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E. Jurčišinová*

AN INTEGRAL FORMULA FOR MULTI-LOOP CALCULATIONS IN QUANTUM FIELD THEORY

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*Institute of Experimental Physics, SAS, Watsonova 47, 04001 Košice, Slovakia

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Интегральная формула для многопетлевых вычислений
в квантовой теории поля

Дано доказательство одной общей интегральной формулы для аналитических вычислений многопетлевых диаграмм Фейнмана в моделях квантовой теории поля.

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An Integral Formula for Multi-Loop Calculations in Quantum Field Theory

The proof of a general integral formula for analytical calculations of multi-loop Feynman diagrams in quantum field theory models is given.

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When applying quantum-field theory methods to arbitrary physical problem (e.g., in elementary particles physics, condensed matter physics, statistical mechanics, critical dynamics, stochastic dynamics, etc.), the necessity to calculate, in general, multi-loop Feynman diagrams usually appears [1–3]. In this respect, the one-loop calculations are always relatively simple, and this fact usually allows one to make full analysis of the problem. Beautiful demonstration of such calculations was given, e.g., in [4], where a formula for reducing n-point Feynman diagrams to scalar integrals was presented. On the other hand, the corresponding calculations become much more complicated when one wants to make similar analysis in two- and/or higher-loop approximation. Despite the fact that in such a case it is usually impossible to make complete analytical calculations, a general integral formula for reducing tensor momentum integrals to scalar ones can also be found. The formula was explicitly shown in [5], where it was present without proof and used in two-loop calculations in field-theoretic model of passive scalar advected by given turbulent environment. The aim of the present work is to prove the formula in detail.

Thus, in what follows, we shall find and prove an analytical representation for the following general tensor integrals:

$$\int \dots \int d\mathbf{k}^{(1)} \dots d\mathbf{k}^{(l)} \frac{k_{i_1}^{(q_1)} k_{i_2}^{(q_2)} \dots k_{i_n}^{(q_n)}}{\left[\sum\limits_{i=1}^{l} \sum\limits_{s=1}^{l} v_{is} \mathbf{k}^{(i)} \cdot \mathbf{k}^{(s)} + 2 \sum\limits_{i=1}^{l} \mathbf{a}^{(i)} \cdot \mathbf{k}^{(i)} + c\right]^{\alpha}}, \tag{1}$$

where l and n are natural numbers, $\mathbf{k}^{(i)}$ and $\mathbf{a}^{(i)}, i = 1, 2, \dots, l$ are vectors in d-dimensional real Euclidean vector space, $k_j^{(i)}$ denotes jth component of the vector $\mathbf{k}^{(i)}$, v_{is} is an arbitrary symmetric $l \times l$ real matrix, c and α are arbitrary real numbers, $\mathbf{x} \cdot \mathbf{y} = \sum\limits_{i=1}^{d} x_i y_i$ is the scalar product, and integrations are taken over d-dimensional Euclidean space.

Before we formulate the corresponding theorem, let us briefly describe the process which leads to tensor integrals (1). For simplicity we shall work in the Euclidean space, but the procedure can be directly generalized to the pseudo-Euclidean space.

Typical l-loop Feynman diagram is proportional to the combination of the integrals of the following type:

$$\int \dots \int d\mathbf{k}^{(1)} \dots d\mathbf{k}^{(l)} \frac{k_{i_1}^{(q_1)} k_{i_2}^{(q_2)} \dots k_{i_n}^{(q_n)}}{A_1^{\alpha_1} A_2^{\alpha_2} \cdots A_m^{\alpha_m}}, \tag{2}$$

where $A_i, i = 1, \dots, m$ are some polynomials of the second order with respect to independent momenta (wave-vectors) $\mathbf{k}^{(i)}, i = 1, \dots, l$ and $\alpha_i, i = 1, \dots, m$ are some, in general, real numbers. Further, by using the well-known Feynman parametrization procedure [1,2], which is given by the following relation:

$$\frac{1}{A_1^{\alpha_1} A_2^{\alpha_2} \cdots A_m^{\alpha_m}} = \frac{\Gamma\Big(\sum\limits_{i=1}^{m} \alpha_i\Big)}{\prod\limits_{i=1}^{m} \Gamma(\alpha_i)} \int\limits_0^1 \dots \int\limits_0^1 du_1 \dots du_m \frac{\delta\Big(\sum\limits_{i=1}^{m} u_i - 1\Big) \prod\limits_{i=1}^{m} u_i^{\alpha_i - 1}}{\Big(\sum\limits_{i=1}^{m} A_i u_i\Big)^{\sum\limits_{i=1}^{m} \alpha_i}}, \tag{3}$$

where $\delta(x)$ is the Dirac δ-function, the integration over momenta $\mathbf{k}^{(i)}, i = 1, \dots, l$ is reduced to integrals of the form presented in (1) with the corresponding symmetric real $l \times l$ matrix v_{is}, l vectors $\mathbf{a}^i$, and some real quantities c and α which are independent of momenta $\mathbf{k}^{(i)}, i = 1, \dots, l$.

Thus, to proceed it is necessary to calculate the general integral (1). It is the subject of the following theorem which represents the integrals of the form (1) in appropriate analytical form which is convenient for further analysis.

Theorem: *Let $\mathcal{V}$ be the d-dimensional Euclidean vector space over the field of real numbers $\mathbb{R}$. Let $l, n \in \mathbb{N}$ (natural numbers), and $\mathbf{k}^{(i)}$, for $i = 1, 2, \dots, l$ be vectors in $\mathcal{V}$. Then for an arbitrary $l \times l$ symmetric real matrix with* $\det v \neq 0$, *arbitrary vectors $\mathbf{a}^{(i)}$ ($i = 1, 2, \dots, l$), and arbitrary $c, \alpha \in \mathbb{R}$, the following general formula holds*:

$$\begin{aligned}
\int \dots \int \frac{d\mathbf{k}^{(1)} \dots d\mathbf{k}^{(l)} k_{i_1}^{(q_1)} k_{i_2}^{(q_2)} \dots k_{i_n}^{(q_n)}}{\left[v_{is} \mathbf{k}^{(i)} \cdot \mathbf{k}^{(s)} + 2\mathbf{a}^{(i)} \cdot \mathbf{k}^{(i)} + c\right]^{\alpha}} &= \frac{(-1)^n \pi^{\frac{dl}{2}} (\det v)^{-\frac{d}{2}}}{\Gamma(\alpha)} \times \\
\times \sum_{p=0}^{\left[\frac{n}{2}\right]} \frac{\Gamma\left(\alpha - \dfrac{dl}{2} - \left[\dfrac{n}{2}\right] + p\right) \left[c - (v^{-1})_{is} \mathbf{a}^{(i)} \cdot \mathbf{a}^{(s)}\right]^{\left[\frac{n}{2}\right] + \frac{dl}{2} - \alpha - p}}{\left(\left[\dfrac{n}{2}\right] - p\right)! \, (2p + n(\mathrm{mod}\ 2))! 4^{\left[\frac{n}{2}\right] - \mathrm{p}}} &\times \\
\times \sum_{P(r_1, r_2, \dots, r_n)} (v^{-1})_{q_1 s_1} a_{i_1}^{(s_1)} (v^{-1})_{q_2 s_2} a_{i_2}^{(s_2)} \dots (v^{-1})_{q_{2p+n(\mathrm{mod}\ 2)} s_{2p+n(\mathrm{mod}\ 2)}} &\times \\
\times a_{i_{2p+n(\mathrm{mod}\ 2)}}^{(s_{2p+n(\mathrm{mod}\ 2)})} \delta_{i_{2p+n(\mathrm{mod}\ 2)+1} i_{2p+n(\mathrm{mod}\ 2)+2}} &\times \\
\times (v^{-1})_{q_{2p+n(\mathrm{mod}\ 2)+1} q_{2p+n(\mathrm{mod}\ 2)+2}} \dots \delta_{i_{n-1} i_n} (v^{-1})_{q_{n-1} q_n}, &
\end{aligned} \tag{4}$$

where summation is taken over all simultaneous permutations of couples of indices $r_j = \{q_j, i_j\}, j = 1, \dots, n$; $k_j^{(s)}$ and $a_j^{(s)}$ are j-th components of the vectors $\mathbf{k}^{(s)}$ and $\mathbf{a}^{(s)}$, respectively; δ_{ij} denotes Kronecker delta; v^{-1} is the inverse matrix; $\lfloor n/2 \rfloor = n/2$ for an even n, and $\lfloor n/2 \rfloor = (n-1)/2$ for an odd n, and over all dummy indices the corresponding summation is assumed.

Proof: To prove formula (4) it is appropriate to use mathematical induction. First of all, the theorem is correct for the scalar case with $n = 0$. In this specific situation the formula is well known (see, e.g., Ref. [3]), namely,

$$\int \dots \int \frac{d\mathbf{k}^{(1)} \dots d\mathbf{k}^{(l)}}{\left[v_{is}\mathbf{k}^{(i)} \cdot \mathbf{k}^{(s)} + 2\mathbf{a}^{(i)} \cdot \mathbf{k}^{(i)} + c\right]^{\alpha}} = \\ = \frac{\pi^{\frac{dl}{2}} (\det v)^{-\frac{d}{2}} \Gamma\left(\alpha - \dfrac{dl}{2}\right)}{\Gamma(\alpha)} \left[c - v^{-1}_{is}\mathbf{a}^{(i)} \cdot \mathbf{a}^{(s)}\right]^{\frac{dl}{2}-\alpha}, \quad (5)$$

where, for simplicity, we use the following suitable notation: $v^{-1}_{is} \equiv (v^{-1})_{is}$. This notation will be used within the whole proof.

Now, by differentiating both sides of equation (5) with respect to $a^{(q_1)}_{i_1}$ and by subsequent replacing $\alpha \to \alpha - 1$, one obtains

$$\int \dots \int d\mathbf{k}^{(1)} \dots d\mathbf{k}^{(l)} \frac{k^{(q_1)}_{i_1}}{\left[v_{is}\mathbf{k}^{(i)} \cdot \mathbf{k}^{(s)} + 2\mathbf{a}^{(i)} \cdot \mathbf{k}^{(i)} + c\right]^{\alpha}} = \\ = (-1) \frac{\pi^{\frac{dl}{2}} (\det v)^{-\frac{d}{2}} \Gamma\left(\alpha - \dfrac{dl}{2}\right)}{\Gamma(\alpha)} \left[c - v^{-1}_{is}\mathbf{a}^{(i)} \cdot \mathbf{a}^{(s)}\right]^{\frac{dl}{2}-\alpha} v^{-1}_{q_1 s} a^{(s)}_{i_1},$$

which is exactly the integral (4) for $n = 1$.

To proceed, it is suitable to define the following notation:

$$C_{(i_1,\dots,i_n;s_1,\dots,s_t)} = \\ = v^{-1}_{q_1 s_1} a^{(s_1)}_{i_1} v^{-1}_{q_2 s_2} a^{(s_2)}_{i_2} \dots v^{-1}_{q_t s_t} a^{(s_t)}_{i_t} \delta_{i_{t+1} i_{t+2}} v^{-1}_{q_{t+1} q_{t+2}} \dots \delta_{i_{n-1} i_n} v^{-1}_{q_{n-1} q_n}. \quad (6)$$

Further, let us suppose that formula (4) is valid for an even $n \in \mathbb{N},\ n \geqslant 0$, i.e.,

$$\int \dots \int d\mathbf{k}^{(1)} \dots d\mathbf{k}^{(l)} \frac{k^{(q_1)}_{i_1} k^{(q_2)}_{i_2} \dots k^{(q_n)}_{i_n}}{\left[v_{is}\mathbf{k}^{(i)} \cdot \mathbf{k}^{(s)} + 2\mathbf{a}^{(i)} \cdot \mathbf{k}^{(i)} + c\right]^{\alpha}} = \\ = \frac{(-1)^n \pi^{\frac{dl}{2}} (\det v)^{-\frac{d}{2}}}{\Gamma(\alpha)} \times \\ \times \sum_{p=0}^{\frac{n}{2}} \frac{\Gamma\left(\alpha - \dfrac{dl}{2} - \dfrac{n}{2} + p\right) \left[c - v^{-1}_{is}\mathbf{a}^{(i)} \cdot \mathbf{a}^{(s)}\right]^{\frac{n}{2}+\frac{dl}{2}-\alpha-p}}{\left(\dfrac{n}{2} - p\right)! \, (2p)! \, 4^{\frac{n}{2}-p}} \times \\ \times \sum_{P(r_1, r_2, \dots, r_n)} C_{(i_1,\dots,i_n;s_1,\dots,s_{2p})}. \quad (7)$$

By differentiating both sides of equation (7) with respect to $a_{i_{n+1}}^{(q_{n+1})}$, one obtains

$$\int \dots \int d\mathbf{k}^{(1)} \dots d\mathbf{k}^{(l)} \frac{k_{i_1}^{(q_1)} k_{i_2}^{(q_2)} \dots k_{i_n}^{(q_n)} k_{i_{n+1}}^{(q_{n+1})}}{\left[v_{is}\mathbf{k}^{(i)} \cdot \mathbf{k}^{(s)} + 2\mathbf{a}^{(i)} \cdot \mathbf{k}^{(i)} + c\right]^{\alpha+1}} =$$
$$= \frac{(-1)^{n+1}\pi^{\frac{dl}{2}}(\det v)^{-\frac{d}{2}}}{\Gamma(\alpha+1)} \times$$
$$\times \sum_{p=0}^{\frac{n}{2}} \frac{\Gamma\left(\alpha + 1 - \dfrac{dl}{2} - \dfrac{n}{2} + p\right)\left[c - v_{is}^{-1}\mathbf{a}^{(i)} \cdot \mathbf{a}^{(s)}\right]^{\frac{n}{2}+\frac{dl}{2}-\alpha-p-1}}{\left(\dfrac{n}{2} - p\right)!\,(2p)!4^{\frac{n}{2}-p}} \times$$
$$\times\ v_{q_{n+1}s}^{-1} a_{i_{n+1}}^{(s)} \sum_{P(r_1,r_2,\dots,r_n)} C_{(i_1,\dots,i_n;s_1,\dots,s_{2p})} + \frac{(-1)^{n+1}\pi^{\frac{dl}{2}}(\det v)^{-\frac{d}{2}}}{\Gamma(\alpha+1)} \times$$
$$\times \sum_{p=0}^{\frac{n}{2}} \frac{\Gamma\left(\alpha - \frac{dl}{2} - \dfrac{n}{2} + p\right)\left[c - v_{is}^{-1}\mathbf{a}^{(i)} \cdot \mathbf{a}^{(s)}\right]^{\frac{n}{2}+\frac{dl}{2}-\alpha-p}}{\left(\dfrac{n}{2} - p\right)!\,(2p)!4^{\frac{n}{2}-p+\frac{1}{2}}} \times$$
$$\times \sum_{P(r_1,r_2,\dots,r_n)} \Big[v_{q_1 q_{n+1}}^{-1}\delta_{i_1 i_{n+1}}\ C_{(i_2,\dots,i_n;s_2,\dots,s_{2p})} + v_{q_2 q_{n+1}}^{-1}\delta_{i_2 i_{n+1}} \times$$
$$\times\ C_{(i_1,i_3,\dots,i_n;s_1,s_3,\dots,s_{2p})} + \dots + v_{q_{2p} q_{n+1}}^{-1}\delta_{i_{2p} i_{n+1}} \times$$
$$\times\ C_{(i_1,\dots,i_{2p-1},i_{2p+1},\dots i_n;s_1,\dots,s_{2p-1})}\Big].$$

Now, using the substitution $\alpha \to \alpha - 1$ and after appropriate algebraic manipulations, one obtains

$$\int \dots \int d\mathbf{k}^{(1)} \dots d\mathbf{k}^{(l)} \frac{k_{i_1}^{(q_1)} k_{i_2}^{(q_2)} \dots k_{i_n}^{(q_n)} k_{i_{n+1}}^{(q_{n+1})}}{\left[v_{is}\mathbf{k}^{(i)} \cdot \mathbf{k}^{(s)} + 2\mathbf{a}^{(i)} \cdot \mathbf{k}^{(i)} + c\right]^{\alpha}} =$$
$$= \frac{(-1)^{n+1}\pi^{\frac{dl}{2}}(\det v)^{-\frac{d}{2}}}{\Gamma(\alpha)} \sum_{p=0}^{\frac{(n+1)-1}{2}} \times$$
$$\times \frac{\Gamma\left(\alpha - \dfrac{dl}{2} - \dfrac{(n+1)-1}{2} + p\right)\left[c - v_{is}^{-1}\mathbf{a}^{(i)} \cdot \mathbf{a}^{(s)}\right]^{\frac{(n+1)-1}{2}+\frac{dl}{2}-\alpha-p}}{\left(\dfrac{(n+1)-1}{2} - p\right)!\,(2p+1)!4^{\frac{(n+1)-1}{2}-p}} \times$$
$$\times \sum_{P(r_1,r_2,\dots,r_{n+1})} C_{(i_1,\dots,i_{n+1};s_1,\dots,s_{2p+1})},$$

which is exactly formula (4) for odd values of $n+1$ (to obtain the exact form as in (4), it is necessary to make the substitution $n+1 \to n$).

Now, let us suppose that formula (4) is valid for an odd $n \in \mathbb{N},\ n \geqslant 1$. In this case, formula (4) takes the following form $\left(\left[\frac{n}{2}\right] = \frac{n-1}{2} \text{ and } n(\operatorname{mod} 2) = 1 \right)$:

$$\int \dots \int d\mathbf{k}^{(1)} \dots d\mathbf{k}^{(l)} \frac{k_{i_1}^{(q_1)} k_{i_2}^{(q_2)} \dots k_{i_n}^{(q_n)}}{\left[v_{is}\mathbf{k}^{(i)} \cdot \mathbf{k}^{(s)} + 2\,\mathbf{a}^{(i)} \cdot \mathbf{k}^{(i)} + c\right]^{\alpha}} =$$

$$= \frac{(-1)^n \pi^{\frac{dl}{2}} (\det v)^{-\frac{d}{2}}}{\Gamma(\alpha)} \times$$

$$\times \sum_{p=0}^{\frac{n-1}{2}} \frac{\Gamma\left(\alpha - \frac{dl}{2} - \frac{n-1}{2} + p\right) \left[c - v_{is}^{-1}\mathbf{a}^{(i)} \cdot \mathbf{a}^{(s)}\right]^{\frac{n-1}{2} + \frac{dl}{2} - \alpha - p}}{\left(\frac{n-1}{2} - p\right)!\,(2p+1)!\,4^{\frac{n-1}{2} - p}} \times$$

$$\times \sum_{P(r_1, r_2, \dots, r_n)} C_{(i_1, \dots, i_n; s_1, \dots, s_{2p+1})}. \tag{8}$$

Again, by differentiating both sides of equation (8) with respect to $a_{i_{n+1}}^{(q_{n+1})}$, one obtains

$$\int \dots \int d\mathbf{k}^{(1)} \dots d\mathbf{k}^{(l)} \frac{k_{i_1}^{(q_1)} k_{i_2}^{(q_2)} \dots k_{i_n}^{(q_n)} k_{i_{n+1}}^{(q_{n+1})}}{\left[v_{is}\mathbf{k}^{(i)} \cdot \mathbf{k}^{(s)} + 2\mathbf{a}^{(i)} \cdot \mathbf{k}^{(i)} + c\right]^{\alpha+1}} =$$

$$= \frac{(-1)^{n+1} \pi^{\frac{dl}{2}} (\det v)^{-\frac{d}{2}}}{\Gamma(\alpha+1)} \times$$

$$\times \sum_{p=0}^{\frac{n-1}{2}} \frac{\Gamma\left(\alpha - \frac{dl}{2} - \frac{n-1}{2} + p + 1\right) \left[c - v_{is}^{-1}\mathbf{a}^{(i)} \cdot \mathbf{a}^{(s)}\right]^{\frac{n-1}{2} + \frac{dl}{2} - \alpha - p - 1}}{\left(\frac{n-1}{2} - p\right)!\,(2p+1)!4^{\frac{n-1}{2} - p}} \times$$

$$\times\ v_{q_{n+1}s}^{-1} a_{i_{n+1}}^{(s)} \sum_{P(r_1, r_2, \dots, r_n)} C_{(i_1, \dots, i_n; s_1, \dots, s_{2p+1})} + \frac{(-1)^{n+1} \pi^{\frac{dl}{2}} (\det v)^{-\frac{d}{2}}}{\Gamma(\alpha+1)} \times$$

$$\times \sum_{p=0}^{\frac{n-1}{2}} \frac{\Gamma\left(\alpha - \frac{dl}{2} - \frac{n-1}{2} + p\right) \left[c - v_{is}^{-1}\mathbf{a}^{(i)} \cdot \mathbf{a}^{(s)}\right]^{\frac{n-1}{2} + \frac{dl}{2} - \alpha - p}}{\left(\frac{n-1}{2} - p\right)!\,(2p+1)!4^{\frac{n-1}{2} - p + \frac{1}{2}}} \times$$

$$\times \sum_{P(r_1,r_2,\ldots,r_n)} \Big[v^{-1}_{q_1 q_{n+1}} \delta_{i_1 i_{n+1}} \, C_{(i_2,\ldots,i_n;s_2,\ldots,s_{2p+1})} + v^{-1}_{q_2 q_{n+1}} \delta_{i_2 i_{n+1}} \times$$
$$\times \, C_{(i_1,i_3,\ldots,i_n;s_1,s_3,\ldots,s_{2p+1})} +$$
$$+ \cdots + v^{-1}_{q_{2p+1} q_{n+1}} \delta_{i_{2p+1} i_{n+1}} \, C_{(i_1,\ldots,i_{2p},i_{2p+2},\ldots i_n;s_1,\ldots,s_{2p})} \Big] .$$

Thus, using the substitution $\alpha \to \alpha - 1$ and rather cumbersome but straightforward algebraic and combinatoric manipulations, one obtains the final result:

$$\int \ldots \int d\mathbf{k}^{(1)} \ldots d\mathbf{k}^{(l)} \frac{k^{(q_1)}_{i_1} k^{(q_2)}_{i_2} \ldots k^{(q_n)}_{i_n} k^{(q_{n+1})}_{i_{n+1}}}{\left[v_{is} \mathbf{k}^{(i)} \cdot \mathbf{k}^{(s)} + 2\mathbf{a}^{(i)} \cdot \mathbf{k}^{(i)} + c \right]^{\alpha}} =$$
$$= \frac{(-1)^{n+1} \pi^{\frac{dl}{2}} (\det v)^{-\frac{d}{2}}}{\Gamma(\alpha)} \times$$
$$\times \sum_{p=0}^{\frac{n+1}{2}} \frac{\Gamma\left(\alpha - \frac{dl}{2} - \frac{n+1}{2} + p \right) \left[c - v^{-1}_{is} \mathbf{a}^{(i)} \cdot \mathbf{a}^{(s)} \right]^{\frac{n+1}{2} + \frac{dl}{2} - \alpha - p}}{\left(\frac{n+1}{2} - p \right)! \, (2p)! 4^{\frac{n+1}{2} - p}} \times$$
$$\times \sum_{P(r_1,r_2,\ldots,r_{n+1})} C_{(i_1,\ldots,i_{n+1};s_1,\ldots,s_{2p})},$$

which is exactly equal to formula (4) for even values of $n+1$ (again, to obtain the exact form as in (4), it is necessary to make the substitution $n+1 \to n$).

Thus, we prove the formula given in Eq. (4) by using the method of mathematical induction in two steps. First, from an assumption of validity of the formula for even values of n we have obtained the formula for odd values of $n+1$, and at the second stage we have obtained the formula for even values of $n+1$ from assumption of validity of the formula for odd values of n. ■

Therefore, by applying the Feynman parametrization formula (3) to the typical l-loop integrals (2) and by subsequently applying the above-proven theorem (4), the integrals (2) are reduced to the integrals over parameters $u_i, i = 1, \ldots, m$ which must be analyzed separately.

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REFERENCES

1. *Ramond P.* Field Theory: A Modern Primer. Westview, 2001.
2. *Zinn-Justin J.* Quantum Field Theory and Critical Phenomena. Oxford: Clarendon, 1989.

3. *Vasil'ev A. N.* Quantum-Field Renormalization Group in the Theory of Critical Phenomena and Stochastic Dynamics. St. Petersburg: St. Petersburg Institute of Nuclear Physics, 1998.

4. *Davydyshev A. I.* A Simple Formula for Reducing Feynman Diagrams to Scalar Integrals // Phys. Lett. B. 1991. V. 263. P. 107.

5. *Chkhetiani O. G., Hnatich M., Jurčišinová E., Jurčišin M., Mazzino A., Repašan M.* Influence of Helicity on Anomalous Scaling of a Passive Scalar Advected by the Turbulent Velocity Field with Finite Correlation Time: Two-Loop Approximation // Phys. Rev. E. 2006. V. 74. P. 036310.

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141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.
E-mail: publish@jinr.ru
www.jinr.ru/publish/