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THE WARD–TAKAHASHI IDENTITIES TO DESCRIBE NUCLEON AND PION ELECTROWEAK TRANSITIONS

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Тождества Уорда–Такахаши в описании электрослабых переходов нуклонов и пионов

Для нуклонов и пионов получены соотношения между пропагаторами и вершинными функциями, описывающими векторные электрослабые переходы, как прямое следствие симметрий, сильных и электрослабых взаимодействий адронов. Существенно, что исследуемая система включает различные сильно взаимодействующие адроны. Электромагнитные поправки к вершинным функциям и пропагаторам адронов учитываются с точностью до e^2. Полученные выводы обсуждаются в связи с вычислением радиационных поправок в описании электрослабых переходов нуклонов и пионов.

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The Ward–Takahashi Identities to Describe Nucleon and Pion Electroweak Transitions

For nucleons and pions, the relations among the propagators and vertex functions to describe the vector electroweak transitions are acquired as immediate corollary of symmetries of the hadron strong and electroweak interactions. A point of value is that the considered system comprises strongly interacting hadrons of different sorts. The electromagnetic corrections to hadron vertex functions and propagators are taken into account up to e^2 order. The sequels are discussed in the light of calculation of the radiative corrections in describing the nucleon and pion electroweak transitions.

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INTRODUCTION

In these days, the investigations in the realm of elementary particles are usually associated with the high-energy physics on the modern super-accelerators. It is so just because the large momentum transfer is generally considered to be quite necessary to inquire into the structure of particles. Notwithstanding, thorough study of the low-energy electromagnetic and weak (EW) interactions of hadrons with the gauge fields, in particular the neutron and pion β-decay, is realized to be appropriate to gain an insight into the elementary particle physics. Such investigations can especially serve to check up conceivable deviations of experimental data from predictions of the Standard Model (SM) which is widely believed to be the real theory of elementary particles [1, 2]. That is why there exists the unwaning interest in studying the semileptonic processes [3]. Yet these differences between SM predictions and experimental measurements can never be expected to amount more than $\sim 1\%$. Consequently, the accuracy of experimental measurements and theoretical calculations has at least to be a few tenth of per cent in order to descry these feasible discrepancies, for otherwise they would be indiscernible. This high-precise study has also to be all-round comprising manifold characteristics of the considered phenomena. By now, the rates of the neutron and pion β-decay have been measured with the highest accuracy, $\sim 0.1\%$ [4, 5]. Other characteristics of these processes are believed to be obtained with the same precision in the relevant experiments before long [3, 6]. Then, in its own right, the theory is required to provide the respective trustworthy computations which ought to be correct at the 10^{-3}-level in order to try and make certain of SM validity. In the case that some ambiguities (for instance, any ad-hoc quantities such as the cut-off parameters and so on) slip in consideration, any uncertainties induced thereby must be elucidated explicitly.

In so far as the accuracy 10^{-2} or better goes, consistent allowance for the radiative corrections (RC) becomes of crucial value in the theoretical treatment of the electroweak processes involving hadrons. In calculating RC to the processes of hadron interaction with the electromagnetic field and to the semileptonic processes, such as the neutron and pion β-decay, one faces three rather different problems. The infra-red (IR) and ultra-violet (UV) divergences inherent in the RC computation are removed amenably to the method elaborated in Ref. [7], and pursuant to the on-mass-shell renormalization scheme [2, 8, 9] in the SM framework (see Refs. [10–12]). The third problem is how to allow for the hadron strong interaction and the hadron compositeness, intrinsic structure of the hadron. Although, in principle, it is just the SM where allowance for the hadron structure and strong interaction ought systematically to be carried out in a given electroweak process, the third problem still persists unsolved, remaining by now an unsettled computational challenge. An attractive and encouraging idea is to sidestep the immediate allowance for hadron compositeness with recourse to the certain general relations between the propagators, scattering amplitudes and vertex functions of hadrons interacting with the gauge fields. The way how to put this idea into effect could be perceived in the widely known work [13], with some kind of the Ward–Takahashi (WT) identities designed to treat the pure vector part of RC to the pion β-decay.

Utilizing the respective WT identities, first set forth in Ref. [13], is pivot of the ensuing calculations [14–16] of RC to the β-decay of hadrons. The manifold posterior papers (see, for instance, Refs. [17] and plenty of others) have merely been reasoning about the issues of the work [16]. Thus, to repose full trust and confidence in the results obtained in this approach, one ought first of all to acquire consistently the WT identities directly dictated by symmetries of the theory. In consequence, the respective WT identities to describe the vector interaction of nucleons and pions with the gauge fields are obtained (Sec. 3) as direct corollary of the global gauge invariance of the pure hadron lagrangian, that provides the total hadron vector current conservation (Sec. 1), and of the local gauge invariance of the lagrangian to describe hadron interactions with the electromagnetic and weak fields (Sec. 2). The evaluation of the electromagnetic corrections to the nucleon and pion vertex functions and propagators is carried out with the accuracy of order e^2. The outcome is discussed (Sec. 4), especially in correlating with what was asserted in Refs. [13–17] concerning the RC computation.

Even in mere deducing the WT identities themselves one gets to realize to what extent they are pertinent to bypass the immediate allowance for hadron compositeness in RC calculation. None the less, the inquiry into the WT identities is of principle value in its own right.

1. THE CONSERVED HADRON CURRENTS

Pursuing the general Lagrange method (see, for instance, Refs. [1, 2, 18, 20–22]), let

$$\{\phi(x)\} \equiv \{\phi_1(x_1), \phi_2(x_2), ...\} \quad \text{and} \quad \{\partial_\mu\phi(x)\} \equiv \{\partial_\mu\phi_1(x_1), \partial_\mu\phi_2(x_2), ...\} \tag{1.1}$$

be sets of generic hadron fields and their first derivatives, and the lagrangian

$$\mathcal{L}_h \equiv \mathcal{L}_h[\{\phi(x)\}, \{\partial_\mu\phi(x)\}] \tag{1.2}$$

describe a system of these strongly interacting fields ϕ which satisfy the Euler–Lagrange equations

$$\partial_\mu \frac{\partial \mathcal{L}_h}{\partial[\partial_\mu \phi_r(x)]} = \frac{\partial \mathcal{L}_h}{\partial \phi_r(x)}. \tag{1.3}$$

The canonically-conjugated momentum is defined

$$\Pi_r(x) = \frac{\partial \mathcal{L}_h}{\partial\Big(\partial_0 \phi_r(x)\Big)}, \tag{1.4}$$

and the equal-time (anti)commutation relations hold

$$\begin{aligned} &\phi_s(x)\phi_r(y) \pm \phi_r(y)\phi_s(x) = \Pi_s(x)\Pi_r(y) \pm \Pi_r(y)\Pi_s(x) = 0, \\ &\phi_s(x)\Pi_r(y) \pm \Pi_r(y)\phi_s(x) = i\delta_{sr} \cdot \delta(\mathbf{x} - \mathbf{y}), \quad x_0 = y_0. \end{aligned} \tag{1.5}$$

When the lagrangian (1.2) is invariant under a gauge transformation

$$\phi'_r(x) \Longrightarrow \phi_r(x) - i\varepsilon^a(x)\ell^a_{rs}\phi_s(x) \tag{1.6}$$

with constant infinitesimal parameters ε^a, i.e., under a global transformation, the corresponding hadronic currents

$$J^a_\mu(x) = \frac{\partial \mathcal{L}_h(x)}{\partial[\partial_\mu \varepsilon^a(x)]} = -i\frac{\partial \mathcal{L}_h(x)}{\partial[\partial_\mu \phi_r(x)]}\ell^a_{rs}\phi_s(x) \tag{1.7}$$

are conserved

$$\partial^\mu J^a_\mu(x) = 0. \tag{1.8}$$

The time components of the currents (1.7) read as

$$J^a_0(x) = -i\Pi_r(x)\ell^a_{rs}\phi_s(x) \tag{1.9}$$

and the commutators come out

$$[J^a_0(x), \phi_r(y)] = -\delta(x - y)\ell^a_{rs}\phi_s(x). \tag{1.10}$$

Let our general objective be studying the strangeness-conserving electroweak transitions of non-strange hadrons. The nucleon and the pion are only non-strange hadrons which are stable against strong decay modes. In this respect, the generic set of the hadronic fields $\{\phi_r(x)\}$ can be considered as consisting of a doublet of nucleon fields $\phi_r(x) \equiv \psi_N(x), N = n, p$, and a triplet of physical pion fields $\phi_r(x) \equiv \pi^a(x),\ a = 0, \pm$, defined as usual through the isovector field $\boldsymbol{\varphi} = \{\varphi_r\},\ \ r = 1, 2, 3,$

$$\pi^0(x) = \varphi_3(x), \quad \pi^\pm(x) = \frac{1}{\sqrt{2}}[\varphi_1(x) \mp i\varphi_2(x)]. \tag{1.11}$$

Strong interactions of these fields $\psi_N(x), \pi^a(x)$ ought to be taken into consideration in due course.

The conserved vector hadron currents are induced as we explore this isospin symmetric hadron system described by the lagrangian $\mathcal{L}_h$ (1.2). Invariance of the nucleon–pion lagrangian $\mathcal{L}_h(x)$ (1.2) under the global $SU(2)$ rotation of the fields

$$\psi'_N(x) \Longrightarrow \psi_N(x) - i\varepsilon_r \frac{\tau_{r\,NN'}}{2}\psi_{N'}(x)\,, \tag{1.12}$$

$$\varphi'_s(x) \Longrightarrow \varphi_s(x) - \epsilon_{\rm str}\varphi_t(x)\varepsilon_r\,, \tag{1.13}$$

$$N, N' = n, p, \quad r, s, t = 1, 2, 3$$

yields, amenably to the general Eqs. (1.6)–(1.8), the conserved hadron isovector current $\boldsymbol{\mathcal{J}}_\mu(x)\,,\ \{\mathcal{J}_{r\,\mu}(x)\}\,(r = 1, 2, 3)$, one current $\mathcal{J}_{r\,\mu}(x)$ of the triplet for each ε_r in Eqs. (1.12), (1.13). The lagrangian $\mathcal{L}_h(x)$ (1.2) of the considered system is also invariant under the global $U_Y(1)$ transformation, with the hypercharge $Y = 1$ for the nucleon isodoublet and $Y = 0$ for the pion isotriplet. Then, the invariance of $\mathcal{L}_h(x)$ under the nucleon field $U_Y(1)$ transformation

$$\psi'_N(x) \Longrightarrow \psi_N(x) - i\varepsilon_0 \mathbf{I}_{NN'}\psi_{N'}(x)\frac{Y}{2}, \ \ Y = 1, \tag{1.14}$$

yields the conserved neutral isoscalar current $\mathcal{J}_{0\,\mu}(x)$. The conserved currents $\mathcal{J}^\mu_r(x)$ and $\mathcal{J}^\mu_0(x)$ in the usual way serve to construct the neutral current

$$J^0_\mu(x) = \mathcal{J}_{3\,\mu}(x) + \mathcal{J}_{0\,\mu}(x), \tag{1.15}$$

and the charge transition currents

$$J^\pm_\mu(x) = \mathcal{J}_{1\,\mu}(x) \pm i\mathcal{J}_{2\,\mu}(x). \tag{1.16}$$

These conserved physical currents $J^0_\mu(x),\ J^\pm_\mu(x)$ simultaneously take their origin in the invariance of the lagrangian $\mathcal{L}_h(x)$ (1.2) under the global gauge transformation (1.6) of the doublet of nucleon fields $\psi_N(x)$ and the triplet of physical pion fields $\pi^a(x)$,

$$\psi'_N(x) \Longrightarrow \psi_N(x) - i\varepsilon^a \ell^a_{NN'}\psi_{N'}(x), \ \ a = 0, \pm, \ \ N, N' = n, p, \tag{1.17}$$

$$\pi'^r(x) \Longrightarrow \pi^r(x) - i\varepsilon^a \ell^a_{rs}\pi^s(x), \ \ a, s, r = 0, \pm, \tag{1.18}$$

with the constant matrices ℓ^a_{rs}

$$\ell^a_{NN'} = \frac{1}{2}\delta_{a0}(\mathbf{I}_{NN'} + \tau^0_{NN'}) + a^2\tau^a_{NN'}, \quad a = 0, \pm, \ \ N, N' = n, p, \tag{1.19}$$

$$\ell^+_{0-} = -\ell^+_{+0} = \ell^-_{-0} = -\ell^-_{0+} = \sqrt{2}, \quad \ell^0_{++} = -\ell^0_{--} = 1, \tag{1.20}$$

where $\tau^0 = \tau_3, \tau^\pm = (\tau_1 \pm i\tau_2)/2$ are the Pauli matrices; all the other elements of the ℓ-matrices are equal to zero. The notations (1.19), (1.20) prove to be convenient in the further evaluations. As one observes, the charged current $J^+_\mu(x)$ increases and $J^-_\mu(x)$ decreases electric charge of a hadron system by unity. $J^\pm_\mu(x)$ are the transition currents between states with charges differing by one. They actually occur in treating the β-decay of hadrons. The neutral current $J^0_\mu(x)$, the combination of the third component of the isovector current and the isoscalar current, is rightly understood to be the electromagnetic current, which stands to describe interaction of a hadronic system with an electromagnetic field. So, hereafter any current with the upper index 0 is implied to be the electromagnetic current, $J^0_\mu(x) \equiv J^{\rm em}_\mu(x)$.

With having recourse to the SM concepts [1, 2, 8, 9, 22], the third component $\mathcal{J}^\mu_3(x)$ of the isotriplet current $\boldsymbol{\mathcal{J}}^\mu(x)$ and the isoscalar current $\mathcal{J}^\mu_0$ can still be combined giving, besides $J^{\rm em}_\mu(x)$, the conserved neutral weak current

$$J^Z_\mu(x) = \mathcal{J}_{3\,\mu}(x)\big(1 - 2s^2_W\big) - \mathcal{J}_{0\,\mu}(x)s^2_W, \;\; s^2_W = 1 - \frac{M^2_W}{M^2_Z}, \tag{1.21}$$

with the coefficients chosen so as this current (1.21) serves to describe the feasible hadron transitions in the neutral Z-boson field [1, 2, 22]. This conserved current J^Z_μ is due to $\mathcal{L}_h(x)$ (1.2) invariance under the global gauge transformation (1.17), (1.18) with $a \to Z$ and

$$\ell^Z_{NN'} = -\frac{1}{2}s^2_W + \frac{1}{2}\tau^0_{NN'}\big(1 - 2s^2_W\big), \tag{1.22}$$

$$\ell^Z_{rs} = \ell^Z_{++} = -\ell^Z_{--} = \big(1 - 2s^2_W\big). \tag{1.23}$$

Here M_Z, M_W are the Z- and W-boson masses.

It is right off to stress that just the very total hadron currents $\mathcal{J}^\mu_r(x), \mathcal{J}^\mu_0(x)$, $J^0_\mu(x) \equiv J^{\rm em}_\mu(x)$, $J^\pm_\mu(x)$, $J^Z_\mu(x)$ are conserved, and not the currents of nucleons and pions separately, as we consider the system of interacting hadrons. The lagrangian $\mathcal{L}_h(x)$ (1.2) is to be invariant under the simultaneous gauge transformation of nucleon and pion fields. And it can not be required to be invariant under the transformation (1.12) or (1.13) of nucleon and pion fields separately.

A particular form of the conserved current $J^a_\mu(x)$ is specified by a concrete form of $\mathcal{L}_h$ (1.2), provided it is invariant under the transformations (1.12)–(1.14), (1.17)–(1.20), (1.22), (1.23). This lagrangian is put into the usual form

$$\mathcal{L}_h(x) = \mathcal{L}_N(x) + \mathcal{L}_\pi(x) + \mathcal{L}^{\rm int}_{\rm str}(x), \tag{1.24}$$

where

$$\mathcal{L}_N(x) = i\bar{\psi}_N(x)[\gamma^\mu\partial_\mu - M_N]\psi_N(x), \tag{1.25}$$

$$\mathcal{L}_\pi(x) = \frac{1}{2}[\partial_\mu \boldsymbol{\varphi} \partial^\mu \boldsymbol{\varphi} - m^2 \boldsymbol{\varphi}\boldsymbol{\varphi}] = \partial_\mu \pi^+ \partial^\mu \pi^- - m^2 \pi^+ \pi^- + \\ + \frac{1}{2}\partial_\mu \pi^0 \partial^\mu \pi^0 - \frac{1}{2} m^2 \pi^{02}, \quad (1.26)$$

and we choose, for the sake of definiteness, the hadron field interaction in the widely applied expedient form

$$\mathcal{L}^{\rm int}_{\rm str} = g_{NN'\pi} \bar{\psi}_{N'}(x)\gamma^5 \Big(\tau^0 \pi^0(x) + \sqrt{2}\pi^+(x)\tau^+ + \sqrt{2}\pi^-(x)\tau^-\Big)\psi_N(x) - \\ - \frac{\lambda}{4}\big(\pi^a \pi^{-a}\big)^2, \quad (1.27)$$

so as the total hadron lagrangian is invariant under the global gauge transformations (1.12)–(1.14), (1.17)–(1.20), (1.22), (1.23), and the total hadron currents are conserved. These total hadron conserved currents are then written as the sum

$$J^a_\mu(x) = j^a_{N\,\mu}(x) + j^a_{\pi\,\mu}(x), \quad a = 0, \pm, Z, \qquad (1.28)$$

of the nucleon

$$j^a_{N\,\mu}(x) = \bar{\psi}_N(x)\gamma_\mu \ell^a_{NN'} \psi_{N'}(x), \quad N, N' = n, p, \qquad (1.29)$$

and pion

$$j^a_{\pi\,\mu}(x) = -i\ell^a_{rs} \partial_\mu \pi^{(-r)}(x) \pi^s(x), \quad r, s = 0, \pm, \qquad (1.30)$$

currents. In the formulae (1.25)–(1.30) and in the akin expressions hereafter, the $\mathcal{N}$-products of field operators are implied.

So far the lagrangian (1.24) involves pure strong interactions, the masses of the members of a certain isomultiplet are equal in the expressions (1.25)–(1.27). Distinctions between the members of an isomultiplet are on account of the hadron interaction with the electromagnetic field introduced afterwards in due course.

As said above, the upper index $a = 0$ by any field and current operators means that they are operators of the electromagnetic field and current. When the multiplier a occurs in any formulae, it is implied having got the value $a = 0$ or $a = \pm$, corresponding to the elecrtomagnetic, J^0_μ, or transition, $J^\pm_\mu$, current with which a comes in conjunction.

In actual fact, any strong hadron interactions are allowable in place of the suggested $\mathcal{L}^{\rm int}_{\rm str}$ (1.27), in particular the interactions generally received in the chiral perturbation theory [23], provided the lagrangian $\mathcal{L}_h$ (1.24) remains invariant under the global gauge transformations (1.12)–(1.14), (1.17)–(1.20), (1.22), (1.23). A distinct form of $\mathcal{L}^{\rm int}_{\rm str}$ does not matter for our further treatment. In the same light, although the expressions (1.24)–(1.27) incorporate explicitly the genuine nucleon and pion fields solely, this lagrangian $\mathcal{L}_h$ (1.24) is in general certain to describe a variety of non-stable hadrons, hadron resonances, with consistent allowance for expedient strong interactions $\mathcal{L}_{\rm str}^{\rm int}$. Thus, having recourse to the expressions (1.24)–(1.27) does not spoil generality of our treatment, in point of actual fact.

We deal with the Heisenberg field operators

$$\phi_r(x) = \mathcal{S}_{\rm str}^+(x_0)\phi_{0\,r}(t,\mathbf{x})\mathcal{S}_{\rm str}(x_0), \tag{1.31}$$

where $\phi_{0\,r}(x)$ stand for field operators in the interaction representation, and the matrix $\mathcal{S}_{\rm str}$ is dictated by $\mathcal{L}_{\rm str}^{\rm int}(x)$:

$$\mathcal{S}_{\rm str}(x_0) = \mathcal{T}\exp\ [i\int\limits_{-\infty}^{x_0} {\rm d}t \int {\rm d}\mathbf{x}\mathcal{L}_{\rm str}^{\rm int}(t,\mathbf{x})], \tag{1.32}$$

with the time ordering operator $\mathcal{T}$. This $\mathcal{L}_{\rm str}^{\rm int}$ incorporates all the sorts of interacting hadrons which are involved into consideration. In the case specified by (1.24)–(1.27), those are nucleons and pions. It stands to reason that the nucleon field operators $\psi_N(x)$ occur in the pion current $j^a_{\pi\,\mu}(x)$ (1.30), and vice versa the pion field operators $\pi^r(x)$ occur in the nucleon current $j^a_{N\,\mu}(x)$ (1.29). Then, a matrix element of $j^a_{\pi\,\mu}(x)$ between pure nucleon states, $\langle N_f \mid j^a_{\pi\,\mu}(x) \mid N_i\rangle$, as well as a matrix element of $j^a_{N\,\mu}(x)$ between pure pion states, $\langle \pi_f \mid j^a_{N\,\mu}(x) \mid \pi_i\rangle$, does not vanish in the general case.

It is here of crucial value to emphasize once more that only the total current $J^a_\mu(x)$ (1.28) is conserved, when we deal with a system of interacting hadrons of different kinds, nucleons and pions in the case considered. The nucleon and pion currents $j^a_{N\,\mu}$ (1.29) and $j^a_{\pi\,\mu}$ (1.30) themselves are not conserved separately, as strong interactions $\mathcal{L}_{\rm srt}^{\rm int}(x)$ are involved into $\mathcal{L}_h(x)$.

2. HADRON INTERACTION WITH THE GAUGE FIELDS

Now we are to construct the electroweak lagrangian $\mathcal{L}_{\rm int}^{\rm EW}(x)$ to describe interaction of the genuine hadron fields ϕ_r in $\mathcal{L}_h$ (1.2) with the electromagnetic field $\mathcal{A}_\mu^{\rm em}(x)$, and with the charged $\mathcal{A}_\mu^\pm(x)$ and neutral $\mathcal{A}_\mu^Z(x)$ fields associated with the $W^\pm$- and Z-boson fields. It originates from requirement of $\mathcal{L}_h$ (1.2) invariance under a local (i.e., with space–time dependent parameters $\varepsilon(x)$) gauge transformation (1.6).

In order for the local gauge transformation (1.6), displaced for the considered case by Eqs. (1.12), (1.13), (1.17)–(1.20), (1.22), (1.23) with the space–time dependent parameters $\varepsilon(x)$, to be an invariance of the lagrangian $\mathcal{L}_h(x)$ (1.2), (1.24), the derivatives $\partial_\mu\phi_i(x)$ in $\mathcal{L}_h(x)$ are known (see, for instance, [1, 2, 18, 20, 21]) to be replaced by the respective covariant derivatives $\mathcal{D}_\mu(x)$ that read for our consideration as

$$\partial_\mu \phi_s(x) \Longrightarrow \mathcal{D}_\mu \phi_s(x) = \partial_\mu \phi_s(x) + ie\mathcal{A}^a_\mu(x)\ell^a_{sr}\phi_r(x), \tag{2.1}$$

where the coefficients ℓ^a_{sr} are given by Eqs. (1.19), (1.20), (1.22), (1.23), and $\mathcal{A}^a_\mu(x)$, with $a = 0, \pm, Z$, are the respective gauge fields. Then the lagrangian $\mathcal{L}^{\rm EW}_{\rm int}$ results in terms of the hadron currents $J^a_\mu(x)$ (1.28) and these fields $\mathcal{A}^a_\mu(x)$. They are known to be related to the physical fields $\mathcal{A}^{\rm em}_\mu(x)$, $W^\pm_\mu(x)$, $Z_\mu(x)$ and the effective coupling constants by the equations

$$\mathcal{A}^0(x) \equiv \mathcal{A}^{\rm em}_\mu(x), \quad e\mathcal{A}^\pm_\mu(x) = \frac{\sqrt{G}M_W}{2^{1/4}}W^\pm_\mu, \quad e\mathcal{A}^Z_\mu(x) = \frac{\sqrt{G}M_Z}{2^{1/4}}Z_\mu, \tag{2.2}$$

with the Fermi interaction constant G, so that to provide the true effective description of nucleon and pion interaction with the electromagnetic and weak fields [1, 2, 18, 23]. It is expedient to treat linear interactions of hadrons with the external fields $\mathcal{A}^\pm_\mu(x)$, $\mathcal{A}^Z_\mu(x)$, which are defined in Eq. (2.2), and alike with the external electromagnetic field $A^{(e)}_\mu(x)$, separated explicitly from $\mathcal{A}^{\rm em}_\mu(x)$,

$$\mathcal{A}^{\rm em}_\mu(x) = A^{(e)}_\mu(x) + A^{\rm em}_\mu(x), \tag{2.3}$$

so as $A^{\rm em}_\mu(x)$ remains to describe the quantum electromagnetic field.

What eventually counts is the effective lagrangian to explore the electromagnetic corrections in describing the nucleon and pion β-decay, and their transitions in external electromagnetic and neutral Z-boson fields

$$\tilde{\mathcal{L}}^{\rm EW}_{\rm int}(x) = \tilde{L}^{\rm em}(x) + \tilde{L}^{1W}(x) + \tilde{L}^{W{\rm em}}(x) + \tilde{L}^{1Z}(x) + \tilde{L}^{Z{\rm em}}(x), \tag{2.4}$$

$$\tilde{L}^{\rm em}(x) = \tilde{L}^{1{\rm em}}(x) + \tilde{L}^{2{\rm em}}(x), \tag{2.5}$$

$$\begin{aligned}\tilde{L}^{1{\rm em}}(x) = -e\mathcal{A}^0_\mu(x)\tilde{J}^{0\,\mu}(x) = -e\mathcal{A}^{\rm em}_\mu(x)\tilde{J}^{{\rm em}\,\mu}(x) = \\ = -e\tilde{J}^{0\,\mu}(x)\Big(A^{(e)}_\mu(x) + A^{\rm em}_\mu(x)\Big),\end{aligned} \tag{2.6}$$

$$\begin{aligned}\tilde{L}^{2{\rm em}}(x) = e^2\mathcal{A}^{\rm em}_\mu\mathcal{A}^{{\rm em}\,\mu}\tilde{\pi}^-(x)\tilde{\pi}^+(x) \approx \\ \approx e^2 A^{\rm em}_\mu A^{{\rm em}\,\mu}\tilde{\pi}^+(x)\tilde{\pi}^-(x) - e^2 A^{(e)\,\mu}(x)\tilde{\jmath}^0_\mu(x),\end{aligned} \tag{2.7}$$

$$\tilde{\jmath}^a_\mu(x) = A^{\rm em}_\mu(x)\tilde{\pi}^s(x)\tilde{\pi}^r(x)\ell^a_{dr}\ell^0_{(-d)s}, \quad d, s, r = 0, \pm, \quad a = 0, \pm, Z, \tag{2.8}$$

$$\tilde{L}^{1W}(x) = -e\mathcal{A}^{a\,\mu}(x)\tilde{J}^a_\mu(x) = -\frac{\sqrt{G}M_W}{2^{1/4}}\tilde{J}^a_\mu(x)W^{a\,\mu}(x), \quad a = \pm, \tag{2.9}$$

$$\begin{aligned}\tilde{L}^{W{\rm em}}(x) = -e^2\mathcal{A}^0_\mu(x)\mathcal{A}^{a\,\mu}(x)\tilde{\pi}^s(x)\tilde{\pi}^r(x)\ [\ell^a_{dr}\ell^0_{(-d)s}] \approx \\ \approx -e^2\tilde{\jmath}^a_\mu(x)\mathcal{A}^{a\,\mu}(x) \approx -e\tilde{\jmath}^a_\mu(x)\frac{\sqrt{G}M_W}{2^{1/4}}W^{a\,\mu}(x), \quad a = \pm,\end{aligned} \tag{2.10}$$

$$\tilde{L}^{1Z}(x) = -\frac{\sqrt{G}M_Z}{2^{1/4}}\tilde{J}^Z_\mu(x)Z^\mu(x), \tag{2.11}$$

$$\tilde{L}^{Z\text{em}}(x) \approx -e\tilde{\jmath}^Z_\mu(x)\frac{\sqrt{G}M_Z}{2^{1/4}}Z^\mu(x). \tag{2.12}$$

All the interactions quadratic in the external fields $A^{(e)}(x)$, $\mathcal{A}^{\pm}_\mu(x)$, $\mathcal{A}^Z_\pm(x)$ are abandoned in $\mathcal{L}^{\text{EW}}(x)$ (2.4). As the interactions $\tilde{L}^{W\text{em}}(x)$, $\tilde{L}^{Z\text{em}}(x)$, (2.10), (2.12) explicitly involve the external fields $\mathcal{A}^{\pm}_\mu(x)$, $\mathcal{A}^Z_\mu(x)$, the field $\mathcal{A}^{\text{em}}_\mu(x)$ is therein replaced by the pure quantum field $\hat{A}^{\text{em}}_\mu(x)$, omitting the external field $A^{(e)}_\mu(x)$. Retaining the terms linear in the external field $A^{(e)}_\mu(x)$, last rewriting Eq. (2.7) gets clear. The contribution of the gauge fields $\mathcal{A}^a_\mu(x)$ themselves $\mathcal{L}_{\text{gauge}}(\mathcal{A}^a, \partial_\mu\mathcal{A}^a)$ into the lagrangian of the treated system will not occur in the further consideration, therefore there is here no need to plunge into its construction and further to add $\mathcal{L}_{\text{gauge}}(\mathcal{A}^a, \partial_\mu\mathcal{A}^a)$ to the total lagrangian $\mathcal{L}_{\text{tot}}$ (2.13). For completeness' sake, we have rewritten Eqs. (2.6)–(2.12) through $\mathcal{A}^{\text{em}}_\mu(x)$, $W^{\pm}_\mu(x)$, $Z_\mu(x)$ and the respective coupling constants, though the particular form of the coefficients in these expressions does not matter in further acquiring the desirable WT identities.

The interactions $\tilde{L}^{1Z}(x)$, $\tilde{L}^{Z\text{em}}(x)$, (2.11), (2.12) are observed to be of the same contents as the interactions $\tilde{L}^{1W}(x)$, $\tilde{L}^{W\text{em}}(x)$, (2.9), (2.10) are, with mere replacing $W^{\pm}(x)$, $J^{\pm}_\mu(x)$, $\jmath^{\pm}_\mu(x)$ by $W^Z(x)$, $J^Z_\mu(x)$, $\jmath^Z_\mu(x)$. Therefore, in order to avoid superfluous writing we treat hereafter the hadron β-decay and the hadron transitions due to interaction with an external electromagnetic field, and we shall further leave out the feasible hadron transitions caused by interactions with the external neutral weak Z-boson field. All the more that these transitions would be actual when the large momenta, $k^2 \sim M^2_Z$, were transferred thereby. Yet this is rather not the case we are interested in for now.

As the total lagrangian to describe the considered system is the sum

$$\mathcal{L}_{\text{tot}}(x) = \tilde{\mathcal{L}}_h[\{\tilde{\phi}\}, \{\partial_\mu\tilde{\phi}\}] + \tilde{\mathcal{L}}^{\text{EW}}_{\text{int}}[\{\tilde{\phi}\}, \{\partial_\mu\tilde{\phi}\}, \{\mathcal{A}^a\}], \tag{2.13}$$

the dependence of all the Heisenberg field operators $\tilde{\phi}_i(x)$ on time, i.e., on x_0, is just determined by this $\mathcal{L}_{\text{tot}}(x)$ (2.13). The sign «tilde», $\sim$, over the hadron Heisenberg field operators and currents in Eqs. (2.4)–(2.12), as well as over all the operators hereafter, designates the additional time dependence caused by $\tilde{\mathcal{L}}^{\text{EW}}(x)$. Let us recall that, as has been mentioned after Eq. (1.24), the $\mathcal{N}$-products of field operators are implied in expressions (2.4)–(2.12), as well as everywhere in the akin formulae.

We purpose to calculate the electromagnetic corrections to the hadron currents, propagators, and vertex functions describing interaction of hadrons with the external gauge fields $A^{(e)}_\mu(x)$, $W^{\pm}_\mu(x)$. The needful expression of the hadron

Heisenberg field operators $\tilde{\phi}_i(x)$ is given in the usual way [22]:

$$\tilde{\phi}_i(x) = S^+_{\rm em}(x_0)\phi_i(x)S_{\rm em}(x_0), \qquad (2.14)$$

$$S_{\rm em}(x_0) = \mathcal{T}\cdot\exp\biggl(i\int\limits_{-\infty}^{x_0} {\rm d}\tau\int {\bf dx}L^{\rm qem}(\tau,{\bf x})\biggr), S_{\rm em}(\infty)\equiv S_{\rm em}, \qquad (2.15)$$

in terms of the hadron interaction with the quantum electromagnetic field $A^{\rm em}_\mu(x)$,

$$L^{\rm qem}(x) = -eJ^{0\,\mu}(x)A^{\rm em}_\mu(x) + e^2A^{\rm em}_\mu(x)A^{\rm em\,\mu}(x)\pi^+(x)\pi^-(x), \qquad (2.16)$$

as seen from Eq. (2.5)–(2.7). Consequently, with allowance for the general equation for S-matrix, one gets

$$\partial_\mu\tilde{\phi}_i(x) = S^+_{\rm em}(x_0)\partial_\mu\phi_i(x)S_{\rm em}(x_0)+ \\ + ig_{\mu 0}S^+_{\rm em}(x_0)\ \ [\phi_i(x),\int {\bf dy}L^{\rm qem}(x_0,{\bf y})]S_{\rm em}(x_0). \qquad (2.17)$$

Then, the current $\tilde{J}^a_\mu(x)$ (1.28) that occurs in Eqs. (2.6), (2.9) proves to be put into the form

$$\tilde{J}^a_\mu(x) = S^+_{\rm em}(x_0)J^a_\mu(x)S_{\rm em}(x_0) - eg_{\mu 0}\tilde{\jmath}^a_0(x), \qquad (2.18)$$

where the first order e-dependence is explicitly set forth in the second term.

To ascertain the desirable WT identities we are to calculate the current divergence

$$\partial^\mu\tilde{J}^a_\mu(x) = S^+_{\rm em}(x_0)\partial^\mu J^a_\mu(x)S_{\rm em}(x_0)+ \\ + iS^+_{\rm em}(x_0)\ \ [J^a_0(x),\int {\bf dy}L^{\rm qem}(x_0,{\bf y})]S_{\rm em}(x_0) - e\partial^0\tilde{\jmath}^a_0(x). \qquad (2.19)$$

As the current $J^a_\mu(x)$ (1.28) is conserved, see Eq. (1.8), the first term here disappears. Then, with the respective application [20–22] of Eqs. (1.5), (1.9), (1.10) for field operators and their derivatives, the divergence (2.19) directly transforms to

$$\partial^\mu\tilde{J}^a_\mu(x) = ieaA^{\rm em}_\nu(x)S^+_{\rm em}(x_0)J^{a\,\nu}(x)S_{\rm em}(x_0) - e\partial^\nu\tilde{\jmath}^a_\nu(x)+ \\ + ie^2aA^{\rm em\,\nu}(x)\tilde{\jmath}^a_\nu(x),\ a=0,\pm, \qquad (2.20)$$

in place of Eq. (1.8). As one might behold, Eq. (2.20) could be said to be, in a way, in line with the theorem asserted in Ref. [25]. The last term in Eq. (2.20) is due to the interaction $\tilde{L}^{2\rm em}$ (2.7). The second term in the divergence (2.20) could be actually associated with the so-called Schwinger terms which are known to occur in the current algebras approach, see, for instance, Refs. [20, 21]. Let us remark that this term turns out to be determined by the divergence $\partial^\mu \tilde{\jmath}^a_\mu(x)$ of that, in a manner of speaking, «current» $\tilde{\jmath}^a_\mu(x)$ which determines the interactions $\tilde{L}^{W\mathrm{em}}(x), \tilde{L}^{2\mathrm{em}}(x)$, (2.10), (2.7), alike the current $\tilde{J}^a_\mu(x)$ determines the interactions $\tilde{L}^{1\mathrm{em}}(x), \tilde{L}^{1W}(x)$, (2.6), (2.9). This point proves to be of value in the presented treatment.

In order to acquire the WT identities, we consider the vacuum expectation value

$$\tilde{\mathcal{K}}^{afi}_\mu(\{\phi\}x, y, z) = \langle 0 \mid \mathcal{T}[\tilde{J}^a_\mu(x)\tilde{\phi}_f(y)\tilde{\phi}_i(z)] \mid 0\rangle \tag{2.21}$$

of the time ordered product of the Heisenberg field operators $\tilde{\phi}_i(x)$ and the currents $\tilde{J}^a_\mu(x)$ (1.28) which determine the interactions $\tilde{L}^{1\mathrm{em}}(x), \tilde{L}^{1W}(x)$, (2.6), (2.9). The divergences of these currents are given by Eq. (2.20). In Eq. (2.21), as well as in all the expressions thereafter, the product $\tilde{\phi}_f(y)\tilde{\phi}_i(z)$ of Heisenberg field operators means $\tilde{\psi}_N(y)\tilde{\bar{\psi}}_{N'}(z)$, $N, N' = n, p$, or $\tilde{\pi}^b(y)\tilde{\pi}^c(z)$, $b, c = 0\pm$, in considering the nucleon isodublet or pion isotriplet, respectively. Fourier transfer (FT) of the quantity (2.21),

$$\mathrm{FT}\Big(\tilde{\mathcal{K}}^{afi}_\mu(\{\phi\}x, y, z)\Big) \equiv \int \mathrm{d}x\mathrm{d}y\mathrm{d}z \exp\ [ikx + ip_f y + ip_i z]\tilde{\mathcal{K}}^{afi}_\mu(\{\phi\}x, y, z), \tag{2.22}$$

is generally received (see, for instance, [18, 19]) to be presented as follows:

$$\mathrm{FT}\Big(\tilde{\mathcal{K}}^{aNN'}_\mu(\{\psi\}x, y, z)\Big) = (2\pi)^4\delta(k{+}p_N{+}p_{N'})\tilde{\Gamma}^a_\mu(\{\psi\}N, p_N; N', -p_{N'}; k)\times \\ \times [-\tilde{G}_N(p_N)\tilde{G}_{N'}(-p_{N'})],\ \ N, N' = n, p,\ \ a = 0, \pm, \tag{2.23}$$

$$\mathrm{FT}\Big(\tilde{\mathcal{K}}^{abc}_\mu(\{\pi\}x, y, z)\Big) = (2\pi)^4\delta(k + p_b + p_c)\tilde{\Gamma}^a_\mu(\{\pi\}b, p_b; -c, -p_c; k)\times \\ \times [-\tilde{G}_b(p_b)\tilde{G}_{-c}(-p_c)],\ a, b, c = 0, \pm, \tag{2.24}$$

in terms of the nucleon and pion propagators

$$i\tilde{G}_N(p) = \mathrm{FT}(\langle 0 \mid \mathcal{T}\{\tilde{\psi}_N(x)\tilde{\bar{\psi}}_{N'}(x')\} \mid 0\rangle)\delta_{NN'}\,, \tag{2.25}$$

$$i\tilde{G}_d(p) = \mathrm{FT}(\langle 0 \mid \mathcal{T}\{\tilde{\pi}^d(x)\tilde{\pi}^{-d}(x')\} \mid 0\rangle), \tag{2.26}$$

and the proper, one-particle irreducible, vertex function (VF) $\tilde{\Gamma}^a_\mu(\{\phi\}f, p_f; i, -p_i; k)$ to describe transitions of a hadron from a state i with a momentum p_i to a state f with a momentum p_f, a momentum k transferred to hadrons thereby.

As understood, these transitions are due to the current $\tilde{J}^a_\mu(x)$ that causes the interactions $\tilde{L}^{1\mathrm{em}}(x)$, $\tilde{L}^{1W}(x)$, (2.6), (2.9). As usual,

$$\tilde{G}_j^{-1}(p) = G_j^{-1}(p) - \tilde{\Sigma}_j(p). \tag{2.27}$$

Here $G_j(P)$ is the free hadron propagator dictated by $\mathcal{L}_N(x)$, $\mathcal{L}_\pi(x)$, (1.25), (1.26), and the self-energy is

$$\tilde{\Sigma}_j(p) = \Sigma_j^h(p) + \tilde{\Sigma}_j^{he}(p)\,, \tag{2.28}$$

where $\Sigma_j^h(p)$ is caused by the pure strong interaction $\mathcal{L}^{\mathrm{int}}_{\mathrm{str}}(x)$ (1.27) in $\mathcal{L}_h(x)$ (1.24), and $\tilde{\Sigma}_j^{he}(p)$ is due to the electromagnetic interactions $\tilde{L}^{1\mathrm{em}}(x)$, $\tilde{L}^{2\mathrm{em}}(x)$, (2.6), (2.7) in $\tilde{\mathcal{L}}^{\mathrm{EW}}(x)$ (2.4), with taking the strong interaction $\mathcal{L}^{\mathrm{int}}_{\mathrm{str}}(x)$ (1.27) into account therein as well. The self-energies $\Sigma_j^h(p)$ are equal for all the members of a given isomultiplet, whereas $\tilde{\Sigma}_j^{he}(p)$ are different for charged and neutral particles. Involving the electromagnetic interaction, i.e., being of order e^2 at least, the quantities $\tilde{\Sigma}_j^{he}(p)$ are obviously considered to be much smaller as compared to $\Sigma_j^h(p)$.

When strong interactions are turned off, i.e., for non-interacting structureless particles, the self-energy reduces in e^2-order to

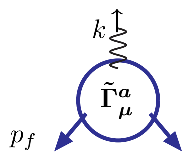
$$\Sigma_N^{(2)}(p) = e^2\delta_{Np}\int\frac{\mathrm{d}q}{(2\pi)^4 i}D_{\lambda\nu}(q)\gamma^\lambda G_N(p+q)\gamma^\nu,\ N = n, p, \tag{2.29}$$

$$\Sigma_d^{(2)}(p) = e^2 d^2\int\frac{\mathrm{d}q}{(2\pi)^4 i}D_{\lambda\nu}(q)G_d(p+q)(2p^\lambda+q^\lambda)(2p^\nu+q^\nu),\ d = 0, \pm, \tag{2.30}$$

for the nucleon and pion, respectively, where

$$D_{\lambda\nu}(q) = i\mathrm{FT}\langle 0 \mid \mathcal{T}[A^{\mathrm{em}}_\lambda(x)A^{\mathrm{em}}_\nu(x')] \mid 0\rangle \tag{2.31}$$

is the usual photon propagator. VF $\tilde{\Gamma}^a_\mu(\{\phi\}f, p_f; i, -p_i; k)$ is depicted by the diagram



. (2.32)

It should be emphasized that here we deal with the very proper («truncated») VF, without external hadronic lines.

The matrix element, dictated by the interactions $L^{1\mathrm{em}}(x)$, $L^{1W}(x)$, (2.6), (2.9), to describe transitions between states i and f of real free hadrons in an external field is given, in accordance with Eqs. (2.23), (2.24), in terms of VF $\tilde{\Gamma}^a_\mu$

$$\mathcal{M}^a_{if}(\{\phi\}p_f, p_i, k) = \langle \phi_f, p_f \mid \int \mathrm{d}x\big(-ie\mathcal{A}^a_\mu(x)\tilde{J}^{a\,\mu}(x)\big) \mid \phi_i, p_i\rangle =$$
$$= -ie\int \frac{\mathrm{d}q}{(2\pi)^4}\mathcal{A}^a_\mu(q)\int \mathrm{d}x\exp(iqx)\langle\phi_f, p_f \mid \tilde{J}^a_\mu(x) \mid \phi_i, p_i\rangle =$$
$$= -ie\mathcal{A}^{a\,\mu}(k)U^+(\phi_f p_f)\tilde{\Gamma}^a_\mu(\{\phi\}f, p_f; i, -p_i; k)U(\phi_i, p_i), \quad (2.33)$$

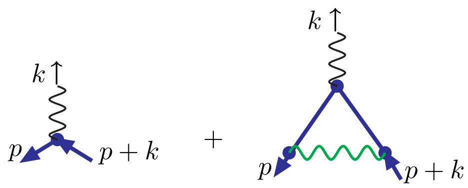
where $\mathcal{A}^a_\mu(k)$ is the component of Fourier transformation of an external field $\mathcal{A}^a_\mu(x)$, $k = p_i - p_f$ is a momentum transferred, and $U^+(\phi_f, p_f), U(\phi_i, p_i)$ are the wave amplitudes of free particles: the Dirac bispinor amplitudes $\bar{u}_N(p_N)$, $u_{N'}(p_{n'})$, $N, N' = n, p$, for nucleons, and the scalar amplitudes $u^*_a(p_a)$, $u_b(p_b)$, $a, b = 0, \pm$, for pions.

For non-interacting, point-like nucleons and pions, i.e., when $S_{\mathrm{str}} = 1$ in Eq. (1.31), we have directly got, to order e^2, the VFs

$$\Gamma^a_{0\,\mu}(\{\psi\}N, p; N', p+k; k) = \gamma^\mu\ell^a_{NN'} + \ell^0_{NN'}e^2\int\frac{\mathrm{d}q}{(2\pi)^4 i}D_{\lambda\nu}(q)\times$$
$$\times\,\gamma^\lambda G_N(p+q)\gamma^\mu G_{N'}(p+k-q)\gamma^\nu, \quad N, N' = n, p, \quad (2.34)$$

$$\Gamma^a_{0\,\mu}(\{\pi\}b, p; -c, p+k; k) = \ell^a_{b(-c)}[2p^\mu + k^\mu] -$$
$$-\,(cb)e^2\int\frac{\mathrm{d}q}{(2\pi)^4 i}D_{\lambda\nu}(q)G_{-c}(p+k-q)G_b(p-q)\times$$
$$\times\,(2p^\lambda - q^\lambda)(2(p+k)^\nu - q^\nu)\ell^a_{b(-c)}[2(p-q)_\mu + k_\mu], \quad a, b, c, = 0\pm, \quad (2.35)$$

which give place to Eqs. (2.23), (2.24) in the general case. The usual diagrams, with the vertices corresponding to the interactions $L^{1\mathrm{em}}(x)$, $L^{1W}(x)$, (2.6), (2.9),



(2.36)

will serve to illustrate these equations. Apparently, the second terms in Eqs. (2.34)–(2.36) are non-vanishing only for interactions of charge particles with an electromagnetic field, i.e., at $a = 0$.

We shall keep in view that these VFs treated in Eqs. (2.22)–(2.36) stand to describe the hadron electroweak transitions caused only by the interactions

$\tilde{L}^{1\mathrm{em}}(x)$, $\tilde{L}^{1W}(x)$, (2.6), (2.9), which involve the current $\tilde{J}^a_\mu(x)$, yet not by the whole interaction $\tilde{\mathcal{L}}^{\mathrm{EW}}(x)$ (2.4). And yet the actual electroweak hadron transitions are certainly caused by the very total interaction $\tilde{\mathcal{L}}^{\mathrm{EW}}(x)$ (2.4). Therefore, VFs (2.22)–(2.36) themselves can not be correlated with the respective experimental data that could serve to parameterizing the vertex functions. Thus, the VF $\tilde{\Gamma}^a_\mu$ has to be accomplished so as to gain a certain total VF $\tilde{\Gamma}^a_{\mathrm{tot}\,\mu}$ to describe the observable electroweak transitions. The transitions caused by the left-over interactions $\tilde{L}^{2\mathrm{em}}(x)$, $\tilde{L}^{W\mathrm{em}}(x)$, (2.7), (2.10) will naturally come into consideration in due course later on.

3. THE RELATIONS BETWEEN HADRON VERTEX FUNCTIONS AND PROPAGATORS

Having acquired the hadron current divergence (2.20) and the electroweak VFs (2.23)–(2.35), we now consider the key expression [19]

$$\mathrm{FT}\left(\partial^\mu_x \tilde{\mathcal{K}}^{afi}_\mu\Big(\{\phi\}x,y,z\Big)\right) \equiv (-ik^\mu)\mathrm{FT}\left(\tilde{\mathcal{K}}^{afi}_\mu\Big(\{\phi\}x,y,z\Big)\right) \equiv$$
$$\equiv \mathrm{FT}\left(\tilde{K}^{0afi}\Big(\{\phi\}x,y,z\Big)\right) + \mathrm{FT}\left(\tilde{K}^{afi}\Big(\{\phi\}x,y,z\Big)\right) \quad (3.1)$$

to obtain the WT identities. Here the quantity $\mathrm{FT}\left(\tilde{\mathcal{K}}^{afi}_\mu\Big(\{\phi\}x,y,z\Big)\right)$ is defined by Eqs. (2.21), (2.22) and

$$\tilde{K}^{0afi}\Big(\{\phi\}x,y,z\Big) = \delta(x_0-y_0)\langle 0\mid \mathcal{T}\{[\tilde{J}^a_0(x),\, \tilde{\phi}_f(y)]\tilde{\phi}_i(z)\}\mid 0\rangle +$$
$$+\,\delta(x_0-z_0)\langle 0\mid \mathcal{T}\{\tilde{\phi}_f(y)[\tilde{J}^a_0(x),\, \tilde{\phi}_i(z)]\}\mid 0\rangle, \quad (3.2)$$

$$\tilde{K}^{afi}\Big(\{\phi\}x,y,z\Big) = \langle 0\mid \mathcal{T}\{\partial^\mu_x\tilde{J}^a_\mu(x)\tilde{\phi}_f(y)\tilde{\phi}_i(z)\}\mid 0\rangle, \quad (3.3)$$

where the current divergence $\partial^\mu_x\tilde{J}^a_\mu(x)$ is put into the expedient form (2.20).

Now, we are to work out the right-hand side of Eq. (3.1) by way of ascertaining the relations between the electroweak VFs and the propagators of hadrons: the WT identities.

With allowance for Eqs. (1.4), (1.5), (1.9), (1.10) for the Heisenberg field operators $\tilde{\phi}(x)$, the first term on right-hand side of Eq. (3.1) transforms to

$$\mathrm{FT}\Big(\tilde{K}^{0aNN'}(\{\psi\}x,y,z,)\Big) = i(2\pi)^4\delta(p_N+p_{N'}+k)\ell^a_{NN'}[\tilde{G}_N(p_N)-$$
$$-\,\tilde{G}_{N'}(-p_{N'})],\ N,N'=n,p, \quad (3.4)$$

for nucleons, and to

$$\mathrm{FT}\Big(\tilde{K}^{0abc}(\{\pi\}x,y,z,)\Big) = i(2\pi)^4\delta(p_b+p_c+k)\ell^a_{b(-c)}[\tilde{G}_b(p_b)- \\ - \tilde{G}_{-c}(-p_c)],\ a,b,c=0,\pm \quad (3.5)$$

for pions.

Terms of different kinds are incorporated into expression (3.3).

As presumed, all the calculations are carried out with the accuracy of e^2-order. With this precision, we can put $S_{\mathrm{em}}=1$ in the contribution to (3.3) from the last (third) term of the divergence (2.20),

$$ie^2a\sqrt{2}\langle 0\mid\{\mathcal{N}[A^{\mathrm{em}}_\nu(x)A^{\mathrm{em}\,\nu}(x)]\}\mid 0\rangle\times \\ \times\langle 0\mid\mathcal{T}\{\mathcal{N}[\pi^0(x)\pi^{-a}(x)]\phi_f(y)\phi_i(z)\}\mid 0\rangle = 0, \quad (3.6)$$

that vanishes apparently as a pure vacuum expectation value of an $\mathcal{N}$-product of electromagnetic field operators occurs therein. (Let us here recall the remark given after Eqs. (1.28)–(1.30)). Then, we are left with the contributions to this expression (3.3) from the first and the second terms of Eq. (2.20)

$$\mathrm{FT}\Big(\tilde{K}^{afi}(\{\phi\}x,y,z,)\Big) = \mathrm{FT}\Big(\tilde{K}^{1afi}(\{\phi\}x,y,z,)\Big) + \mathrm{FT}\Big(\tilde{K}^{2afi}(\{\phi\}x,y,z,)\Big), \quad (3.7)$$

$$\mathrm{FT}\Big(\tilde{K}^{1afi}(\{\phi\}x,y,z,)\Big) = iea\mathrm{FT}\Big(\langle 0|\mathcal{T}\{A^{\mathrm{em}}_\nu(x)J^{a\nu}(x)\phi_f(y)\phi_i(z)S_{\mathrm{em}}\}|0\rangle\Big), \quad (3.8)$$

$$\mathrm{FT}\Big(\tilde{K}^{2afi}(\{\phi\}x,y,z,)\Big) = -e\mathrm{FT}\Big(\langle 0|\mathcal{T}\{\partial^\mu_x\tilde{\jmath}^a_\mu(x)\tilde{\phi}_f(y)\tilde{\phi}_i(z)\}|0\rangle\Big). \quad (3.9)$$

First we consider the quantity (3.8). Retaining the terms $\sim e^2$, it is rewritten as follows

$$\mathrm{FT}\Big(\tilde{K}^{1afi}(\{\phi\}x,y,z,)\Big)\approx \\ \approx ae^2\mathrm{FT}\Big(\langle 0\mid\mathcal{T}\{A^{\mathrm{em}}_\nu(x)J^{a\,\nu}(x)\phi_f(y)\phi_i(z)\int \mathrm{d}x' A^{\mathrm{em}}_\lambda(x')J^{0\,\lambda}(x')\}\mid 0\rangle\Big) = \\ = a\int\frac{\mathrm{d}q}{i(2\pi)^4}D_{\mu\nu}(q)\cdot T^a_{1\,\mu\nu}(\{\phi\}f,p_f;i,p_i;k-q,q), \quad (3.10)$$

where the Fourier transfer of the vacuum expectation value of the product of currents (1.28) and field operators $\phi_r(x)$ is introduced

$$T^a_{1\,\mu\nu}(\{\phi\}f,p_f;i,p_i;k-q,q) = \\ = \int\mathrm{d}x\int\mathrm{d}y\int\mathrm{d}z\int\mathrm{d}x'\exp[ix(k-q)+ix'q+iyp_f+izp_i]\times \\ \times e^2\langle 0\mid\mathcal{T}\{J^{a\,\nu}(x)J^{0\,\mu}(x')\phi_f(y)\phi_i(z)\}\mid 0\rangle. \quad (3.11)$$

This quantity (3.11) is generally received to be given (see, for instance, [18]) as follows:

$$T^a_{1\,\mu\nu}(\{\psi\}N,p_N;N',p_{N'};k-q,q) = \\ = (2\pi)^4\delta(p_N+p_{N'}+k)[-\tilde{G}_N(p_N)\tilde{G}_{N'}(-p_{N'})]\times \\ \times e^2 M^a_{1\,\mu\nu}(\{\psi\}N,p_N;N',-p_{N'};k-q,q), \quad N,N'=n,p,a=0,\pm \quad (3.12)$$

$$T^a_{1\,\mu\nu}(\{\pi\}b,p_b;c,p_c;k-q,q) = (2\pi)^4\delta(p_b+p_c+k)[-\tilde{G}_b(p_b)\tilde{G}_{-c}(-p_c)]\times \\ \times e^2 M^a_{1\,\mu\nu}(\{\pi\}b,p_b;-c,-p_c;k-q,q), \quad a,b,c=0,\pm \quad (3.13)$$

in terms of the proper VFs $M^a_{1\,\mu\nu}(\{\phi\}f,p_f;i,-p_i;k_1,k_2)$ to describe transitions of nucleons and pions from a state i with momentum p_i to a state f with momentum p_f, momenta $k_1 = k-q$ and $k_2 = q$ transferred thereby. As seen from Eq. (3.11), these transitions are caused by the product of two currents, $J^a_\nu(x)$, $a = 0,\pm$, and $J^0_\mu(x') \equiv J^{\rm em}_\mu(x')$, determining the interactions $\tilde{L}^{1W}(x)$ (2.9) and $\tilde{L}^{1\rm em}$ (2.6). Evidently, the quantities $M^a_{1\,\mu\nu}$ do not involve electromagnetic interactions, which are separated explicitly in Eqs. (3.10)–(3.13) to order e^2, the multiplier e^2 in front of $M^a_{1\,\mu}$.

If anything, it goes as a matter of course that the very expressions (3.10), (3.11), which stand to introduce this quantity $M^a_{1\,\mu\nu}$, are gauge invariant. Indeed, when we replace $A^{\rm em}_\mu(x) \to \partial_\mu$, expression (3.10) apparently gets equal to zero due to the vector current conservation (1.28), (1.8).

Equations (3.12), (3.13) are pronouncedly understood to define the proper VFs $M^a_{1\mu\nu}$ in such a way that they do not include the corrected external hadronic states.

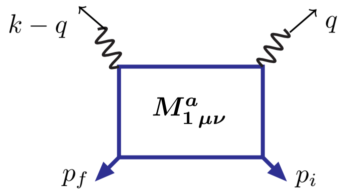
In turn, the quantity (3.8) is rewritten in terms of these VFs $M^a_{1\,\mu\nu}$:

$$\mathrm{FT}\Big(K^{1aNN'}(\{\psi\}x,y,z)\Big) = (2\pi)^4\delta(p_N+p_{N'}+k)[-\tilde{G}_N(p_N)\tilde{G}_{N'}(-p_{N'})]\times \\ \times e^2 a\int\frac{\mathrm{d}q}{i(2\pi)^4}D_{\mu\nu}(q)M^a_{1\mu\nu}(\{\psi\}N,p_N;N',-p_{N'};k-q,q), \quad (3.14)$$

$$\mathrm{FT}\Big(K^{1abc}(\{\pi\}x,y,z)\Big) = (2\pi)^4\delta(p_b+p_c+k)[-\tilde{G}_b(p_b)\tilde{G}_{-c}(-p_c)]\times \\ \times e^2 a\int\frac{\mathrm{d}q}{i(2\pi)^4}D_{\mu\nu}(q)M^a_{1\mu\nu}(\{\pi\}b,p_b;-c,-p_c;k-q,q), \quad (3.15)$$

for transitions within nucleon and pion isomultiplets, respectively.

VFs $M^a_{1\,\mu\nu}$ are depictured by the typical diagram:



$$. \qquad (3.16)$$

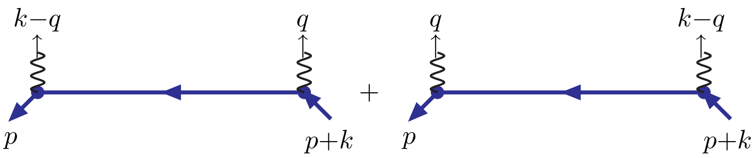
There is to emphasize that here we deal with the very proper, «truncated», VF $M^a_{1\,\mu\nu}$, alike VF (2.22)–(2.24), (2.32), without external hadron lines.

For non-interacting structureless nucleons and pions, i.e., with $S_{\rm str} = 1$ in Eq. (1.31), direct evaluation gives, with the accuracy $\sim e^2$,

$$\begin{aligned} e^2 M^a_{0\,1\,\mu\nu}(\{\psi\}N, p_N; N', -p_{N'}; k-q, q) = \\ = ie^2[\ell^a_{NN''}\gamma^\mu G_{N''}(-p_{N'} - q)\gamma^\nu \ell^0_{N''N'} + \\ + \ell^0_{NN''}\gamma^\nu G_{N''}(p_N + q)\ell^a_{N''N'}\gamma^\mu], \quad p_N + p_{N'} + k = 0, \end{aligned} \qquad (3.17)$$

$$\begin{aligned} e^2 M^a_{0\,1\,\mu\nu}(\{\pi\}b, p_b; -c, -p_c; k-q, q) = \\ = -ie^2\sqrt{2}[(2p_c + q + k)_\mu (2p_c + q)_\nu G_c(p_c + q)\delta_{b0}\delta_{ac} + \\ + (2p_b + q + k)_\nu (2p_b + q)_\mu G_b(p_b + q)\delta_{c0}\delta_{ab}], \\ p_b + p_c + k = 0, \quad a, b, c = 0, \pm, \end{aligned} \qquad (3.18)$$

which are replaced by Eqs. (3.12), (3.13) in the general case. These Eqs. (3.17), (3.18) could be displayed by the familiar diagrams:



$$. \qquad (3.19)$$

For free real particles in initial and final states, VF $M^a_{1\,\mu\nu}(\{\phi\}f, p_f; i, -p_i; q_1, q_2)$ serves to determine the «Compton scattering amplitude» caused by the interactions $\tilde{L}^{1\rm em}(x)$, $\tilde{L}^{1W}(x)$, (2.6), (2.9) (but not by $\tilde{L}^{2\rm em}(x), \tilde{L}^{W\rm em}$ (2.7), (2.10)) in the external gauge fields $A^{(e)}_\nu(x)$, $\mathcal{A}^a_\mu(x)$, $(a = 0, \pm)$, with momenta

q_1, q_2 transferred to hadrons,

$$\mathcal{M}_1^{a\,(\mathrm{ext})}(\{\phi\}f, p_f; i, p_i; q_1, q_2) =$$
$$= e^2 \int \mathrm{d}x \int \mathrm{d}x' \langle \phi_f, p_f \mid \mathcal{T}\{J^{a\,\nu}(x) J^{0\,\mu}(x')\} \mid \phi_i, p_i\rangle \mathcal{A}^a_\mu(x) A^{(e)}(x') =$$
$$= e^2 \int \frac{\mathrm{d}q_1}{(2\pi)^4} \int \frac{\mathrm{d}q_2}{(2\pi)^4} \mathcal{A}^a_\mu(q_1) A^{(e)}_\nu(q_2)(2\pi)^4 \delta(p_f - p_i + q_1 + q_2) \times$$
$$\times U^+(\phi_f, p_f) M^a_{1\,\mu\nu}(\{\phi\}f, p_f; i, -p_i; q_1, q_2) U(\phi_i, p_i), \quad (3.20)$$

where $A^{(e)}_\nu(q_2)$, $\mathcal{A}^a_\mu(q_1)$ are the components of Fourier transformation of the external gauge fields, and $U(\phi_i, p_i)$, $U^+(\phi_f, p_f)$ are the wave amplitudes of free particles: the Dirac bispinors $u_N(p_N)$, $\bar{u}_{N'}(p_{n'})$, $N, N' = n, p$, for nucleons, and the scalar amplitudes $u_b(p_b)$, $u^*_a(p_a)$, $a, b = 0, \pm$, for pions, alike in Eq. (2.33). In much the same manner, when hadrons interact with quantum gauge fields $W^\pm_\mu(x)$, VF $M^a_{1\,\mu\nu}(\{\phi\}f, p_f; i, -p_i; q_1, q_2)$ will give the free $W^\pm$-boson photoproduction amplitude

$$\mathcal{M}^a_{1\,\mu\nu}(\{\phi\}f, p_f; i, p_i; q_1, q_2) = iU^+(\phi_f, p_f) \cdot \frac{\sqrt{G} M_W}{2^{1/4}} w^\pm_\mu(q_2) \times$$
$$\times M^a_{1\,\mu\nu}(\{\phi\}f, p_f; i, -p_i; q_1, q_2) U(\phi_i, p_i) \cdot e e_\nu(q_1), \quad (3.21)$$

where e_ν stands for the polarization vector of a photon and $w^\pm_\mu$ of a $W^\pm$-boson, and e and $\frac{\sqrt{G} M_W}{2^{1/4}}$ are the respective coupling constants.

As understood, the aforesaid quantities (3.10)–(3.21), which stem from Eq. (3.8), are purely due to the currents $J^a_\mu(x)$ (1.28) that determine the interactions $\tilde{L}^{1\mathrm{em}}(x)$, $\tilde{L}^{1W}(x)$, (2.6), (2.9), and yet the interactions $\tilde{L}^{2\mathrm{em}}(x)$, $\tilde{L}^{W\mathrm{em}}(x)$, (2.7), (2.10) are obviously not involved, alike in treating VF $\tilde{\Gamma}^a_\mu$ in Eqs. (2.22)–(2.36).

Now, we turn to calculating the quantity (3.9). With allowance for Eqs. (1.4), (1.5), (1.9), (1.10) for the Heisenberg field operators $\tilde{\phi}(x)$, it transforms to

$$\mathrm{FT}\Big(\tilde{K}^{2afi}(\{\phi\}x, y, z,)\Big) = eik^\nu \mathrm{FT}\Big(\langle 0 \mid \mathcal{T}\{\tilde{\jmath}^a_\nu(x) \tilde{\phi}_f(y) \tilde{\phi}_i(z)\} \mid 0\rangle\Big) =$$
$$= eik^\nu \mathrm{FT}\Big(\langle 0 \mid \mathcal{T}\{\jmath^a_\nu(x) \phi_f(y) \phi_i(z) \mathcal{S}_{\mathrm{em}}\} \mid 0\rangle\Big). \quad (3.22)$$

As one recognizes, this quantity (3.22) shows up to be determined by the same «current» $\tilde{\jmath}^a_\mu(x)$ that determines the interactions $\tilde{L}^{2\mathrm{em}}(x)$, $\tilde{L}^{W\mathrm{em}}(x)$, (2.7), (2.10) in the lagrangian $\tilde{\mathcal{L}}^{\mathrm{EW}}$, (2.4), alike the current $\tilde{J}^a_\mu(x)$ determines $\tilde{L}^{1\mathrm{em}}(x)$, $\tilde{L}^{1W}(x)$, (2.6), (2.9). This point is noteworthy to perceive the physical contents of the treatment.

Alike Eq. (2.22), which serves to define the proper VFs (2.23), (2.24), (2.32), the quantity (3.22) is presented as follows:

$$\mathrm{FT}\Big(\tilde{K}^{2aNN'}(\{\psi\}x,y,z)\Big)=$$
$$=i(2\pi)^4\delta(p_N+p_{N'}+k)k^\nu[\tilde{G}_N(p_N)\tilde{G}_{N'}(-p_{N'})]\times$$
$$\times\,\tilde{\Gamma}^a_{2\,\nu}(\{\psi\}N,p_N;N',-p_{N'};k),\ N,N'=n,p,\quad(3.23)$$

$$\mathrm{FT}\Big(\tilde{K}^{2abc}(\{\pi\}x,y,z)\Big)=$$
$$=i(2\pi)^4\delta(p_b+p_c+k)k^\nu\ [\tilde{G}_b(p_b)\tilde{G}_{-c}(-p_c)]\times$$
$$\times\,\tilde{\Gamma}^a_{2\,\nu}(\{\pi\}b,p_b;-c,-p_c;k),\quad a,b,c=0,\pm,\quad(3.24)$$

through the proper VF $\tilde{\Gamma}^a_{2\,\nu}(\{\phi\}f,p_f;i,-p_i;k)$ to describe transitions of a hadron from a state i with momentum p_i to a state f with momentum p_f, a momentum k transferred thereby. As determined by Eq. (3.22), these transitions are now due to the «current» $\tilde{\jmath}^a_\mu(x)$, which causes the very interactions $\tilde{L}^{2\mathrm{em}}(x)$, $\tilde{L}^{W\mathrm{em}}(x)$, (2.7), (2.10), in place of the current $\tilde{J}^a_\mu(x)$ and the interactions $\tilde{L}^{1\mathrm{em}}(x)$, $\tilde{L}^{1W}(x)$, (2.6), (2.9), associated with VFs $\tilde{\Gamma}^a_\mu$ (2.23), (2.24), (2.32). In turn, the matrix element, dictated by $\tilde{L}^{2\mathrm{em}}(x)$, $\tilde{L}^{W\mathrm{em}}(x)$, (2.7), (2.10), which describes transitions between states i and f of free real hadrons, is given in terms of VF $\tilde{\Gamma}^a_{2\,\nu}(\{\phi\}f,p_f;i,-p_i;k)$ as follows:

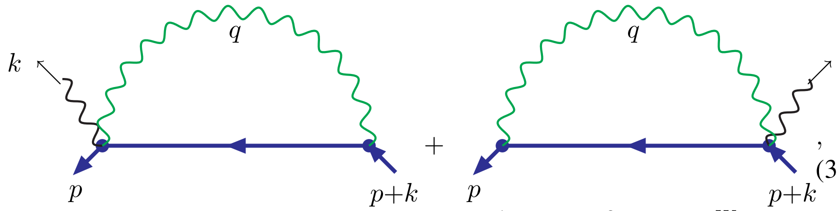
$$\mathcal{M}^a_{2\,if}(\{\phi\}p_f,p_i,k)=-ie\mathcal{A}^{a\,\mu}(k)U^+(\phi_f,p_f)\tilde{\Gamma}^a_{2\,\mu}(\{\phi\}f,p_f;i,-p_i;k)U(\phi_i,p_i),\quad(3.25)$$

in place of Eq. (2.33).

Let us mention that VF $\Gamma^a_{2\,\nu}(\{\psi\}N,p_N;N',-p_{N'};k)$ evidently vanishes for non-interacting structureless particles, and VF $\tilde{\Gamma}^a_{2\,\nu}(\{\pi\}b,p;-c,p+k;k)$ in this case directly reduces to

$$\Gamma^a_{0\,2\,\nu}(\{\pi\}b,p;-c,p+k;k)=$$
$$=-e^2\int\frac{\mathrm{d}q}{i(2\pi)^4}D_{\mu\nu}(q)\Big\{G_c(p+k+q)(2p+2k+q)^\mu[2c\delta_{a0}\delta_{b(-c)}-a\sqrt{2}\delta_{b0}\delta_{ac}]+$$
$$+\,G_b(p+q)(2p+q)^\mu[2b\delta_{a0}\delta_{b(-c)}-a\sqrt{2}\delta_{c0}\delta_{ab}]\Big\},$$
$$a,b,c=0,\pm,\ p_b+p_c+k=0,\quad(3.26)$$

that could be depictured by the diagrams:



, (3.27)

where vertices correspond to the interactions $L^{1\mathrm{em}}(x)$, $L^{2\mathrm{em}}(x)$, $L^{W\mathrm{em}}(x)$, (2.6), (2.7), (2.10).

At the same time, it is expedient to rewrite Eqs. (3.9), (3.22), retaining terms $\sim e^2$, in the form

$$\begin{aligned}
&\mathrm{FT}\Big(\tilde{K}^{2afi}(\{\phi\}x, y, z,)\Big) \approx \\
&\quad\approx e^2 k^\nu \mathrm{FT}\Big(\langle 0|\mathcal{T}\{j^a_\nu(x)\phi_f(y)\phi_i(z)\Big)\int \mathrm{d}x' A^{\mathrm{em}}_\mu(x') J^{\mathrm{em}\,\mu}(x')\}|0\rangle = \\
&\quad\quad = k^\nu \int \frac{\mathrm{d}q}{i(2\pi)^4} D_{\nu\mu}(q) T^a_{2\,\mu}(\{\phi\}f, p_f; i, p_i; k-q, q), \qquad (3.28)
\end{aligned}$$

where we define

$$\begin{aligned}
&T^a_{2\,\mu}(\{\phi\}f, p_f; i, p_i; k-q, q) = \\
&= \int \mathrm{d}x \int \mathrm{d}y \int \mathrm{d}z \int \mathrm{d}x' \exp[ix(k-q) + ix'q + iyp_f + izp_i] \cdot [\ell^a_{dr}\ell^0_{(-d)s}] \times \\
&\quad\quad \times e^2 \langle 0 \mid \mathcal{T}\{J^{\mathrm{em}}_\mu(x')\pi^s(x)\pi^r(x)\phi_f(y)\phi_i(z)\} \mid 0\rangle. \qquad (3.29)
\end{aligned}$$

It can be observed that the quantity $T^a_{2\,\nu}$ (3.29) is akin to the quantity $T^a_{\mu\nu}$ (3.11). In much the same way as Eqs. (3.12), (3.13) define the proper VFs $M^a_{1\,\mu\nu}(\{\phi\}f, p_f; i, -p_i; k_1, k_2)$, the following analogous equations

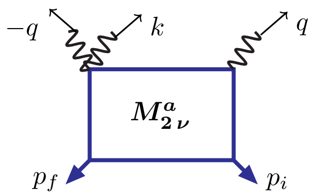
$$\begin{aligned}
&T^a_{2\,\mu}(\{\psi\}N, p_N; N', p_{N'}; k-q, q) = \\
&\quad\quad = (2\pi)^4\delta(p_N + p_{N'} + k)[-\tilde{G}_N(p_N)\tilde{G}_{N'}(-p_{N'})] \times \\
&\quad\quad\quad \times e^2 M^a_{2\mu}(\{\psi\}N, p_N; N', -p_{N'}; k-q, q), \qquad (3.30)
\end{aligned}$$

$$\begin{aligned}
&T^a_{2\,\mu}(\{\pi\}b, p_b; c, p_c; k-q, q) = \\
&\quad\quad = (2\pi)^4\delta(p_b + p_c + k)[-\tilde{G}_b(p_b)\tilde{G}_{-c}(-p_c)] \times \\
&\quad\quad\quad \times e^2 M^a_{2\,\mu}(\{\pi\}b, p_b; -c, -p_c; k-q, q), \qquad (3.31)
\end{aligned}$$

serve to draw into consideration the very special proper VF $M^a_{2\,\nu}(\{\phi\}f, p_f; i, -p_i; k_1, k_2)$ to describe hadron transitions from a state i with momentum p_i to a state

f with momentum p_f, momenta $k_1,\ k_2$ transferred to hadrons thereby. These transitions are caused by the product of the current $J^{\rm em}_{\nu}(x)$, which determines the interaction $\tilde{L}^{1\rm em}(x)$ (2.6), and the operator $\pi^s(x)\pi^r(x)[\ell^a_{dr}\ell^0_{(-d)s}]$, which determines, in turn, the interactions $\tilde{L}^{2\rm em}(x)$, $\tilde{L}^{W\rm em}(x)$, (2.7), (2.10). The quantity $M^a_{2\,\nu}$ evidently does not involve electromagnetic interactions separated explicitly in Eqs. (3.28)–(3.31).

This VF $M^a_{2\,\nu}$ could, conceivably, be depictured by the diagram:



. (3.32)

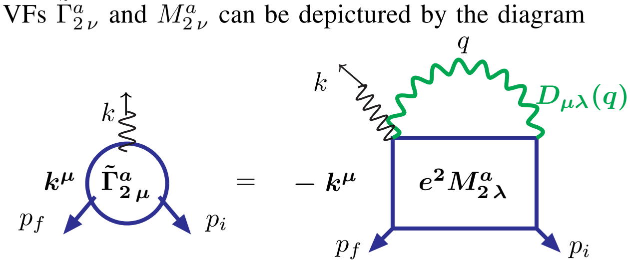
One recognizes from Eqs. (3.29)–(3.32) that VF $M^a_{2\,\nu}$ could serve to describe certain conceivable physical processes with real free hadrons in initial and final states. For instance, one could discuss a free real hadron transition from a state i to a state f with emitting (or absorbing) two photons and a $W^{\pm}$-boson, with certain momenta and polarizations.

With allowance for Eqs. (3.28)–(3.31), VFs $\tilde{\Gamma}^a_{2\,\nu}$ defined by Eqs. (3.22)–(3.24) are rewritten, with the accuracy $\sim e^2$, in terms of these quantities $M^a_{2\,\nu}$ as follows:

$$
\begin{aligned}
&k^{\nu}\tilde{\Gamma}^a_{2\,\nu}(\{\psi\}N,p_N;N',-p_{N'};k)=\\
&=-e^2k^{\nu}\int\frac{\mathrm{d}q}{(2\pi)^4}D_{\nu\lambda}(q)M^a_{2\,\lambda}(\{\psi\}N,p_N;N',-p_{N'};k-q,q),\ \ N,N'=n,p,
\end{aligned}
\tag{3.33}
$$

$$
\begin{aligned}
&k^{\nu}\tilde{\Gamma}^a_{2\,\nu}(\{\pi\}b,p_b;-c,-p_c;k)=\\
&=-e^2k^{\nu}\int\frac{\mathrm{d}q}{(2\pi)^4}D_{\nu\lambda}(q)M^a_{2\,\lambda}(\{\pi\}b,p_b;-c,-p_c;k-q,q),\ \ a,b,c,=0,\pm,
\end{aligned}
\tag{3.34}
$$

where dependence on the photon propagator $D_{\mu}(q)$ is set forth explicitly. In conjunction, VFs $\tilde{\Gamma}^a_{2\,\nu}$ and $M^a_{2\,\nu}$ can be depictured by the diagram



. (3.35)

For non-interacting point-like pions, Eqs. (3.34), (3.35) directly reduce to (3.26), (3.27). As the operator $\jmath^a_\mu(x)$ (2.8) in this case involves the pure pion field operators, VF (3.33) apparently vanishes.

In correlating Eqs. (2.23), (2.24), (2.32)–(2.36), which determine VF $\tilde{\Gamma}^a_\mu$, with Eqs. (3.23)–(3.27), (3.32)–(3.35), which determine VF $\tilde{\Gamma}^a_{2\,\nu}$, one realizes that these VFs inherently differ from each other, stemming from different interactions in $\tilde{\mathcal{L}}^{\mathrm{EW}}(x)$ (2.4). VF $\tilde{\Gamma}^a_\mu$ involves the electromagnetic corrections $\sim e^2$ to the uncorrected value, whereas VF $\tilde{\Gamma}^a_{2\,\nu}\sim e^2$ completely emerges due to electromagnetic interactions. So, the last would apparently vanish in the nucleon case, when strong interactions were turned off.

Now, all the terms in the identity (3.1) have been acquired and their physical purport elucidated. Upon inserting all the calculated quantities (2.23), (2.24), (3.4), (3.5), (3.14), (3.15), (3.23), (3.24), (3.33), (3.34) into the identity (3.1), we arrive at the relations among the electroweak VFs and propagators of nucleons and pions, respectively:

$$\begin{aligned}
k^\nu \tilde{\Gamma}^a_\nu(\{\psi\}N, p_N; N', -p_{N'}; k) &= \ell^a_{NN'}[\tilde{G}^{-1}_{N'}(-p_{N'}) - \tilde{G}^{-1}_N(p_N)] + \\
&+ e^2 a \int \frac{\mathrm{d}q}{(2\pi)^4} D_{\mu\nu}(q) M^a_{1\,\mu\nu}(\{\psi\}N, p_N; N', -p_{N'}; k-q, q) + \\
&+ k^\mu e^2 \int \frac{\mathrm{d}q}{(2\pi)^4} D_{\mu\nu}(q) M^a_{2\,\nu}(\{\psi\}N, p_N; N', -p_{N'}; k-q, q), \quad (3.36) \\
p_N + p_{N'} + k = 0, \quad a = 0, \pm, \quad N, N' = n, p,
\end{aligned}$$

$$\begin{aligned}
k^\nu \tilde{\Gamma}^a_\nu(\{\pi\}b, p_b; -c, -p_c; k) &= \ell^a_{b(-c)}[\tilde{G}^{-1}_{-c}(-p_c) - \tilde{G}^{-1}_b(p_b)] + \\
&+ e^2 a \int \frac{\mathrm{d}q}{(2\pi)^4} D_{\mu\nu}(q) M^a_{1\,\mu\nu}(\{\pi\}b, p_b; -c, -p_c; k-q, q) + \\
&+ k^\mu e^2 \int \frac{\mathrm{d}q}{(2\pi)^4} D_{\mu\nu}(q) M^a_{2\,\nu}(\{\pi\}b, p_b; -c, -p_c; k-q, q), \quad (3.37) \\
p_b + p_c + k = 0, \quad a, b, c = 0, \pm.
\end{aligned}$$

These equations are to be rewritten, displacing the last terms to the left-hand sides and treating the total VF

$$\tilde{\Gamma}^a_{\mathrm{tot}\,\nu} = \tilde{\Gamma}^a_\nu + \tilde{\Gamma}^a_{2\,\nu}, \quad (3.38)$$

to describe the electroweak transitions caused by all the interactions involved into $\tilde{\mathcal{L}}^{\mathrm{EW}}(x)$ (2.4). Then, we arrive at

$$\begin{aligned}
k^\nu \tilde{\Gamma}^a_{\mathrm{tot}\,\nu}(\{\psi\}N, p_N; N', -p_{N'}; k) &= \ell^a_{NN'}[\tilde{G}^{-1}_{N'}(-p_{N'}) - \tilde{G}^{-1}_N(p_N)] + \\
&+ e^2 a \int \frac{\mathrm{d}q}{(2\pi)^4} D_{\mu\nu}(q) M^a_{1\,\mu\nu}(\{\psi\}N, p_N; N', -p_{N'}; k-q, q), \quad (3.39)
\end{aligned}$$

$$k^\nu \tilde{\Gamma}^a_{\mathrm{tot}\,\nu}(\{\pi\}b, p_b; -c, -p_c; k) = \ell^a_{b(-c)}[\tilde{G}^{-1}_{-c}(-p_c) - \tilde{G}^{-1}_b(p_b)] + \\ + e^2 a \int \frac{\mathrm{d}q}{(2\pi)^4} D_{\mu\nu}(q) M^a_{1\,\mu\nu}(\{\pi\}b, p_b; -c, -p_c; k-q, q). \quad (3.40)$$

The relations of this kind are generally referred to as the generalized WT identities. As expounded, we have ascertained them with the e^2 accuracy.

For the sake of elucidation, the expressions of VFs for structureless non-interacting hadrons were purposely presented at every stage in the course of the aforesaid consideration, see Eqs. (2.34), (2.35), (2.29), (2.30), (3.17), (3.18), (3.26). Now, having them at our disposal, it is just straightforward matter to become convinced that Eqs. (3.36)–(3.40) get trivial in the case of free point-like particles.

4. CONSEQUENCES OF THE WT IDENTITIES AND DISCUSSION

The identities (3.36)–(3.40) hold true for any strong interactions $\mathcal{L}^{\mathrm{int}}_{\mathrm{str}}(x)$ that cause the hadron propagators and VFs therein, provided the total vector hadron currents $J^a_\mu(x)$ are conserved (1.8). So, when, in treating RC to the electroweak hadron transitions, one evaluates the propagators and VFs amenably to some plausible approach, for instance, in the framework of the chiral perturbation theory [24, 26], the results are to satisfy the identities (3.36)–(3.40). Such an available test of consistency is thought to be of use to repose full confidence in the computations carried out within the effective field theory [27].

From the outset, what is to be highlighted is that the left-hand sides of Eqs. (3.39), (3.40) are just the total VFs to describe the electroweak transitions caused by all the electroweak interactions $\tilde{\mathcal{L}}^{\mathrm{EW}}(x)$ (2.4), and not just solely by $\tilde{L}^{1\mathrm{em}}(x)$, $\tilde{L}^{1W}(x)$, (2.6), (2.9), which are due to the very currents $\tilde{J}^a_\mu(x)$ giving rise to VFs $M^a_{1\,\mu\nu}$. As to the «currents» $\tilde{\jmath}^a_\mu(x)$, which cause the electroweak interactions $\tilde{L}^{2\mathrm{em}}(x)$, $\tilde{L}^{W\mathrm{em}}(x)$, (2.7), (2.10), they are not involved into these quantities $M^a_{1\,\mu\nu}$ determining right-hand sides of Eqs. (3.39)–(3.40).

In this regard, the approach asserted in the work [13], Eqs. (9.10)–(9.14), and thereupon in the ensuing computations [14–16], do not evidently go with our plain straightforward consideration.

In so far as we explore a system of strong interacting hadrons of different kinds, nucleons and pions, the WT identities hold true only for the total VF $\tilde{\Gamma}^a_{\mathrm{tot}\,\nu}$, and not separately for the $\tilde{\Gamma}^a_\mu$ itself.

The WT identities (3.36)–(3.40) simultaneously treat the weak and electromagnetic transitions both of nucleons and pions in the unified way. That may safely be pointed out as being a noteworthy distinction from what was done in Refs. [13–16].

Just for the very total electromagnetic VFs, i.e., for $a = 0$ in Eqs. (3.39), (3.40), the WT identities are

$$k^\nu \tilde{\Gamma}^0_{\mathrm{tot}\,\nu}(\{\psi\}N, p_N; N', p_N + k; k) = \\ = \ell^0_{NN'}[\tilde{G}^{-1}_{N'}(p_N + k) - \tilde{G}^{-1}_{N}(p_N)], \quad N, N' = n, p, \quad (4.1)$$

$$k^\nu \tilde{\Gamma}^0_{\mathrm{tot}\,\nu}(\{\pi\}b, p_b; -c, p_b + k; k) = \\ = \ell^0_{b(-c)}[\tilde{G}^{-1}_{-c}(p_b + k) - \tilde{G}^{-1}_{b}(p_b)], \quad b, c = 0, \pm, \quad (4.2)$$

which evidently manifest that the gauge invariance holds. Of course, one directly recognizes these equations as the familiar WT identities in the electrodynamics. Yet they now deal with the hadrons, strongly interacting composed particles. For the neutron, n, and neutral pion, π^0, Eqs. (4.1), (4.2) reduce to

$$k^\nu \tilde{\Gamma}^0_{\mathrm{tot}\,\nu}(\{\psi\}n, p_n; n, p_n + k; k) = k^\nu \tilde{\Gamma}^0_{\mathrm{tot}\,\nu}(\{\pi\}0, p_0; 0, p_0 + k; k) = 0. \quad (4.3)$$

Certainly, it does on no account mean that VFs

$$\tilde{\Gamma}^0_{\mathrm{tot}\,\nu}(\{\psi\}n, p_n; n, p_n + k; k) \text{ and } \tilde{\Gamma}^0_{\mathrm{tot}\,\nu}(\{\pi\}0, p_0; 0, p_0 + k; k)$$

themselves are equal to zero at arbitrary k. In fact, these quantities have by now been well measured in manifold experiments and evaluated in various plausible models.

In obtaining the generalized WT identities (3.36)–(3.40), the momentum k transferred by a considered transition has been nowhere presumed to be small in any sense, and therefore the outcome, i.e., Eqs. (3.36)–(3.40) and all the following Eqs. (4.1), (4.2), (4.3) . . . , holds true at arbitrary k values. Quite the contrary was the approach explicated in Ref. [13] (and thereupon utilized in Refs. [14–16]) in that the key starting point, see Eq. (9.11) in [13], was the momentum transferred which tended to zero, $k \to 0$.

What is procured by the principal Eqs. (3.36)–(3.40) is just the scalar product $k^\nu \tilde{\Gamma}^a_{\mathrm{tot}\,\nu}$ of the VF $\tilde{\Gamma}^a_{\mathrm{tot}\,\nu}$ and the momentum transferred k^ν, yet obviously not the VF $\tilde{\Gamma}^a_{\mathrm{tot}\,\nu}$ itself. For a simple apt illustration, let us point out what was said around Eq. (4.3). For that matter, we added any quantities of the form

$$\Gamma_{C\,\mu}(k) = (k_\mu - g_{\mu\alpha}k_\alpha) \cdot C(k^2) \text{ or } \Gamma_{C\,\mu}(k) = (\gamma_\mu\gamma^\alpha - \gamma^\alpha\gamma_\mu)k_\alpha \cdot C(k^2) \quad (4.4)$$

to a distinct VF $\tilde{\Gamma}^a_{\mathrm{tot}\,\mu}$, Eqs. (3.36)–(3.40) undergo no modifications at all.

The relations among the hadron propagators $\tilde{G}_i(p_i)$ and VFs $\tilde{\Gamma}^a_{\mathrm{tot}\,\nu}(k)$ and $M^a_{1\,\mu\nu}(k)$ themselves could be hoped to be of crucial value in calculating RC to the electroweak processes, in particular at an infinitesimal momentum transferred [14–17]. That is why now we try and explore Eqs. (3.36)–(3.40) at the limit

$k \to 0$, on the purpose to specify VFs $\tilde{\Gamma}^a_{\mathrm{tot}\,\nu}(0)$. Up to now, our consideration has been rigorous, to order e^2. It has been based on the symmetry principles and on the general relations of the Lagrange method. There have been made no loose presuppositions concerning the quantities involved into Eqs. (3.36)–(3.40). Yet, in order to proceed to acquire VFs $\tilde{\Gamma}^a_{\mathrm{tot}\,\nu}(0)$, we now presume all the terms in Eqs. (3.39), (3.40) to be regular at $k_\alpha = 0$. At least, let they and their first derivatives do really exist, have got finite values, at $k_\alpha = 0$. Furthermore, let the derivative in respect to k_α of the second terms on the right-hand sides of Eqs. (3.39), (3.40) be drawn under integral sign, so as the differentiation could be put onto the function $M^a_{1\mu\nu}(k)$ which resides in the respective integrand. Then, upon expanding all the terms in Eqs. (3.39), (3.40) in a power series in k_α and equating the coefficients by corresponding k_α powers, we would arrive, on these suppositions, at the following relations between the hadron VFs and propagators:

$$0 = \ell^a_{NN'}[\Sigma^{he}_N(p) - \Sigma^{he}_{N'}(p)]+ \\ + ae^2 \int \frac{\mathrm{d}q}{(2\pi)^4} D_{\mu\nu}(q) \cdot M^a_{1\mu\nu}(\{\psi\}N, p; N', p; -q, q), \quad (4.5)$$

$$0 = \ell^a_{b(-c)}[\Sigma^{he}_b(p)] - \Sigma^{he}_{-c}(p)]+ \\ + ae^2 \int \frac{\mathrm{d}q}{(2\pi)^4} D_{\mu\nu}(q) \cdot M^a_{1\,\mu\nu}(\{\pi\}b, p; -c, p; -q, q), \quad (4.6)$$

$$\tilde{\Gamma}^a_{\mathrm{tot}\,\lambda}(\{\psi\}N, p; N'p; 0) = \ell^a_{NN'}\gamma_\lambda \frac{1}{2}[Z^{-2}_{N'}(p) + Z^{-2}_N(p)]+ \\ + ae^2 \int \frac{\mathrm{d}q}{(2\pi)^4} D_{\mu\nu}(q) \partial_{k\,\lambda} M^a_{1\,\mu\nu}(\{\psi\}N, p; N', p+k; k-q, q)\,|_{k=0} \quad (4.7)$$

$$\tilde{\Gamma}^a_{\mathrm{tot}\,\lambda}(\{\pi\}b, p; -c, p; 0) = (2p_\lambda)\ell^a_{b(-c)} \frac{1}{2}[Z^{-2}_b(p) + Z^{-2}_{-c}(p)]+ \\ + ae^2 \int \frac{\mathrm{d}q}{(2\pi)^4} D_{\mu\nu}(q) \partial_{k\,\lambda} M^a_{1\,\mu\nu}(\{\pi\}b, p; -c, p+k; k-q, q)\,|_{k=0}, \quad (4.8)$$

$$N, N' = n, p, \quad a, b, c = 0, \pm,$$

where the familiar definitions are introduced

$$Z^{-2}_N(p) = \frac{\partial}{\partial \hat{p}} G^{-1}_N(p), \ \hat{p} \equiv \gamma^\mu p_\mu; \qquad Z^{-2}_b(p) = \frac{\partial}{\partial p^2} G^{-1}_b(p). \quad (4.9)$$

Equations (4.5), (4.6) apparently get trivial for a hadron system in an electromagnetic field, i.e., for $a = 0$, and they could, in general, serve to acquire the differences between mass operators of the neutral and charged members of a certain isomultiplet.

Next, to reduce further these formulae, let us, in turn, presume the formal approximations

$$Z_N^{-1}(p) \approx \sqrt{1 - \frac{\partial \Sigma_N^h(p)}{\partial \hat{p}}} \left(1 - \frac{1}{2}\frac{\partial \tilde{\Sigma}_N^{he}(p)}{\partial \hat{p}}\Big(1 - \frac{\Sigma_N^h(p)}{\partial \hat{p}}\Big)^{-1}\right), \tag{4.10}$$

$$Z_b^{-1}(p) \approx \sqrt{1 - \frac{\partial \Sigma_b^h(p)}{\partial p^2}} \left(1 - \frac{1}{2}\frac{\partial \tilde{\Sigma}_b^{he}(p)}{\partial p^2}\Big(1 - \frac{\Sigma_b^h(p)}{\partial p^2}\Big)^{-1}\right), \tag{4.11}$$

with recalling what was reasoned of concerning Eqs. (2.27), (2.28). Of course, one recognizes these quantities $Z_N(p),\ Z_b(p)$ giving at $\hat{p} = M_N, \quad p^2 = m_b^2$, i.e., on the mass shell, the renormalization constants of hadron states [1, 2, 18],

$$\mathcal{U}_N(p) = Z_N(p)u_N(p)\mid_{\hat{p}=M_N}, N = n, p,\ \ \mathcal{U}_b(p) = Z_b(p)u_b(p)\mid_{p^2=m_b^2}, b = 0, \pm, \tag{4.12}$$

where $u_N(p), u_b(p)$ are the wave amplitudes of real free nucleons and pions, and $\mathcal{U}_N(p),\ \mathcal{U}_b(p)$ stand for the respective renormalized amplitudes. Pursuing this approach, Eqs. (4.7), (4.8) are rewritten in the form

$$\begin{aligned} &Z_N(p)\tilde{\Gamma}^a_{\mathrm{tot}\,\lambda}(\{\psi\}N, p; N', p; 0)Z_{N'}(p) \approx \\ &\qquad \approx \ell^a_{NN'}\gamma_\lambda + ae^2 \int \frac{\mathrm{d}q}{(2\pi)^4} D_{\mu\nu}(q) Z_N(p) \frac{\partial}{\partial k_\lambda} \times \\ &\qquad\qquad \times M^a_{1\,\mu\nu}(\{\psi\}N, p; N', p+k; k-q, q)\mid_{k=0} Z_{N'}(p), \end{aligned} \tag{4.13}$$

$$\begin{aligned} &Z_b(p)\tilde{\Gamma}^a_{\mathrm{tot}\,\lambda}(\{\pi\}b, p; -c, p; 0)Z_{-c}(p) \approx \\ &\qquad \approx \ell^a_{b(-c)}(2p_\lambda) + ae^2 \int \frac{\mathrm{d}q}{(2\pi)^4} D_{\mu\nu}(q) Z_b(p) \frac{\partial}{\partial k_\lambda} \times \\ &\qquad\qquad \times M^a_{1\,\mu\nu}(\{\pi\}b, p; -c, p+k; k-q, q)\mid_{k=0} Z_{-c}(p). \end{aligned} \tag{4.14}$$

With allowance for Eqs. (4.10)–(4.12), these expressions would lead to the relations among matrix elements of VFs between the real hadron states, at zero momentum transferred,

$$\begin{aligned} &\bar{\mathcal{U}}_N(p)\tilde{\Gamma}^a_{\mathrm{tot}\,\lambda}(\{\psi\}N, p; N', p; 0)\mathcal{U}_{N'}(p) \approx \\ &\qquad \approx \ell^a_{NN'}\bar{u}_N(p)\gamma_\lambda u_{N'}(p) + ae^2\bar{\mathcal{U}}_N(p) \int \frac{\mathrm{d}q}{(2\pi)^4} D_{\mu\nu}(q) \frac{\partial}{\partial k_\lambda} \times \\ &\qquad\qquad \times M_{1\,\mu\nu}(\{\psi\}N, p; N', p+k; k-q, q)\mid_{k=0} \mathcal{U}_{N'}(p), \end{aligned} \tag{4.15}$$

$$\mathcal{U}_b^*(p)\tilde{\Gamma}^a_{\mathrm{tot}\,\lambda}(\{\pi\}b,p;-c,p;0)\mathcal{U}_{-c}(p) \approx$$
$$\approx \ell^a_{b(-c)}u_b^*(p)(2p_\lambda)u_{-c}(p) + ae^2\mathcal{U}_b^*(p)\int\frac{\mathrm{d}q}{(2\pi)^4}D_{\mu\nu}(q)\frac{\partial}{\partial k_\lambda}\times$$
$$\times M_{1\,\mu\nu}(\{\pi\}b,p;-c,p+k;k-q,q)\,|_{k=0}\,\mathcal{U}_{-c}(p). \quad (4.16)$$

Obviously, these identities are in perfect agreement with the requirement of gauge invariance, as all the foregoing were. In particular, they at $a=0$ reduce to the familiar relations that hold in the electrodynamics.

At first thought, the relations (4.15), (4.16) could now seem to be akin to Eq. (9.14) of Ref. [13] and to the analogous expressions in Refs. [14–17], for example, Eq. (10) in Ref. [14], Eq. (4.3) in Ref. [15]. Yet even at this stage, after all the aforesaid simplifying assumptions, such a kinship is actually ostensible because the quantities involved into (4.15), (4.16) and into Eq. (9.14) in Ref. [13] are, in fact, of different physical contents. This inference emerges in correlating the thread of our treatment with reasoning that leads to Eq. (9.14) in Ref. [13]. In particular, one must behold that in the sum (3.38) only the first term $\tilde{\Gamma}^a_\mu$ is due to the current $\tilde{J}^a_\mu$ (1.28), which causes VFs $M^a_{1\,\mu\nu}$ (3.11)–(3.13), whereas the second term $\tilde{\Gamma}^a_{2\,\mu}$ is determined by the quantity $\tilde{\jmath}^a_\mu$ that does not come into $M^a_{1\,\mu\nu}$.

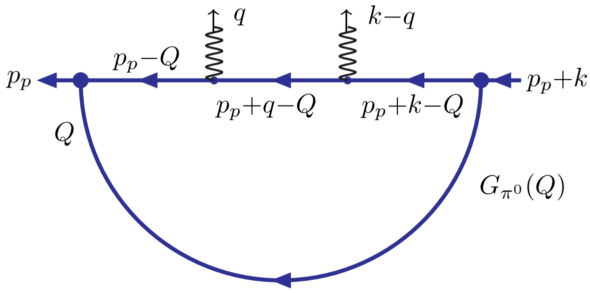
Next, it is to the point having a good look at how the quantities

$$M^a_{1\,\mu\nu}(\{\psi\}N,p_N;N',-p_{N'};k-q,q) \text{ and } M^a_{1\,\mu\nu}(\{\pi\}b,p_b;-c,-p_c;k-q,q)$$

defined in Eqs. (3.11)–(3.13) depend on the momenta q and k. Obviously, the momentum conservation $p_N+p_{N'}+k=0, \quad p_b+p_{-c}+k=0$ dictated by the respective δ-functions in Eqs. (3.12)–(3.18), (3.36)–(3.40) does not concern the variable of integration q that is perfectly independent of the momenta k,p_i,p_f. So, the functions $M^a_{1\,\mu\nu}$ determined by Eqs. (3.11)–(3.13) do separately depend on the variables $k-q$ and q themselves, and there is no way to transform the dependence on the variable q into the dependence on the variable k or $k-q$. Consequently, the derivatives $\partial M^a_{1\,\mu\nu}(k-q,q)/\partial k_\alpha$ which occur in Eqs. (4.7), (4.8) cannot be transformed into the derivatives $\partial M^a_{1\,\mu\nu}(k-q,q)/\partial q_\alpha$. As a mere pertinent elucidation, let us consider the quantity $M^a_{1\,\mu\nu}(\{\psi\}p,p_p;n,p_p+k;k-q,q)$ obtained in the case that the pion–nucleon interaction (1.27) is taken into account in the lowest feasible, second, order. It incorporates, along with others, the contribution

$$g^2_{NN\pi}\int\frac{\mathrm{d}Q}{(2\pi)^4}\bar{u}_p(p_p)\gamma^5G_p(p_p-Q)\gamma_\mu G_p(q+p_p-Q)\times$$
$$\times\gamma_\nu G_n(p_p+k-Q)\gamma^5u_n(p_p+k)G_{\pi^0}(Q) \quad (4.17)$$

of the process depicted by the diagram:



. (4.18)

As seen, the contribution (4.17) to $M^a_{1\mu\nu}(\{\psi\}N, p_N; N', -p_{N'}; k-q, q)$ does apparently depend on q and k separately. What is here significant, as well as in the general case considered, is that the strong interaction between hadrons of different sorts is involved, at least in the lowest order.

Thus, there is no way to rearrange the second terms on the right-hand sides in Eqs. (4.7), (4.8), (4.13)–(4.16) so that they would contain the integrands

$$D_{\mu\nu}(q)\frac{\partial M^a_{1\mu\nu}(k-q,q)}{\partial q_\alpha} \quad \text{in place of} \quad D_{\mu\nu}(q)\frac{\partial M^a_{1\mu\nu}(k-q,q)}{\partial k_\alpha}, \tag{4.19}$$

and then the differentiation in respect to q_α could be put onto $D_{\mu\nu}(q)$, by integrating by parts.

So, the integrands in Eqs. (4.7)–(4.16) can not be transformed to the form

$$\begin{gathered}\frac{\partial D_{\mu\nu}(q)}{\partial q_\alpha} M^a_{1\mu\nu}(\{\psi\}N, p_N; N', -p_{N'}; k-q, q)\,|_{k=0},\\ \text{and}\\ \frac{\partial D_{\mu\nu}(q)}{\partial q_\alpha} M^a_{1\mu\nu}(\{\pi\}b, p_b; -c, -p_{-c}; k-q, q)\,|_{k=0},\end{gathered} \tag{4.20}$$

as opposed to what was asserted in the work [13] concerning the integrand in Eq. (9.14), and then was posited in the ensuing computations in Refs. [14–16], as a matter of fact. The presumptive transformation of the integrands in Eqs. (4.15), (4.16) to the form (4.20) is a substantial point in pursuing the approach of Refs. [13–17]. This was quite necessary to arrive at the essential inference, proclaimed in [16, 17], that RC to the β-decay amplitude would merely have been multiple to the uncorrected amplitude itself. In turn, this very result would have promoted to avoid the immediate allowance for hadron compositeness in RC computing. Actually, it does not hold true.

All the presented discussion persuades us that the handy issues of Refs. [14–17] can not hold with the actual calculation of RC to the hadron

β-decay. As understood, there sees no strict way from the principle WT identities (3.36)–(3.40), dealing with the scalar product $k^\mu\tilde{\Gamma}^a_\mu(k)$ at arbitrary k-values, to the relations (4.15), (4.16) between the propagators G_h and VFs $\tilde{\Gamma}^a_\mu(0)$, $M_{1,\lambda\nu}(0)$ themselves at $k_\mu = 0$, which could be conjectured to promote the RC computation in treating the neutron and pion β-decay. Moreover, it is even not clear how to estimate the accuracy of several loose presuppositions needful to pass this way. All the more so, there is no reason to warrant the transformation of the integrands in Eqs. (4.15), (4.16) as was discussed just above, around Eqs. (4.17)–(4.20). Therefore, even if the RC computation underlain by Eqs. (4.15), (4.16) was somehow carried out, all the more with utilizing transformations (4.19), (4.20), the inescapable ambiguities inherent in the outcome could not be safely estimated.

At every stage of the consideration presented in the work, especially in Sec. 4, we have been realizing that the relations asserted and utilized in Refs. [13–17] are not in line with the plain findings of the consistent inquiry into the actual relations among the propagators and electroweak VFs of strongly interacting nucleons and pions. As seen, the differences especially take their rise in respect of needful accounting for strong interactions in a system incorporating hadrons of different sorts. Evidently, the plain inferences of the presented consistent consideration can not uphold the approach upon which the RC calculations [14–17] are based. Thus, high as one appreciates the scientific significance of advances of the investigations [14–17], one cannot help avowing, bold though it may seem, that these RC investigations are flawy. It especially concerns the very high computation accuracy, $\sim 0.01\%$, proclaimed in the aforesaid papers [16,17]. This assertion is all the more deceptive that the hadron axial-current interaction with the gauge fields intrudes into the RC computation even in the pion β-decay case, i.e., $0^-\!\to\!0^-$ transition [20,21], though the e-zero-order matrix element of an axial current evidently vanishes in this case. Needless to say, how still more complex and tangle the calculation becomes in the neutron β-decay case that the interaction caused by the axial current is involved just on the e-zero-order level. Resort to the WT identities could at best have promoted to cope with the strong interaction effects in calculating RC to the pure vector interactions in hadron semileptonic decays. To get over the hardships of allowance for hadron compositeness in the case that axial current coming into a considered process, some untenable vague reasoning was expounded [16] that is not persuasive, especially in so far as the accuracy $\sim 1\%$ or better goes. In particular, there proves no reason for presuming RC to the β-decay amplitude to be simply proportional to the uncorrected amplitude. So, even if one had somehow completely circumvented, with a conceivable recourse to any WT identities, the actual allowance for hadron compositeness in computing RC to the pure vector interaction, the total RCs to any semileptonic process could not have been safely obtained in such a way with the sufficient accuracy, against what was asserted in Refs. [16, 17]. One cannot help realizing the inescapable tangible ambiguities inherent into the outcome of the computations [14–17]. All the more in respect that the UV divergences which occur in RC calculating were not treated amenably to the up-to-date on-mass-shell renormanization scheme in the SM framework [1, 2, 8, 9], but they were removed by the obsolete «cut-off» precept, as a matter of fact, with the additional ambiguities intruded thereby.

In fine, in calculating RC to the electroweak processes, the consistent trustworthy allowance for the hadron compositeness and strong interaction still persists pending. Sure enough, this does not mean to say that the WT identities cannot be of service in treating RC. Obtained the relations (3.36)–(3.40), we are on the point of utilizing them to advance in calculating RC in due course. In this respect, the chiral perturbation theory [24, 26, 27] is thought to be resort to. For now, all-round renewal of the RC calculation gets the current objective. As to our mind, there sees no option but parameterizing the effects of hadron structure and strong interactions. The parameters entailed thereby are to be specified by due processing the needful high precision experimental data on various characteristics of the electroweak processes.

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