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BALL SOLITONS IN KINETICS OF THE FIRST ORDER MAGNETIC PHASE TRANSITION

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Шаровые солитоны в кинетике магнитных фазовых переходов первого рода

Представлена теория шаровых магнитных солитонов, возникающих в результате флуктуаций энергии при спин-флоп-переходе, индуцированном магнитным полем в антиферромагнетиках с одноосной анизотропией. Такие солитоны возможны в широком диапазоне по амплитуде и энергии, включая и отрицательные значения энергии относительно исходного состояния. Когда антиферромагнетик находится в метастабильном состоянии, шаровые солитоны рождаются с наибольшей вероятностью, если их энергия близка к нулю. Проанализирована эволюция этих солитонов, при которой они непрерывным образом превращаются в макроскопические домены новой магнитной фазы, осуществляя таким образом полную фазовую перестройку в кристалле.

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Ball Solitons in Kinetics of the First Order Magnetic Phase Transition

The theory of magnetic ball solitons (BS), arising as a result of the energy fluctuations at the spin-flop transition induced by a magnetic field in antiferromagnets with uniaxial anisotropy, is presented. Such solitons are possible in a wide range of amplitudes and energies, including the negative energy relative to an initial condition. When such an antiferromagnet is in a metastable condition, ball solitons are born with the greatest probability if the energy of solitons is close to zero. Evolution of these solitons, at which they develop into macroscopic domains of a new magnetic phase, is analyzed, thus carrying out full phase reorganization.

The investigation has been performed at the Frank Laboratory of Neutron Physics, JINR.

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INTRODUCTION

For the last decades there have been published many works, where decisions of the nonlinear differential equations are devoted to the theoretical analysis of localized excitation in magnetic ordering systems (see, for example, the review [1]). The behaviour of magnetizations of sublattices in a crystal is described generally by a nonlinear equation of the Landau–Lipschitz type. The decisions of this equation have been found for some solitonic excitations. However, in case of three-dimensional solitons, usually it is not possible to advance essentially in analytical consideration of solitonic decisions. Besides, generally the energy of such solitons is too great to count on an opportunity of their spontaneous creation.

A special situation is with the first order phase transitions. Near to points of lability, i. e., of the boundaries of existence of metastability, the energy of such solitons decreases abnormally and, hence, they can arise spontaneously, as a result of thermal fluctuations. Moreover, the mathematical problem of their description becomes essentially simpler. For some specific cases, the phenomenological vector equation of Landau–Lipschitz can be reduced to the approximated scalar equation that differs from the usual linear equations describing spin waves by additional members of decomposition by degrees of some component of magnetization vector only.

Magnetic solitons with spherical symmetry (ball solitons, BS), which can arise in crystals with magnetic ordering during the phase transitions induced by a magnetic field, were considered for three cases: the spin-flop transition in antiferromagnet with a uniaxial anisotropy [2–5], the first order transition in ferromagnet [6], the first order transition in antiferromagnet with a weak ferromagnetism of the hematite type [7].

In the first and second cases of phase transitions, in addition to the amplitude the solitons have the second parameter: the precession frequency. Therefore, in these cases each value of an external field relates to soliton conditions that occupy some range on energy. It was supposed that such BS could play an essential role in kinetics of the first order magnetic phase transitions.

However, the results of the [2–5] articles are correct only at small amplitudes of solitons arising at spin-flop transition. In the given paper, a more complete BS analysis concerning the solitons with any parameters is presented, beginning from solitons of the minimum amplitude and up to macroscopic domains of a new phase. The character of their further evolution relayed to the dissipation of energy is considered.

MAIN CHARACTERISTICS OF BALL SOLITONS

To analyze magnetic solitons in an antiferromagnet with uniaxial anisotropy, we used the following expression for the macroscopic energy (as in [5]):

$$W = 2M_0 \int \left\{ \frac{B}{2} |\mathbf{m}|^2 + \frac{C}{2} (\mathbf{lm})^2 + \frac{K_1}{2} |l_\perp|^2 - \frac{K_2}{4} |l_\perp|^4 - m_z H + \right.$$
$$\left. + \frac{\alpha}{2} \left[\left(\frac{\partial l}{\partial X} \right)^2 + \left(\frac{\partial l}{\partial Y} \right)^2 \right] + \frac{\alpha_z}{2} \left(\frac{\partial \mathbf{l}}{\partial Z} \right)^2 \right\} dX dY dZ. \tag{1}$$

Here $K_1 > 0,\ K_2 > 0$; magnetic field H is directed along the anisotropy axis Z; **m** and **l** are non-dimensional ferromagnetism and antiferromagnetism vectors; $l_\perp = l_x + i l_y,\ m_\perp = m_x + i m_y$; we neglect the gradients of the **m** vector; the absolute value of the vector **l** at $H = 0$ equals to 1, M_0 is the magnetization of each sublattice. The equations of motion with dissipative terms (in the Gilbert form) for the **l** and **m** vectors, taking into account the energy dissipation, are

$$\frac{M_0 \hbar}{\mu_B} \frac{\partial \mathbf{l}}{\partial t} = \mathbf{m} \times \frac{\delta W}{\delta \mathbf{l}} + \mathbf{l} \times \frac{\delta W}{\delta \mathbf{m}} + \Gamma \frac{M_0 \hbar}{\mu_B} \left(\mathbf{m} \times \frac{\partial \mathbf{l}}{\partial t} + \mathbf{l} \times \frac{\partial \mathbf{m}}{\partial t} \right), \tag{2}$$

$$\frac{M_0 \hbar}{\mu_B} \frac{\partial \mathbf{m}}{\partial t} = \mathbf{m} \times \frac{\delta W}{\delta \mathbf{m}} + \mathbf{l} \times \frac{\delta W}{\delta \mathbf{l}} + \Gamma \frac{M_0 \hbar}{\mu_B} \left(\mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} + \mathbf{l} \times \frac{\partial \mathbf{l}}{\partial t} \right). \tag{3}$$

The solutions of Eqs. (2), (3) have the form

$$l_\perp (\mathbf{r}, \tau) = q (\mathbf{r}, \tau) e^{i(\omega \tau - \kappa x)}, \tag{4}$$

$$m_\perp (\mathbf{r}, \tau) = p (\mathbf{r}, \tau) e^{i(\omega \tau - \kappa x)}, \tag{5}$$

for the simplicity it is supposed that the excitations advance along the x-axis.

Therefore, from (2) – (5) we have the following system of equations:

$$k_1^{-0.5} (\omega + h) p = - \left(1 - \frac{k_2}{k_1} q^2 \right) l_z q + l_z \Delta q - q \Delta l_z - l_z \kappa^2 q +$$
$$+ \gamma \left[\left(q \frac{\partial l_z}{\partial \tau} - l_z \frac{\partial q}{\partial \tau} \right) - m_z \frac{\partial p}{\partial \tau} \right], \tag{6}$$

$$\frac{\partial p}{\partial \tau} = \sqrt{k_1} \left(2 l_z \kappa \frac{\partial q}{\partial x} + \gamma (l_z + m_z) \omega q \right), \tag{7}$$

$$
\begin{aligned}
-k_1^{-0.5}(\omega+h)\,q = \left(1-\frac{k_2}{k_1}q^2\right)m_z q + \frac{1}{k_1}l_z p - \frac{1}{k_1}m_z q + p\Delta l_z - \\
- m_z\Delta q + m_z\kappa^2 q - \gamma\left[\left(p\frac{\partial l_z}{\partial\tau} - l_z\frac{\partial p}{\partial\tau}\right) - m_z\frac{\partial q}{\partial\tau}\right], \quad (8)
\end{aligned}
$$

$$
\frac{\partial q}{\partial\tau} = \sqrt{k_1}\left(2m_z\kappa\frac{\partial q}{\partial x} + \gamma\,(l_z+m_z)\,\omega p\right), \tag{9}
$$

$$
\frac{\partial l_z}{\partial\tau} = -2\sqrt{k_1}p\left(\kappa\frac{\partial q}{\partial x} + \gamma\omega q\right). \tag{10}
$$

In (6)–(10), the differentiation is carried out with respect to the dimensionless time $\tau = 2\mu_B\,(K_1 B)^{0.5}\,\hbar^{-1}t$ and dimensionless coordinates $x = K_1^{0.5}\alpha^{-0.5}X$, $y = K_1^{0.5}\alpha^{-0.5}Y$, $z = K_1^{0.5}\alpha_z^{-0.5}Z$; $h = HB^{-0.5}K_1^{-0.5}$, $k_1 = K_1/B$, $k_2 = K_2/B$, $\gamma = \Gamma 2\mu_B k_1^{-0.5}\hbar^{-1}$.

The relaxation time of ferromagnetic moment is less for some orders than the time of the relaxation for an antiferromagnetic vector. Therefore, for m_z component at $l_z \neq 0$ we use its quasi-equilibrium value, which can be obtained from $\dfrac{\delta W}{\delta m_z} = 0$ equation:

$$
\begin{aligned}
m_z \cong m_{z0} = \frac{1}{B+Cl_z^2}\left[H_z - \frac{C}{2}l_z\left(l_\perp m_\perp^* + l_\perp^* m_\perp\right)\right] \cong \\
\cong -\frac{pq}{l_z} + \frac{\delta}{l_z^2}\left(\sqrt{k_1}h + \frac{pq}{l_z}\right), \quad (11)
\end{aligned}
$$

where $\delta = B/(C+B) = \chi_\parallel/\chi_\perp$ is the ratio of magnetic susceptibilies.

Further, we divide the processes connected with solitons into two stages, which can be considered independently to some extent. First of them is the birth of soliton as a result of fluctuation of the energy, the second one is the further evolution of soliton. Such a division is fair if we believe that the lifetime of soliton is much more, than the time necessary for its formation. Actually, we divide the system of Eqs. (6)–(10) into two subsystems: Eqs. (6) and (8) and Eqs. (7), (9) and (10), and then we analyze them consistently. This operation is possible if the inequalities of the type $\gamma\left|\dfrac{\partial q}{\partial\tau}\right| << q$, $\gamma\left|\dfrac{\partial l_z}{\partial\tau}\right| << l_z$ are carried out.

Thus, at first we consider a case of solitons without their change because of the motion and without taking into account the energy dissipation, i. e., from the (6)–(10) system only Eqs. (6) and (8), without the parts containing $\dfrac{\partial q}{\partial\tau}$, $\dfrac{\partial p}{\partial\tau}$ and $\dfrac{\partial l_z}{\partial\tau}$, are used. Taking into consideration $k_1 << 1$, $k_2 q^2 << 1$, $\left(q^2+l_z^2\right) \cong 1$

in Eq. (8) and that for long-wave oscillations $k_1\Delta q << q,\ k_1\Delta l_z << l_z$, we obtain from (8) and (11) the following dependence:

$$p \cong -\sqrt{k_1}\,(\omega+h)\,ql_z+\frac{\delta\sqrt{k_1}q}{l_z}\left(h-(\omega+h)\,q^2\right)-k_1\sqrt{k_1}\,(\omega+h)\,l_zq^3\kappa^2. \quad (12)$$

Substituting (12) into (6), we obtain the equation

$$\Delta q-\frac{q}{l_z}\Delta l_z=\left[1-(\omega+h)^2+\frac{\delta\,(\omega+h)\,h}{l_z^2}+\kappa^2\right]q- \\ -\left[\frac{k_2}{k_1}+\frac{\delta\,(\omega+h)^2}{l_z^2}+k_1\,(\omega+h)^2\,\kappa^2\right]q^3. \quad (13)$$

The decision $q=p\equiv 0,\ m_z=\dfrac{B}{(B+C)\,l_z^2}\cong\dfrac{B}{B+C}$ of Eqs. (6)–(11) corresponds to the low-field equilibrium state of a crystal.

Particular cases of the (6)–(10) system describe different excitations of the magnetic system:

a) At $\Delta q=0,\ \Delta l_z=0$ we obtain a quasi-homogeneous state, and at a small q $(q\to 0)$ the corresponding equation $(\omega+h)^2-\delta\,(\omega+h)\,h-1=0$ connects the precession frequencies and the field value at an antiferromagnetic resonance (at $\delta=0$: $\omega_1=(1-h)$ and $\omega_2=-(1+h)$).

b) At a small $q=\mathrm{const}$, we have a usual spin wave.

It is possible to obtain decisions of Eq. (13), that correspond to spatial-localized excitations:

c) Quasi-one-dimensional soliton, when the q value changes only along one direction; considering time dependence in the equations it is possible to receive expressions for moving quasi-one-dimensional soliton.

d) Solitons with a cylindrical symmetry.

e) Solitons with a spherical symmetry (ball solitons, BS).

In the given paper, the BS decisions are considered only. For such solitons Eq. (13) can be writted in the form

$$\frac{d^2q}{dr^2}+\frac{2}{r}\frac{dq}{dr}+\frac{q}{1-q^2}\left(\frac{dq}{dr}\right)^2=\left[1-(\omega+h)^2+\kappa^2+\delta\,(\omega+h)\,h\right]q- \\ -\left[1-(\omega+h)^2+\kappa^2+\frac{k_2}{k_1}+\delta\,(\omega+h)^2\right]q^3+\left[\frac{k_2}{k_1}+k_1\,(\omega+h)^2\,\kappa^2\right]q^5. \quad (14)$$

The BS configurations, that are localized decisions of Eq. (14), for several ω values at $h=0.99$ are shown in Fig. 1. The dependences of BS energy (using

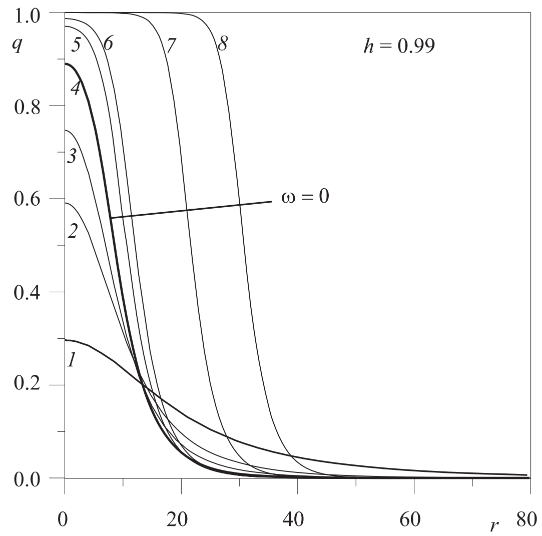

Fig. 1. The BS configurations if $h = 0.99$, $K_1 = 700$ Oe, $K_2 = 140$ Oe, $\kappa = 0$, $\delta = 0$, are for the following values of ω: *1* — +0.0095; *2* — +0.0075; *3* — +0.005; *4* — 0; *5* — −0.0075; *6* — −0.0114; *7* — −0.0025; *8* — −0.003

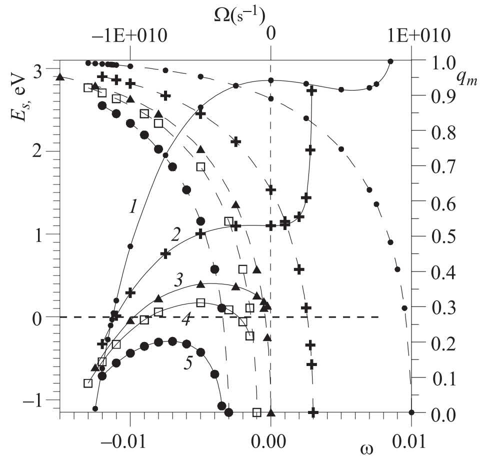

Fig. 2. Dependences of the BS energy and amplitude on the precession frequency if $B = 4.9 \cdot 10^6$ Oe, $K_1 = 700$ Oe [8], $K_2 = 140$ Oe, $M_0 = 0.33 \cdot 10^{-6}$ meV $\cdot$ Oe^{-1} $\times$ Å^{-3}, $\alpha = 3 \cdot 10^6$ Oe $\cdot$ Å^2 for the following values of h: *1* — 0.99; *2* — 0.997; *3* — 1; *4* — 1.001; *5* — 1.003 ($\delta = 0$, $\kappa = 0$). Dotted curves with the same symbols show the amplitudes q_m of these solitons. In the upper scale, the frequency values are shown for Cr_2O_3 case

formula (1)) and its amplitude $q_m \equiv q\,(r=0)$ on the precession frequency for some values of the field have been shown in Fig. 2.

Precession of the magnetic moments is the reason for existence of solitonic conditions and their relative stability in a wide range of amplitudes, from zero up to unit, and energy, including the negative values of energy relative to the initial condition. If there was no precession, there would be only one solitonic decision of the equations, at $\omega = 0$. As is seen from Fig. 2, at $h > 1$ the BS are possible too. Mechanical analogue of the ball precessing solitons is a rotating children's top.

For Cr_2O_3 and the majority of other substances close by their properties to antiferromagnets with uniaxial anisotropy, absolute values of the BS energy, in a wide range of precession frequency, considerably exceed the value $k_B T$ for the temperatures corresponding to the magnetic ordering of crystals. Therefore, the probability of spontaneous origin of such solitons is extremely small. Only BS with a small energy, i. e., on the area in Fig. 2, where $E_s\,(\omega)$ curves pass through the zero value, are of interest for kinetics of the phase transition.

BS AS NEW STRUCTURES ARISING IN THE BIFURCATION POINTS

In Fig. 3 the fragment of the $E\,(\omega)$ and $q_m\,(\omega)$ dependences from Fig. 2 is presented at $h = 0.99$. Parameters of a soliton in the b-point are as follows: $E_s = 0$, $\omega \cong -0.01126$, $q_m = 0.986377$. In other words, in this point the BS activation energy equals to zero relative to the low-field state of the crystal, but the amplitude b_m is close to the $q = 1$ value for the high-field phase (for the initial, low-field phase: $q = 0$). Correlation between the low-field phase and the solitonic states has been shown schematically in Fig. 4. A segment of horizontal line at $E = 0$, $h \leqslant 1$ corresponds to the low-field phase (if $\delta = 0$). At $h < h_{\mathrm{eq}}$, where $h_{\mathrm{eq}}^2 = 1 - K_2/2K_1$, this state is equilibrium, but at $h > h_{\mathrm{eq}}$ it becomes metastable. In turn, for each value of a magnetic field in the $h_{\mathrm{eq}} < h < 1$ range, we have a continuous spectrum of solitonic states differing by their amplitude, frequency of precession and energy. These states have been represented by several segments of vertical lines. Inclined dotted line at $h < h_{\mathrm{eq}}$ limits the energy values of BS states. In the range $h_{\mathrm{eq}} < h < 1$ the lines of BS states cross a line of the low-field phase, i. e., in this range at each value of the field we have a point of bifurcation, in which transition of the system to solitonic states is possible.

Let us notice that the BS states exist at small $(h-1) > 0$ values too, i. e., when the initial phase state is not stable absolutely. Hence, creation of BS's is possible at disintegration of the initial phase, when the system is absolutely unstable. Besides, in some range of values of a magnetic field the energy of such BS can be positive. It is visible in Fig. 2, for example, that at $h = 1.001$ the creation of solitons, having zero energy, is possible with two various frequencies of precession.

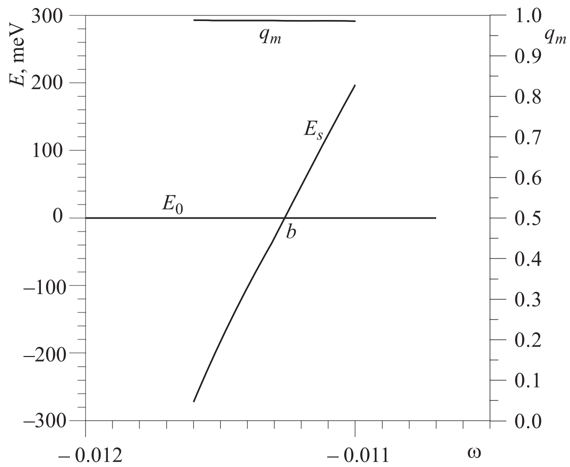

Fig. 3. Fragment of Fig. 2 for $h = 0.99$. Horizontal line at $E = 0$ corresponds to the energy $E_0 = 0$ of the low-field phase (it does not depend on ω value); $E_s(\omega)$ curve is the BS energy at $h = 0.99$; $q_m(\omega)$ curve is corresponding amplitudes of solitons

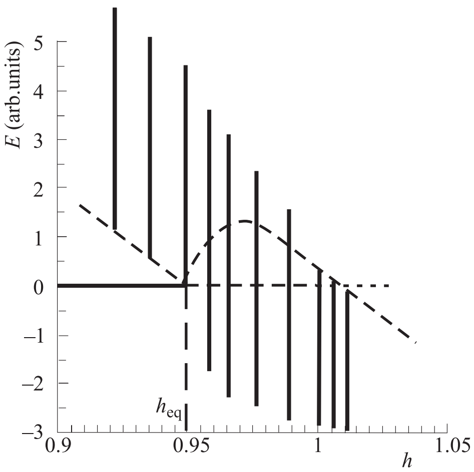

Fig. 4. Schematic representation of a correlation between the low-field state (the horizontal piece $E = 0$ at $h \leqslant 1$) and solitonic conditions (vertical pieces of lines). In the range $h_{\mathrm{eq}} < h < 1$ we have the bifurcation points, close to which BS may origin

Probability of the spontaneous creation of BS is related to the fluctuations of energy (and their configuration) at non-zero temperatures and is proportional to probability of these fluctuations. Probability of fluctuation in the equilibrium state of a system is expressed by Einstein formula

$$P_{\text{fl}} \approx \exp\left(\frac{\Delta S}{k_B}\right), \tag{15}$$

where $\Delta S < 0$ is the entropy change corresponding to the fluctuation. Applicability of this expression for fluctuations in non-equilibrium states has been postulated by I. Prigogine (see, for example, [9]) and strictly proved in [10]. Applying this expression for fluctuations in metastable state ($h_{\text{eq}} < h < 1$) of our magnetic system, i. e., around the bifurcation points, we can use the following expression for the probability of BS creation with the E_s energy:

$$P_s = A_1 \exp\left(\frac{-E_s}{k_B T}\right). \tag{16}$$

Here A_1 is the configuration coefficient in general depending on ω, E_s and configuration of a soliton. Generally, we have to use differing configuration coefficients for positive and negative energy of BS: A_{1+} for $E_s > 0$ and A_{1-} for $E_s < 0$. However, it can be assumed that close to the zero value of energy: $A_{1-} \cong A_{1+} = A_1$. Apparently, $A_{1+},\ A_{1-} << 1$.

In our case, formula (16) describes, first of all, the probability of appearance of precessing solitons with the specified energy. According to [10], formula (15)

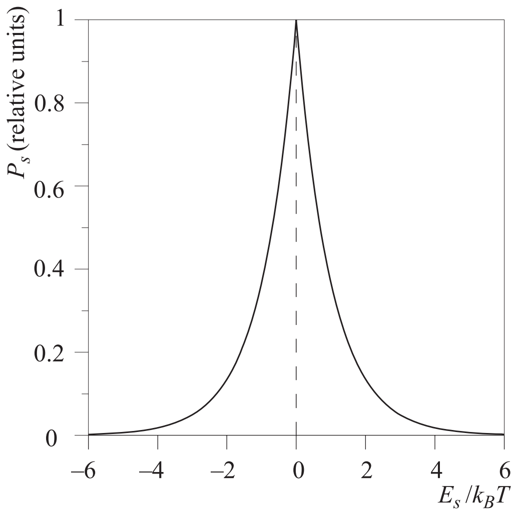

Fig. 5. Approximate dependence of probability of spontaneous BS creation on the energy near the point of bifurcation

can be used for probabilities in non-equilibrium conditions only under certain conditions, the feasibility of which in our case is provided that spontaneous appearance of BS does not lead to essential change of the low-field phase state since the BS volume is much less than the volume of a crystal, and the time, which is spent for its creation, is much less than the lifetime of solitons. At the same time, considering insignificant quantity of solitons, their small sizes in comparison with the volume of a crystal and slow evolution of solitons, it is possible to suppose, that formula (16) represents the distribution of solitons on the energy in quasi-equilibrium condition existing in an initial stage of phase reorganization. Exponential dependence of P_s on the BS energy has been shown in Fig. 5.

TIME EVOLUTION OF SOLITONS AND THE CHANGE OF THEIR FORM DURING THE MOVEMENT

At first, we consider the change of soliton because of the energy dissipation. Let us assume that BS of the $q\,(r,\,0)$ form corresponding to Eq. (14) has arisen. The dissipation of energy leads to the subsequent change of a BS configuration, in accordance with equations (9), (11) and (12) (at $\kappa = 0,\ \gamma \neq 0$; we suppose in (9) that $m_z << l_z$):

$$\frac{\partial q}{\partial \tau} \cong -k_1 \gamma \omega \left(\omega + h\right) q \left(1 - q^2\right). \tag{17}$$

The value

$$t_s \cong \frac{1}{\gamma k_1 \omega \left(\omega + h\right)} \frac{\hbar}{2\mu_B \sqrt{K_1 B}} \tag{18}$$

can be accepted as the evolution parameter of solitons, caused by the precession and energy dissipation. In Fig. 6 dependences of the $\omega^{-1}\left(\omega + h\right)^{-1}$ value and evolution parameter t_s on the precession frequency are shown. The t_s values

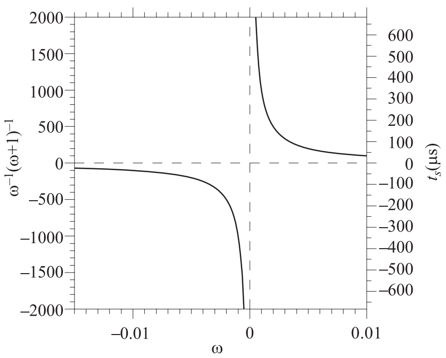

Fig. 6. Dependence of the $\omega^{-1}(\omega + h)^{-1}$ value and evolution parameter t_s on ω value at $h \cong 1$, $\gamma = 0.02$ [8], $K_1 = 700$ Oe, $B = 4.9 \cdot 10^6$ Oe

corresponding to Cr_2O_3 are given in the right scale. According to (17), the character of BS change is determined by a sign of the precession frequency. At $\omega > 0$ the solitons decrease and disappear, but at $\omega < 0$ they grow and turn to the macroscopic domains of the high-field phase.

If $h < h_{\mathrm{eq}}$, the BS states also exist, but the probability of creation of such solitons is extremely small (see Fig. 4). Besides, for these BS the frequency $\omega > 0$ and $t_s > 0$, therefore, they fade and disappear. If $h > h_{\mathrm{eq}}$, the parameter of evolution is positive too above the dotted line, but below this line it is negative, i. e., only these solitons (at $t_s < 0$) grow and turn to domains of the high-field phase.

For sufficiently small time values we have the following correlation (it follows from (17)):

$$\frac{dq}{q\,(1-q^2)} \cong -k_1\gamma\omega\,(\omega+h)\,d\tau. \tag{19}$$

Integrating the (19) correlation, we obtain

$$q\,(r,\tau) = \frac{q\,(r,0)\exp\left(-\tau/\tau_s\right)}{\sqrt{(1-q^2\,(r,0)) + q^2\,(r,0)\exp\left(-2\tau/\tau_s\right)}} = \\ = \frac{q\,(r,0)\exp\left(-t/t_s\right)}{\sqrt{(1-q^2\,(r,0)) + q^2\,(r,0)\exp\left(-2t/t_s\right)}}, \tag{20}$$

where $\tau_s \cong \left[\gamma k_1 \omega\,(\omega+h)\right]^{-1}$.

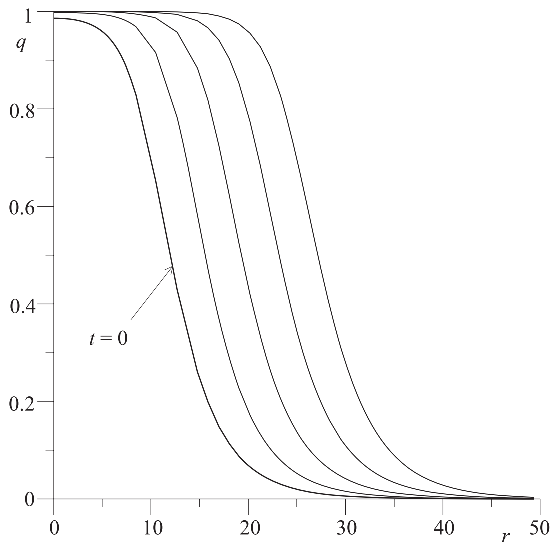

Fig. 7. Time change of the initial BS through $\Delta t = 30$ μs time intervals. Here $h = 0.99$, $\omega = -0.01126$, $\gamma = 0.02$, $K_1 = 700$ Oe, $E_s(t=0) = 1.325$ meV, $t_s = -30.8$ μs

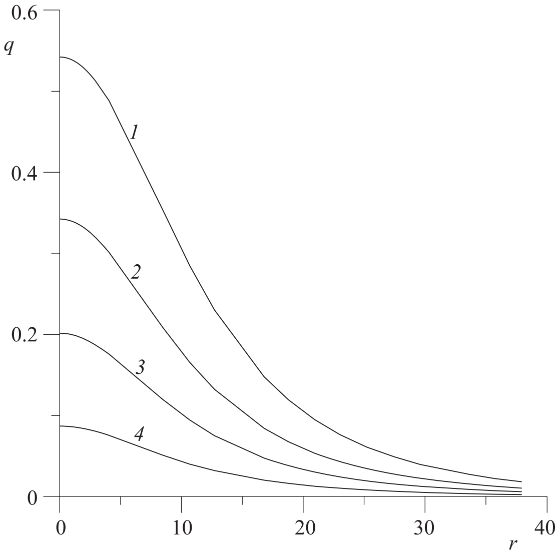

Fig. 8. Time change of the initial BS for the following time: *1* — 0; *2* — 200; *3* — 400; *4* — 700 μs. Here $h = 0.997$, $\omega = 0.001$, $\gamma = 0.02$, $K_1 = 700$ Oe, $E_s(t = 0) = 1113$ meV, $t_s = 350$ μs

Certainly, the (20) dependence can be used only at sufficiently small time values. This formula expresses only the tendency, direction for transformation of BS with $\omega < 0$ in macroscopic domain of the high-field phase and for disappearance of BS with $\omega > 0$. In Figs. 7 and 8 the time changes of two BS are shown (it can be noted that at least in Cr_2O_3 the spontaneous birth of BS with $\omega > 0$ is practically impossible).

As has been noted above, at the BS spontaneous birth a change of the entropy $\Delta S < 0$. At the subsequent evolution of a soliton due to dissipation the entropy is increasing. For BS with $\omega < 0$ the equilibrium state is the high-field phase.

Now, we consider the influence of movement of a soliton on its form. Using the (9), (11) and (12) expressions for $\gamma = 0$, $\delta = 0$, we obtain the following expression:

$$\frac{\partial q}{\partial \tau} = 2k_1 \left(\omega + h\right) \kappa q^2 \frac{\partial q}{\partial x}. \tag{21}$$

Let us present the $q\left(x, y, z, \tau\right)$ function as $q\left(x, y, z, \tau\right) = q_s\left(x_s, y, z, \tau\right)$, where $x_s = \left(x - v_0 \tau\right)$. In such a case we have

$$-v_0 \frac{\partial q_s}{\partial x_s} + \frac{\partial q_s}{\partial \tau} = 2k_1 \left(\omega + h\right) \kappa \frac{\partial q_s}{\partial x_s} - 2k_1 \left(\omega + h\right) \kappa \left(1 - q^2\right) \frac{\partial q_s}{\partial x_s}. \tag{22}$$

One can suppose that

$$v_0 = -2k_1 (\omega + h) \kappa \tag{23}$$

is the velocity of movement of a soliton as a whole. Then, the expression $\dfrac{\partial q_s}{\partial \tau} = -2k_1 (\omega + h) \kappa \left(1 - q^2\right) \dfrac{\partial q_s}{\partial x_s}$ or, in spherical coordinates,

$$\frac{\partial q_s}{\partial \tau} = v_0 \left(1 - q^2\right) \frac{\partial q_s}{\partial r_s} \sin\theta \cos\varphi \tag{24}$$

(here r_s is the radial coordinate in the system of moving soliton) describes the deformation of a soliton because of its movement along the x-axis. For our BS, $\dfrac{\partial q_s}{\partial r} < 0$. Consequently, the BS frontal side is decreasing, i. e., it becomes steeper at the movement of soliton, but the back side becomes sloper in the same extent. The BS, energy does not change during its movement.

CREATION OF LAYER-LIKE BS AND LAYER-BALL DOMAINS

In addition to the considered BS, «simple» solitons, there are other decisions of Eq. (14) (of the «matreshka» type). For comparison, in Fig. 9 the configurations of the first three BS are shown for $h = 0.999$, $\omega = 0$. Each change of the sign

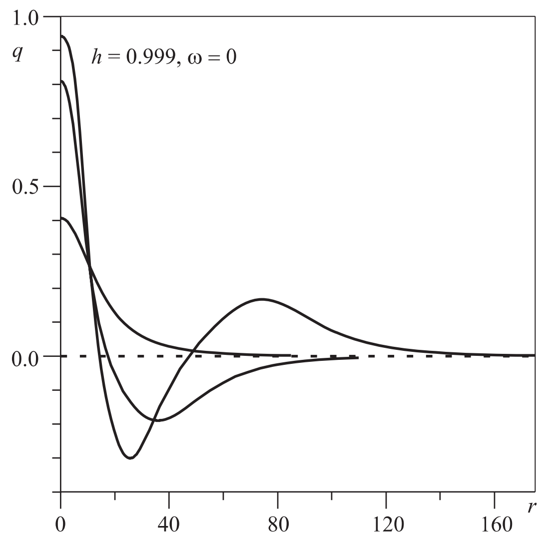

Fig. 9. Configurations of the «simple» ($n = 1$), «double» ($n = 2$) and «triple» ($n = 3$) solitons for $h = 0.999$, $\omega = 0$

of q corresponds to the $180°$ change of the precession phase. The energies and amplitudes of the «simple», «double» and «triple» solitons vs ω are shown in Fig. 10.

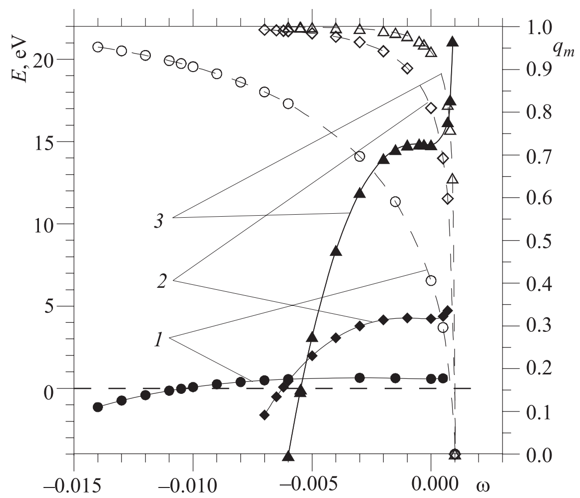

Fig. 10. Dependences of energies and amplitudes (latter ones are shown by dotted curves) in ω for «simple» (*1*), «double» (*2*) and «triple» (*3*) solitons at $h = 0.999$

As is seen from Fig. 10, the energy curves of these BS intersect the $E = 0$ line too. For probabilities of the «double» and «triple» solitons we can use the expressions similar to (16):

$$P_s^{\mathrm{dbl}} = A_2 \exp\left(\frac{-E_s^{\mathrm{dbl}}}{k_B T}\right), \tag{25}$$

$$P_s^{\mathrm{trpl}} = A_3 \exp\left(\frac{-E_s^{\mathrm{trpl}}}{k_B T}\right). \tag{26}$$

The A_n $(n = 1, 2, 3, ...)$ coefficients depend on the form of solitons and are a measure of complexity of their configuration. There are reasons to believe that $A_3 < A_2 < A_1 << 1$.

It is interesting to see the evolution of such solitons. The change of «double» and «triple» solitons are shown for $h = 0.999$ in Figs. 11 and 12.

From «double» BS the ball antiferromagnetic domain is formed with the radius of $R \cong 15\alpha^{0.5} K_1^{-0.5} \cong 1000$ Å, in which the antiferromagnetic vector is directed opposite to the antiferromagnetic vector of the basic volume of a crystal (in the real space this domain has a form of ellipsoid of rotation).

In the case of «triple» BS the ball antiferromagnetic domain with the radius of $R \cong 1000$ Å is surrounded by an envelope about 1000 Å of thickness, and

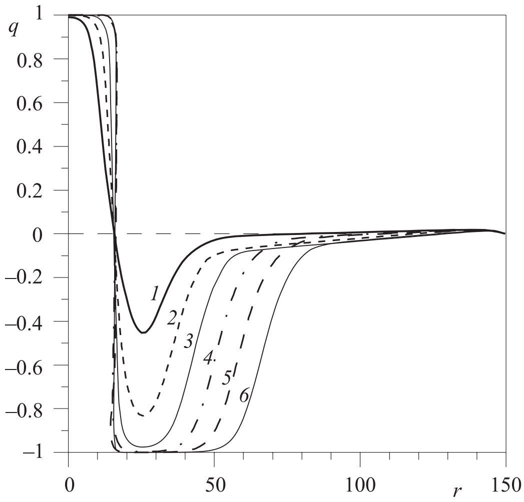

Fig. 11. Evolution of «double» soliton for $h = 0.999$, $\omega = -0.0062$ (in this case $E_s^{\mathrm{dbl}}(t = 0) = 73.3$ meV, $t_s = -55.54$ μs). Values of t are as follows: *1* — 0; *2* — 60; *3* — 120; *4* — 180; *5* — 240; *6* — 300 μs

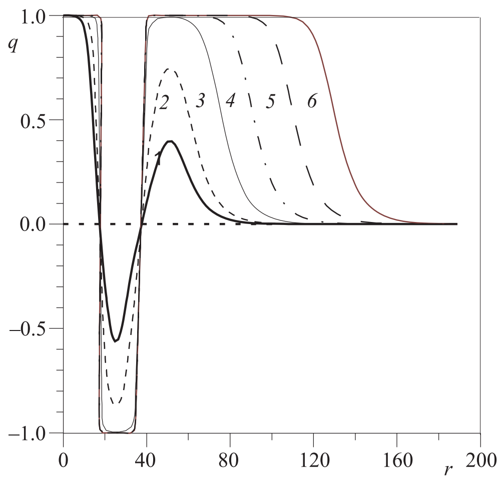

Fig. 12. Evolution of «triple» soliton for $h = 0.999$, $\omega = -0.00548$ (in this case $E_s^{\mathrm{dbl}}(t = 0) = -51.9$ meV, $t_s = -62.76$ μs). Values of t are as follows: *1* — 0; *2* — 60; *3* — 180; *4* — 300; *5* — 450; *6* — 600 μs

all that is in the great bulk of high-field antiferromagnetic state of a crystal. Accordingly, the signs of antiferromagnetic vectors alternate two times.

CONCLUSIONS

1. When an antiferromagnet at the spin-flop transition is in the metastable state, to each value of a magnetic field there corresponds a continuous spectrum of BS states, differing by the precession frequency and energy. These states include a wide range by amplitude, from zero up to the maximal value, equal to unit, and by energy, including negative values.

2. The BS probability increases anomalously when the energy is close to zero (relative to the energy of the initial low-field phase).

3. Character of the BS evolution depends on the sign of precession frequency. If $\omega > 0$, the solitons decrease by amplitude and disappear. If $\omega < 0$, BS grow continuously and become macroscopic domains of the high-field phase. If one excludes the influence of the «parasitic» centers on the process of the first order phase transition (such as clusters of impurity, domain walls, dislocations, boundaries with other environment), kinetics of the phase reorganization will be determined by the creation and evolution of BS. So, a new mechanism of the phase reorganization at the first order transition has been presented.

4. The creation of a more complex BS, the so-called «double» and «triple» solitons, is possible. The evolution of these solitons leads to the formation of «double» and «triple» magnetic domains with the symmetry of ellipsoid of rotation, correspondingly.

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