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TRACK RECONSTRUCTION IN THE CBM TRD

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A description, results and the current status of the track-finding routines developed for the Transition Radiation Detector (TRD) of the CBM experiment are presented. The track-finding algorithm is based on the Kalman Filter and track following methods. Two different approaches have been used: a stand-alone TRD track finder (using only TRD information) and an algorithm based on the information from tracks found in preceding detectors. Performances of the algorithms are presented. A detector layout study has been performed in order to optimize the detector set-up while keeping high reconstruction efficiency.

The investigation has been performed at the Laboratory of Information Technologies, JINR.
INTRODUCTION

In this note we give a description, results and the current status of the LIT track-finding routines which were developed for the Transition Radiation Detector (TRD) of the CBM experiment [1]. Two algorithms of track finding will be described: a stand-alone TRD track finder (using only TRD hit information) and an algorithm based on the information from tracks found in the Silicon Tracking System (STS).

The note is organized as follows: in Sec. 1 we introduce the CBM TRD detector, formulate the track-finding problem and give a general overview of the track reconstruction procedure in the TRD. In Sec. 2 the track-fitting procedure based on the Kalman Filter technique is described. Section 3 describes the two approaches for track finding in the TRD. The performance of the algorithm is presented in Sec. 4 as well as the results of the TRD layout study. A summary and an outlook are given in Conclusion.

1. THE TRANSITION RADIATION DETECTOR OF CBM

The CBM TRD is intended for tracking and identification of high-energy electrons and positrons which are used to reconstruct $J/\psi$ mesons. The geometrical representation of the TRD in the CBM software framework CBMROOT is described in [1]. The TRD is situated between the RICH and TOF detectors (see Fig. 1). It consists of several identical layers, each formed of different materials with a total thickness of 6 cm. The layers are grouped in three stations with 3 or 4 layers per station. These stations are placed at 5, 7 and 9 m downstream from the target. The TRD has a pad read-out with coordinate resolution 0.03–0.05 cm across and 0.27–3.3 cm along the pad depending on the distance from the beam. Pads are rotated on 90° from layer to layer.

The input for track reconstruction in the TRD is the hits in the detector layers. They are represented as three-dimensional space points. In case of simulated data, the hits are obtained from Monte-Carlo information by a digitization procedure. The TRD track finder groups several hits into TRD tracks, which are subsequently fitted to provide a precise estimate of the track parameters. For the evaluation of the track-finding performance, the found TRD tracks are matched to Monte-Carlo
The CBM set-up with the TRD detectors stations tracks using the correspondence of TRD hits to Monte-Carlo points. The flow chart of the TRD track reconstruction is shown in Fig. 2.

The TRD provides three-dimensional information (space points). The track reconstruction performance depends strongly on the quality of measurements in the TRD. The configuration and position of the detector stations are also crucial for successful track reconstruction. The track-finding algorithm has also to take into account disadvantages of the detector such as noise, inefficiencies, measurement errors, as well as the amount of detector material.

The main problems for tracking in the TRD are due to:
1) large track multiplicity;
2) measurement errors (up to 3.3 cm);
3) multiple scattering;
4) noise from secondary electron detection.

2. TRACK FIT

The goal of the track fit procedure is the most accurate estimation of the track parameters and their covariance matrix on the basis of the TRD measurements. The STS-based algorithm requires, in addition, the information from the STS detector.

2.1. Application of the Kalman Filter. The idea to consider a track as a collection of sequential measurements (hits) paved the way for the application of the Kalman Filter as a track fit procedure in high-energy physics, especially in
Fig. 2. The flow chart of the TRD track reconstruction
cases of substantial multiple scattering [2, 3]. The Kalman Filter is a recursive estimator, the principle of which is to add measurements one after another to the track bank, updating every time the track state on the current node. That means that only the estimated state from the previous node and the current measurement are needed to compute the estimate for the current state. This method is mathematically equivalent to the extended least-squares fit, which takes into account parameter covariances.

The Kalman Filter technique has many advantages. One of them is that it can be used for pattern recognition and is convenient for simultaneous track finding and fitting. It is remarkably fast, even in presence of a large number of measurements and notable multiple scattering due to its linearity and, mainly, because of its operating with low-dimensional matrices of parameters instead of multi-dimensional matrices of measurements. Another advantage is that after track fitting by Kalman Filter, one obtains not only the track-parameter estimates at the beginning and at the end of the trajectory, but also estimates along the whole trajectory of the particle, which closely follow the real path. Besides, it also provides a natural way to include process noise such as multiple scattering and energy losses.

We suppose a track can be represented by a straight-line segment tangent to the particle trajectory. The parameters of these segments formed the track state vector. It is determined by the $x$ and $y$ positions and tangent directions $t_x$ and $t_y$. We add to its components, furthermore, $q/p$, where $q$ — the particle’s charge and $p$ — its momentum, include the momentum measurement, obtained from the track curvature in the magnetic field. In the CBM experiment, it is natural to parameterize the track state as a function of $z$, since the produced tracks are strongly forward-focussed. Thus, we have five parameters, and the state vector is chosen as

$$
\mathbf{x} = \begin{pmatrix}
  x \\
  y \\
  t_x \\
  t_y \\
  \frac{q}{p}
\end{pmatrix},
$$

where $t_x = \frac{\partial x}{\partial z}$ and $t_y = \frac{\partial y}{\partial z}$. The corresponding errors form the covariance matrix $C$.

It is convenient to define the track states at the detector stations planes. The combination of a measurement (hit) and the track state vector on the corresponding detector layers is called a node.

The transport from node $k - 1$ to node $k$ is described by

$$
\mathbf{x}_k = f_k (\mathbf{x}_{k-1}) + \mathbf{w}_k,
$$

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where \( f_k \) is the track propagation function, and \( w_k \) is a process noise. The function \( f_k \) can be written as

\[
f_k (x_{k-1}) = F_k x_{k-1},
\]

where \( F_k \) is the transport matrix. For a straight track, which we suppose to have in TRD, it simplifies to

\[
F_k = \begin{pmatrix}
1 & 0 & \Delta z & 0 & 0 \\
0 & 1 & 0 & \Delta z & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix},
\]

where \( \Delta z = z_k - z_{k-1} \).

The relation between a measurement (hit) \( m_k \) and the state vector \( x_k \) is described by

\[
m_k = h_k (x_k) + e_k,
\]

where \( h_k \) is the projection function, and \( e_k \) is the measurement noise. The function \( h_k \) is defined as

\[
h_k (x_k) = H_k x_k,
\]

where \( H_k \) is the measurement matrix. As TRD directly measures \( x \) and \( y \) coordinates, \( H_k \) in this case is described by the \( 2 \times 5 \) matrix:

\[
H_k = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0
\end{pmatrix}.
\]

Process noise \( w_k \) and measurement noise \( e_k \) are unknown random disturbances assumed to have Gaussian distributions with zero expectation values. The corresponding covariance matrices for \( w_k \) and \( e_k \) are \( Q_k \) and \( V_k \). In case of TRD, which measures two-dimensional independent coordinates, \( V_k \) is a \( 2 \times 2 \) matrix:

\[
V_k = \begin{pmatrix}
\text{err}_x^2 & 0 \\
0 & \text{err}_y^2
\end{pmatrix},
\]

where \( \text{err}_x \) and \( \text{err}_y \) are the known measurement errors of the detector. The process noise \( Q_k \) will be discussed later.

The Kalman Filter has two distinct phases: Predict and Update. The predict phase uses the estimate of the track state from the previous node to produce an estimate of the current track state. On the update phase, measurement information from the current node is used to refine this prediction to obtain a new, more accurate estimate.
**Predict.** The prediction of the state vector and the corresponding covariance matrix at the $k$th node is obtained from the previous node, $k-1$, using the propagation relation:

$$x_k^{k-1} = F_k x_{k-1}^{k-1},$$

$$C_k^{k-1} = F_k C_{k-1}^{k-1} F_k^T + Q_k,$$

where $x_k^{k-1}$ and $C_k^{k-1}$ are the predicted state vector and its covariance matrix, respectively. The process noise $Q_k$ is added to the predicted covariance matrix $C_k^{k-1}$.

Residuals and their covariance matrix are given by

$$r_k^{k-1} = m_k - H_k x_k^{k-1},$$

$$R_k^{k-1} = V_k + H_k C_k^{k-1} H_k^T.$$

The contribution from this measurement to the total predicted $\chi^2$ is

$$(\chi^2_+)_k^{k-1} = (r_k^{k-1})^T (R_k^{k-1})^{-1} r_k^{k-1}.$$

The track fit requires an initial estimate of the track state $x_0$ in order to make the first prediction. This estimate is provided by the track-finding algorithm. For the TRD track-finding procedure itself, the estimation routine is described in Sec. 3.

**Update.** The track state is updated using the measurement (hit) information at the $k$th node. The filtered track state and its covariance matrix are

$$x_k = x_k^{k-1} + K_k r_k^{k-1},$$

$$C_k = (1 - K_k H_k) C_k^{k-1},$$

where the $5 \times 1$ gain matrix $K_k$ is obtained as

$$K_k = C_k^{k-1} H_k^T (V_k + H_k C_k^{k-1} H_k^T)^{-1} = C_k^{k-1} H_k^T (R_k^{k-1})^{-1}.$$

The filtered values for the residuals and corresponding covariances are

$$r_k = m_k - h_k (x_k) = (1 - H_k K_k) r_k^{k-1},$$

$$R_k = (1 - H_k K_k) V_k = V_k - H_k C_k H_k^T,$$

the contribution to the total filtered $\chi^2$ being

$$(\chi^2_+)_k = r_k^T R_k^{-1} r_k.$$
The prediction and filter steps are repeated until all measurements (hits) are added to the track. The last track state gives us the best estimation, which accumulates the information from all measurements of the track.

The alternation of prediction and filter steps are represented graphically in Fig. 3. It shows the scattering of a particle in the material between node $k-1$ and $k$. The track filter takes this into account by increasing the predicted error on the state vector $x_{k-1}^k$ by $Q_k$. Then, on the update step, the track state is being pulled to the true trajectory by being updated with the measurement (hit) information.

**Process Noise.** Energy losses are caused mainly by ionization during a charged particle’s passage through detector matter. The propagator has to take this into account for the momentum component of the state vector. The corresponding corrections are obtained from the Bethe–Bloch equation:

$$\frac{-dE}{dx} = 4\pi N_A e^2 m_e c^2 z Z A \frac{1}{\beta^2} \left[ \frac{1}{2} \ln \frac{2m_e c^2 \beta^2 T_{\text{max}}}{(1-\beta^2)} - \beta^2 - \frac{\delta}{2} \right],$$

using the approximation

$$-\frac{dE}{dx} = c_{\text{ion}} \frac{Z}{A},$$

where $c_{\text{ion}}$ includes all constant factors and is to be tuned in order to have no bias in the fitted momentum. The difference in energy before and after the material layer is given by

$$\Delta E = -c_{\text{ion}} \rho l \frac{Z}{A},$$

where $l$ is the distance traversed by the particle in the material with density $\rho$. This correction does not affect the covariance matrix.
A different correction is applied for electrons, as they lose energy by bremsstrahlung. In this case, the energy loss is calculated as
\[ \Delta E = -E \left( 1 - e^{-\frac{x}{X_0}} \right). \]

The covariance matrix has to be correspondingly corrected. The variance of the momentum in covariance matrix of process noise is given by
\[ Q_{55} = \left( \frac{q}{p} \right)^2 \left( e^{\frac{\ln 3}{X_0}} - e^{-2\frac{x}{X_0}} \right). \]

**Multiple Scattering.** Track fit routines have also to take into account the effect of multiple scattering, in particular for the relatively thick material layers of the TRD. The amount of multiple scattering is determined by the material which was passed by the particle. As multiple scattering is a random process, corrections affect only the process noise matrix \( Q \). They are slightly different for thin and thick material layers. For scattering in thin material layers, only variances of the direction components are taken into account, and are calculated as
\[ Q_{33} = (1 + t_x^2)(1 + t_y^2)\Theta_0^2, \]
\[ Q_{44} = (1 + t_y^2)(1 + t_x^2)\Theta_0^2, \]
\[ Q_{34} = t_x t_y(1 + t_x^2 t_y^2)\Theta_0^2, \]
where the projected scattering angle \( \Theta_0 \) is evaluated from
\[ \Theta_0 = \frac{13.6 \text{ MeV}}{\beta p c} \sqrt{-\frac{x}{X_0}} \left[ 1 + 0.038 \ln \frac{x}{X_0} \right]. \]

In this case, the process noise matrix has to be calculated in the center of the medium.

In a thick material layer, multiple scattering affects also the position elements \( x \) and \( y \). Hence, also the variances of the position elements have to be computed. The process noise in a layer of thickness \( \Delta z \) is therefore given by the symmetric matrix:
\[ Q(\Delta z) = \begin{pmatrix} Q_{33} \Delta^2 \frac{x}{X_0} & Q_{34} \Delta^2 \frac{x}{X_0} & Q_{34} \Delta^2 \frac{y}{X_0} & 0 \\ \vdots & Q_{44} \Delta^2 \frac{x}{X_0} & Q_{44} \Delta^2 \frac{y}{X_0} & 0 \\ \vdots & \vdots & Q_{33} & Q_{34} \Delta^2 \frac{y}{X_0} \\ \vdots & \vdots & \vdots & Q_{44} \Delta^2 \frac{y}{X_0} \\ \vdots & \vdots & \vdots & \vdots & 0 \end{pmatrix}. \]

One should note that in the case of scattering in thick layers the noise has to be added at the exit point of the material layer.

If a track passes through several layers of material, the state is extrapolated from one layer to the next.
2.2. Extrapolation of the State Vector and Covariance Matrix. The Kalman Filter requires a precise method to transport the state vector and its covariance matrix from an initial position $z_0$ to a new position $z$. Such a method has to take into account an inhomogeneous magnetic field, as present in the STS, and the detector materials.

The trajectory of a charged particle in the magnetic field satisfies the equation of motion caused by the Lorentz force:

$$\frac{dp}{dt} = Cqv \times B,$$

where $p$ in GeV/c is the momentum, $q$ (dimensionless) is the particle charge, $B$ in kGauss is the magnetic field fluence, $v$ in cm/s is the velocity and parameter $C$ in (GeV/c)kG$^{-1}$cm$^{-1}$ is proportional to the velocity of light equal to $C = 2.99792458 \cdot 10^{-4}$. One can show that the equation of motion could be written as

$$\frac{d}{dz} \begin{pmatrix} x \\ y \\ t_x \\ t_y \\ q/p \end{pmatrix} = \begin{pmatrix} t_x \\ t_y \\ C \cdot (q/p) \cdot A_x \\ C \cdot (q/p) \cdot A_y \\ 0 \end{pmatrix},$$

where $A_x$ and $A_y$ are defined by

$$A_x = \sqrt{1 + t_x^2 + t_y^2 (t_x t_y B_x - (1 + t_x^2) B_y + t_y B_z)},$$

$$A_y = \sqrt{1 + t_x^2 + t_y^2 ((1 + t_y^2) B_x - t_x t_y B_y - t_x B_z)}.$$

To transport the state vector one has to solve this ordinary differential equation. This was done by using the fourth-order Runge–Kutta method, where we implement the adaptive step-size control with step doubling [4].

The Kalman Filter application requires also the transportation of the covariance matrix, for which the evaluation of the derivatives of the state vector components with respect to their initial values is needed. This procedure is described in [5].

3. TRACK FINDING IN THE TRD

In the TRD one can point out two tracking approaches:

1. In the first one, named STS-based approach, one uses the information from the vertex detector STS, i.e., initially one has to reconstruct STS tracks, then propagate them through RICH detector to the first TRD layer and
extract necessary information for initializing TRD tracks from STS tracks. In this case, one has the initial direction of the particle and rather accurate estimation of the particle momentum, which is used to evaluate multiple scattering effects.

2. In the second, **stand-alone** approach, one assumes that there is no information from other detectors except TRD itself. In this case, the problem is more complicated, since the momentum and particle directions are unknown. One has to initiate the search somehow.

There is no magnetic field in TRD, therefore, one can consider a track as a straight line, although the influence of the stray magnetic field should be taken into account while a track is propagated from STS.

The flow chart of track-finding algorithm in the TRD is shown in Fig. 4. Both approaches are similar; the main difference is in the search initialization. Let us consider this procedure in detail.

For the STS-based approach, the initialization consists in setting up the TRD track parameters (state vector, covariance matrix, momentum, etc.) from parameters of STS tracks, which have to be found before, i.e., a bank of TRD tracks is to be created (the number of TRD tracks is the same as for found STS tracks). Then, STS track parameters are copied to the TRD track parameters and initialized TRD tracks are propagated to the first TRD layer, using Kalman Filter and also Runge–Kutta method (to take into account effects of the stray magnetic field).

For stand-alone version, initialization consists in creating track candidates and estimating rough track parameters. It can be done by using the information of the first TRD station (3 or 4 layers), i.e., one should fulfill the search through all possible combinations of track candidates, which could pass through hits on the first TRD station, in order to pick out acceptable ones for further track-following. This is done in the following way. Straight line parameters are calculated for each hit on the first TRD layer using the direction to the target region (point with coordinates $x = 0$, $y = 0$, $z = 0$) and then, using these parameters, the position of the track candidate on the next TRD layer is predicted:

$$x_{\text{pred}} = \left( \frac{x_0}{z_0} \right) \cdot z_1,$$

$$y_{\text{pred}} = \left( \frac{y_0}{z_0} \right) \cdot z_1,$$

where $x_0$, $y_0$, $z_0$ are hit coordinates on the first TRD layer and $z_1$ is $z$ position of the second layer. Next, corridors are set aside the predicted position independently in both views $XOZ$ and $YOZ$. Each corridor width is defined previously, using Monte-Carlo information (deviations between true track position and predicted
straight line parameters are recalculated with information from hits occurred in the corridor, and an intersection point with the next TRD layer is calculated using the view target region and hit on the second layer for the $O_X$ view, while for the $O_Y$ view the target and the hit on the first layer are used. This is caused due to the difference in measurement accuracies for $x$ and $y$ coordinates, which for the TRD are significantly alternated from layer to layer. Therefore, only coordinates which have an acceptable precision, are used. This procedure is repeated until the last layer in the first station is reached. After that, one has to estimate the initial parameters of the track candidates. If the number of layers in the first station
equals 4, the state vector is calculated in the following way:

\[ \mathbf{x} = \begin{pmatrix} a z_0 + b \\ y_0 \\ a \\ (y_2 - y_0) / (z_2 - z_0) \\ q/p \end{pmatrix}, \]

where \( a = (x_3 - x_1) / (z_3 - z_1) \), \( b = x_1 - a z_1 \), \( y_0, z_0, x_1, z_1, y_2, z_2, x_3, z_3 \) are hit coordinates, \( p \) is chosen according to the track-finding iteration between 0.6 and 1 GeV, and \( q = 1 \). If the number of layers is 3, then \( a \) and \( b \) are evaluated by the least-squares method, using \( x \) and \( z \) coordinates of the target region, the first, the second and the third hits. Diagonal elements of the covariance matrix are initially set with the large numbers.

After such an initialization procedure one obtains track candidates with known initial parameters. Further search consists of spatial following of the tracks through remaining stations. This is done in the following way. After the track propagation to the first layer in the next station, a corridor is set aside, whose value is computed on the basis of covariance matrix, evaluated in Kalman Filter, and measurement errors (assuming that covariance matrix and measurement errors are independent and normally distributed):

\[ dx = k_x \sqrt{\sigma_x^2 + \text{err}_x^2}, \]

\[ dy = k_y \sqrt{\sigma_y^2 + \text{err}_y^2}, \]

where \( dx \) and \( dy \) are corridors width in XOZ and YOZ accordingly, \( k_x \) and \( k_y \) are coefficients, which are chosen from 3 to 5, according to the track-finding iteration number, \( \sigma_x \) and \( \sigma_y \) are errors for \( x \) and \( y \) from the covariance matrix, \( \text{err}_x \) and \( \text{err}_y \) are measurement errors. Each track is propagated to the next layer, independently, for all hits appeared in the corridor. That is repeated until the last layer in the current station is reached. Then, all obtained track candidates are refitted with Kalman Filter (prediction + update) and accepted according to the following rule: if two or more tracks have the same hits, the track, which has the least \( \chi^2 \), is kept, others are rejected. After that, all accepted tracks are propagated to the next station and the procedure is repeated until the last station is achieved. If the total iteration number is more than one, then hits, belonging to tracks, found on the previous iteration, are deleted from hits array and the next iteration is executed, using only residual hits, although with wider corridors. As the output there is an array with found TRD tracks.
4. PERFORMANCE

Performance was tested with the following simulation parameters:

1) 1000 Au–Au collisions at 25 GeV;
2) CbmRoot release JUN06;
3) TRD geometry with
   a) 9 layers (3 × 3, 3 layers per station);
   b) 10 layers (4–3–3, 4 in the first station and 3 in the others);
   c) 12 layers (3 × 4, 4 layers per station);
4) first TRD station at 5 m;
5) active shielding magnetic field.

Efficiency in dependence on momentum is shown in Fig. 5.

![Fig. 5. Track-finding efficiency in dependence on particle momentum for standard TRD geometry with 12 layers for: a) STS TRD track finder; b) stand-alone TRD tracking](image)

A reconstructed track is regarded as correctly found, if it has more than 70% of true hits of the corresponding Monte-Carlo track.

The performance of the algorithm is presented in the table. «Reference» contains all tracks, which originate from the target region and have momentum more than 1 GeV. «All» are all reconstructable tracks. «Vetrex» — tracks which originate close to the target region. «Non-vertex» contains tracks which originate far from the target region. Wrongly found tracks are called «Ghost» and, if a track was found more than once, it is called «Clone».

One can see that 10 layers design looks as the most optimal from the cost and performance considerations, since there are:
### TRD track-finding efficiency

<table>
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<th>Stand-alone</th>
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<td>0.0</td>
</tr>
<tr>
<td>Time, s</td>
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<td>3.7</td>
</tr>
</tbody>
</table>

1) the best or the same efficiency for both track finders;

2) the best time consumption for stand-alone one;

3) the lowest ghost rate for both finders.

Such a result can be explained as follows. Four layers in the first TRD station give rather accurate calculation of the straight line parameters and because of the decreasing number of layers from 4 to 3 in the next stations, one gets less combinatoric, multiple scattering and noise from secondary electrons.

«3 × 3» geometry has the lowest time consumption for STS TRD track finder, but it is not suitable for stand-alone track finding.

### CONCLUSION

An efficient algorithm for the CBM TRD track reconstruction has been developed; it is based on two different approaches: a stand-alone track finder (using only TRD information) and an algorithm based on the information from vertex tracks.

After, a detector layout study has been performed in order to optimize the detector set-up while keeping high reconstruction efficiency. It was shown that the detector layout can be optimized. An alternative 10 layer detector layout was presented. Performance study of the track-finding algorithm for that set-up results in the best efficiency.

All developed algorithms were tested on large statistics of simulated events and then were included into the CBM framework for common use.
REFERENCES


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