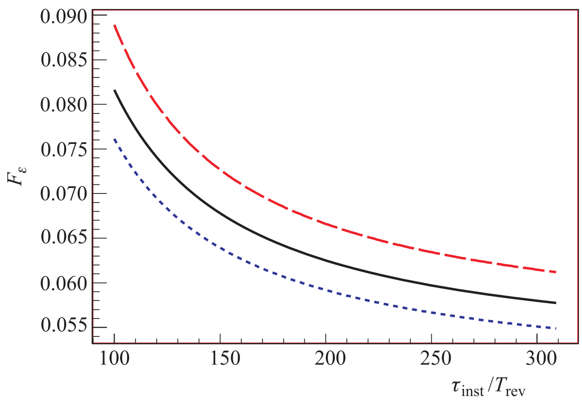

Fig. 2. Dependence of F_ε on $\tau_{\mathrm{inst}}/T_{\mathrm{rev}}$ for $\tau_{\mathrm{d}}/T_{\mathrm{rev}} = 40$, $\tau_{\mathrm{dec}}/T_{\mathrm{rev}} = 750$, $|g_3|(\Delta r_0)^2 = 0.2$; $g_3 = 0$ (solid curve), $g_3 > 0$ (dotted curve), $g_3 < 0$ (dashed curve)

If $|\xi| \ll 1$ then F_ε in (20) takes the simple form:

$$F_\varepsilon = \frac{\tau}{\tau_{\mathrm{dec}}\sqrt{1+\xi}}.$$

Therefore, the form factor F_ε depends on the magnitude of the injection error in case of nonlinear feedback systems. Let us assume that coherent transverse amplitude distribution of bunches after injection into a synchrotron is given by

$$\bar{a}_i(t=0) = \left(1 + 0.05 \cdot \sin\left(\frac{2\pi(i-1)}{K_{\mathrm{b}}}\right)\right) \cdot \bar{a}_1(t=0), \qquad (21)$$

where K_{b} is the number of bunches ($1 \leqslant i \leqslant K_{\mathrm{b}}$). Due to the decoherence process, the transformation from the initial coherent amplitudes $\bar{a}_i$ to the incoherent oscillations is observed. Data on relative emittance blow-up for the distribution function (21) with linear and nonlinear transverse feedback systems are shown in Fig. 3. Dependences of $\Delta\varepsilon_i/\Delta\varepsilon_1$ on the bunch's number i were calculated in accordance with (6) for F_ε from (20) and (14). It is clear from data in Fig. 2 that the final distribution of diameter growth of the bunch's cross section after damping coincides with the initial distribution of injection errors in case of the linear feedback. It should be noted that this rule is the global property of a linear system. However, nonlinear damping changes the transverse distribution

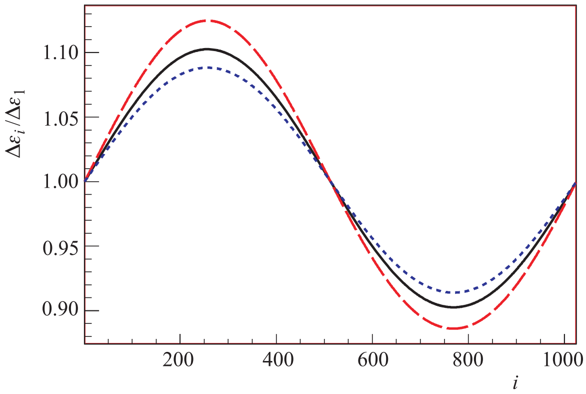

Fig. 3. Relative emittance blow-up $\Delta\varepsilon_i$ for different bunches i normalized to its magnitude $\Delta\varepsilon_1$ for the first bunch. Drawing symbols and parameters of decay process are the same as in Fig. 2

function of bunches. So, the «smoothing» effect is observed in nonlinear regime with the positive cubic term $g_3 > 0$ in the feedback transfer function. In the other case of $g_3 < 0$ the «blow-up» effect is observed for distribution of bunches.

CONCLUSION

The description presented above for the emittance blow-up from beam injection errors in synchrotrons with transverse feedback systems demonstrates the increase of the damping decrement of the beam coherent oscillations and the decrease of the coherent transverse amplitude spread of different bunches in case of the damper with a positive cubic term in the feedback transfer function. It should be emphasized that this nonlinear regime is ensured by the nonlinear transfer function in the feedback loop only. The TFS corrects the transverse momentum of the bunch in the kicker in accordance with the bunch's displacement in the beam position monitor at the previous moment of time. This resonance condition is provided by electronics in the feedback loop. The beam position monitor and the damper kicker operate as devices with linear characteristics. So, BPM measures the position of the centre gravity of the bunch. Because of the linear characteristic of the BPM sensitivity, the position measurement does not depend on the transverse size of the bunch. The electromagnetic field in the damper kicker is the uniform one, and DK changes the transverse momentum of all particles in the bunch's cross section independently of their magnitude. Therefore, BPM and DK operate with a bunch like with a point particle. Digital electronics in feedback loop of TFS allow modifying its linear characteristic by means of changing algorithms in the digital signal processing unit. If the nonlinear transfer function in the feedback loop is employed, then the coherent transverse amplitude distribution of bunches can be changed as time proceeds. Therefore, experiments with nonlinear TFS give the unique opportunity for studying nonlinear phenomena with macroobjects (bunches): the current value of the kicker's force corresponds to the nonlinear phenomenon, but the influence on the particles of the bunch in the kicker is the linear phenomenon because the electromagnetic field in the kicker is the uniform field.

It should be noted that high order modes are exited in the nonlinear regime of TFS. So, the cubic kick excites the third harmonic of oscillations whose magnitude is proportional to $\bar{a}^3$ [4]. It is clear from (18) that the nonlinear term g_3 has no affect on the exponent terms in $\bar{a}$ at the first level of approximation. Hence, the third mode decreases faster than the first one. However, the stability of a beam with a nonlinear feedback transfer function should be further studied.

Concluding, it is necessary to emphasize that integration of traditional approaches for devices with linear characteristics and digital computer technologies in feedback loops of transverse dampers for obtaining nonlinear regimes opens new opportunities for research in accelerator physics.

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