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SPATIAL DEPENDENCE OF PAIR CORRELATIONS (NUCLEAR SCISSORS)

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Бальбуцев Е. Б., Малов Л. А. E4-2009-148
Пространственная зависимость парных корреляций
(ядерные ножницы)

Решение зависящих от времени уравнений Хартри–Фока–Боголюбова методом моментов функции Вигнера приводит к появлению низколежащих мод. Для описания этих возбуждений необходимо точное знание аномальной матрицы плотности. Показано, что расчеты с потенциалом Вудса–Саксона удовлетворяют таким требованиям.

Работа выполнена в Лаборатории теоретической физики им. Н. Н. Боголюбова ОИЯИ.

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Balbutsev E. B., Malov L. A. E4-2009-148
Spatial Dependence of Pair Correlations
(Nuclear Scissors)

The solution of time-dependent Hartree–Fock–Bogoliubov equations by the Wigner function moments method leads to the appearance of low-lying modes whose description requires accurate knowledge of the anomalous density matrix. It is shown that calculations with the Woods–Saxon potential satisfy this requirement.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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INTRODUCTION

The problem of the spatial dependence of the pairing field is at the moment the object of the highest interest of nuclear theorists [1–3] because of the necessity to explain the properties of nuclei disposed far from the beta-stability line. We met this problem when studying the nuclear scissors mode. It is known [4,5] that one must take into account pair correlations to describe correctly nuclear scissors, therefore Time-Dependent Hartree–Fock–Bogoliubov (TDHFB) equations should be the natural instrument to work with. We apply the method of Wigner Function Moments (WFM), or phase space moments, for their solution. To this end we write TDHFB equations in phase space and calculate their various second rank moments which serve as the collective variables of the method. In such a way one derives dynamical equations for quadrupole moments, angular moments and other neutron and proton variables. The relative motion of neutron and proton angular moments generates the scissors mode.

Along with isovector excitations (scissors and the Isovector Giant Quadrupole Resonance (IVGQR)) the derived equations describe also isoscalar modes — the IsoScalar Low-Lying Excitation (ISLLE) and giant quadrupole resonance. The appearance of ISLLE is quite interesting, because it is originated only by quantum corrections to the semiclassical limit of TDHFB equations, that says about their complicate structure. Naturally, one cannot use semiclassical expressions for the anomalous density and pairing field to describe such excitations — one needs quantum mechanical expressions, which can be found by solving static HFB equations, which is the subject of this paper.

1. PAIRING

Pair correlations are taken into account by working with TDHFB equations. Their detailed form is

$$\begin{aligned}
i\hbar\dot{\hat{\rho}} &= \hat{h}\hat{\rho} - \hat{\rho}\hat{h} - \hat{\Delta}\hat{\kappa}^{\dagger} + \hat{\kappa}\hat{\Delta}^{\dagger}, \\
-i\hbar\dot{\hat{\kappa}} &= -\hat{h}\hat{\kappa} - \hat{\kappa}\hat{h}^{*} + \hat{\Delta} - \hat{\Delta}\hat{\rho}^{*} - \hat{\rho}\hat{\Delta}, \\
-i\hbar\dot{\hat{\rho}}^{*} &= \hat{h}^{*}\hat{\rho}^{*} - \hat{\rho}^{*}\hat{h}^{*} - \hat{\Delta}^{\dagger}\hat{\kappa} + \hat{\kappa}^{\dagger}\hat{\Delta}, \\
-i\hbar\dot{\hat{\kappa}}^{\dagger} &= \hat{h}^{*}\hat{\kappa}^{\dagger} + \hat{\kappa}^{\dagger}\hat{h} - \hat{\Delta}^{\dagger} + \hat{\Delta}^{\dagger}\hat{\rho} + \hat{\rho}^{*}\hat{\Delta}^{\dagger},
\end{aligned} \tag{1}$$

where $\hat{\rho}$ and $\hat{\kappa}$ are normal and abnormal density matrices, respectively, $\hat{\Delta}$ is the pairing gap. We work with the Wigner transformation [6] of these equations. For example, the first one reads

$$i\hbar\dot{f} = i\hbar\{h, f\} - \Delta\kappa^* + \kappa\Delta^* - \frac{i\hbar}{2}\{\Delta,\, \kappa^*\} + \frac{i\hbar}{2}\{\kappa,\, \Delta^*\} - \\ - \frac{\hbar^2}{8}[\{\{\kappa,\, \Delta^*\}\} - \{\{\Delta,\, \kappa^*\}\}] + \ldots, \quad (2)$$

where functions $h,\ f,\ \Delta,$ and κ are Wigner transforms of $\hat{h},\ \hat{\rho},\ \hat{\Delta},$ and $\hat{\kappa}$, respectively, $\{f,\, g\}$ is the Poisson bracket of functions $f(\mathbf{r},\, \mathbf{p})$ and $g(\mathbf{r},\, \mathbf{p})$; the dots stand for terms proportional to higher powers of $\hbar$.

To study the quadrupole collective motion with $K^\pi = 1^+$ in axially symmetric nuclei, it is necessary to calculate moments of Eq. (2) (+ three other equations) with the weight functions:

$$W = xz, \quad p_x p_z, \quad zp_x + xp_z \equiv \hat{L}, \quad \text{and} \quad zp_x - xp_z \equiv \hat{I}_y.$$

This procedure yields 16 equations for collective variables $\int d^3p \int d^3r W f$ and $\int d^3p \int d^3r W \kappa$. However, due to symmetry considerations 8 of them turn out trivial. Applying the approximation [5] $\delta\kappa_+(\mathbf{r},\, \mathbf{p}) \ll \delta\kappa_-(\mathbf{r},\, \mathbf{p})$, one is able to reduce the problem to a set of only six dynamical equations (strictly speaking, 12 ones: 6 for protons and 6 for neutrons). Making the standard approximation [5] to decouple isovector and isoscalar subsets, we find that in the case of harmonic oscillator with QQ interaction the isovector subset has two integrals of motion allowing one to reduce the eigenvalue problem to a quadratic equation. Its two solutions

$$E_\pm^2 = \mathcal{D}_\omega \pm \sqrt{\mathcal{D}_\omega^2 - [8\Delta\tilde{\Delta}\epsilon^2 + 4(\hbar\bar{\omega})^4\delta^2](1-\alpha) + 24\alpha\kappa_0\tilde{\Delta}k_0\hbar^4/m^2} \quad (3)$$

describe the energy E_+ of the IVGQR and the energy E_- of scissors. Here $\epsilon^2 = \hbar^2\bar{\omega}^2(1 + \frac{\delta}{3})$, $\bar{\omega} = \omega/(1 + \frac{2}{3}\delta)$, ω — an oscillator frequency, κ_0 — the isoscalar strength constant of QQ interaction, $k_0 = 4\int d^3p \int d^3r \kappa^0(\mathbf{r},\, \mathbf{p})/(2\pi\hbar)^3$, $\kappa^0(\mathbf{r},\, \mathbf{p})$ — the ground state anomalous density, α is the parameter connecting isovector and isoscalar strength constants ($\kappa_1 = \alpha\kappa_0$), $\mathcal{D}_\omega = 2\Delta\tilde{\Delta} + \epsilon^2(2-\alpha)$, $2\tilde{\Delta} = |V_0| I_{pp}^{\kappa\Delta}$,

$$I_{pp}^{\kappa\Delta}(\mathbf{r},\, p) = \frac{r_p^3}{\sqrt{\pi}\hbar^3} \mathrm{e}^{-\gamma p^2} \int \kappa^0(\mathbf{r},\, p') \big[\phi_0(2\gamma pp') - \\ - 4\gamma^2 p'^4 \phi_2(2\gamma pp')\big] \mathrm{e}^{-\gamma p'^2} p'^2 dp', \quad (4)$$

$$\gamma = r_p^2/4\hbar^2, \quad \phi_0(x) = \frac{1}{x} sh(x), \quad \phi_2(x) = \frac{1}{x^3}[(1 + \frac{3}{x^2}) sh(x) - \frac{3}{x} ch(x)],$$

r_p and V_0 are parameters of the pair interaction $v(|\mathbf{p}-\mathbf{p}'|) = -|V_0|(r_p\sqrt{\pi})^3 \times \mathrm{e}^{-|\mathbf{p}-\mathbf{p}'|^2/r_p^2}$. It is worth noting that contrary to the case without pairing [7] the energy E_- does not go to zero for deformation $\delta = 0$. The calculation of transition probabilities shows that this mode of a spherical nucleus can be excited by an electric field and it is not excited by a magnetic field.

In the **isoscalar** case we have two solutions:

$$E_\pm^2 = 2\Delta\tilde{\Delta} + \epsilon^2 \pm \sqrt{(2\Delta\tilde{\Delta} + \epsilon^2)^2 + 24\kappa_0\tilde{\Delta}k_0\hbar^4/m^2}, \tag{5}$$

$E_+ \equiv E_{\mathrm{is}}$ being the energy of the isoscalar giant quadrupole resonance and $E_- \equiv E_{\mathrm{ISL}}$ being the energy of the ISLLE. It is important to note that the very existence of the ISLLE relies on two factors: 1) pair correlations and 2) quantum correction. As it is seen, the value of E_{ISL} is strongly dependent on the value of integral k_0, which, in its turn, is completely determined by the properties of the anomalous density $\kappa^0(\mathbf{r}, \mathbf{p})$, so it would be natural to study these properties before starting the systematic calculations of E_{ISL}.

2. ANOMALOUS DENSITY MATRIX

The anomalous density matrix is defined [6] as

$$\kappa(\mathbf{r}_1, s_1; \mathbf{r}_2, s_2) = \sum_{k>0} u_k v_k [\phi_k(\mathbf{r}_1, s_1)\phi_{\bar{k}}(\mathbf{r}_2, s_2) - \phi_{\bar{k}}(\mathbf{r}_1, s_1)\phi_k(\mathbf{r}_2, s_2)], \tag{6}$$

where $k \equiv n, l, j, m$ is the set of shell model quantum numbers and $\bar{k}$ is that of the time conjugate state. Bogoliubov coefficients u, v are defined in a usual way: $v_k^2 = \dfrac{1}{2}\left(1 - \dfrac{\tilde{\epsilon}_k}{\sqrt{\tilde{\epsilon}_k^2 + \Delta_k^2}}\right)$, $u_k^2 = 1 - v_k^2$ with $\Delta_k = -\sum\limits_{k'>0} \bar{v}_{k\bar{k},k'\bar{k}'} u_{k'} v_{k'}$, $\bar{v}_{ij,mn} = v_{ij,mn} - v_{ij,nm}$ and $\tilde{\epsilon}_k = \epsilon_k - \lambda$. ϵ_k is a single-particle energy and λ is a chemical potential. Inserting expressions for u, v into the formula for Δ, one finds the set of gap equations:

$$\Delta_k = -\frac{1}{2}\sum_{k'>0} \bar{v}_{k\bar{k},k'\bar{k}'} \frac{\Delta_{k'}}{\sqrt{\tilde{\epsilon}_{k'}^2 + \Delta_{k'}^2}}. \tag{7}$$

The solution of these equations for the test nucleus ^{134}Ba is shown in Fig. 1.

Matrix elements $v_{k\bar{k},k'\bar{k}'}$ of the Gaussian pair interaction $v(\mathbf{r}_1, \mathbf{r}_2) = -V_0\mathrm{e}^{-|\mathbf{r}_1-\mathbf{r}_2|^2/r_p^2}$ were calculated with the Woods–Saxon single-particle wave functions [9], the values $V_0 = 23$ MeV and $r_p = 1.7$ fm being used.

This nucleus was chosen because it is the lightest one (with the smallest deformation) of the deformed nuclei where the scissors mode is observed. Assuming artificially its deformation $\delta = 0$, we can compare our results with the numerous results of other authors for ^{120}Sn.

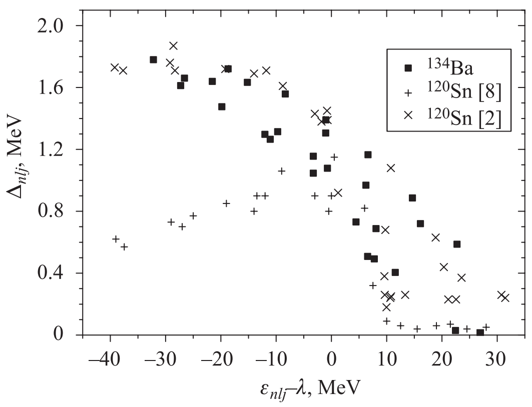

Fig. 1. Δ_{nlj} for ^{134}Ba and for ^{120}Sn [2] and [8]

As it is seen, our results are in very good agreement with that of the paper [2] obtained with the realistic interaction Argonne v_{18} (see also [8]). Comparison with the results of calculations with surface density-dependent delta interaction [2] demonstrates rather good agreement in the vicinity of the Fermi Surface (FS) and strong disagreement below FS (which, naturally, does not have any meaning).

Having Δ_k, one can calculate $\kappa(\mathbf{r}_1,\, s_1;\, \mathbf{r}_2,\, s_2)$. A square of the anomalous density $|\kappa(R,\, s)|^2$, averaged over angle between $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ and $\mathbf{s} = \mathbf{r}_1 - \mathbf{r}_2$, is shown in Fig. 2 as a function of R and s.

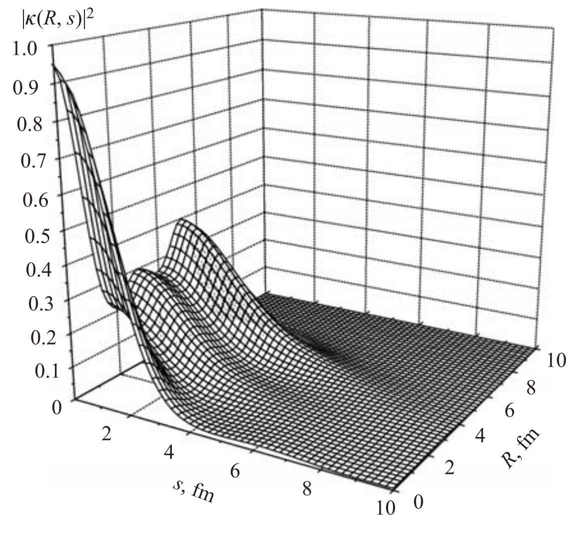

Fig. 2. $|\kappa(R, s)|^2$ for ^{134}Ba; scale has been multiplied by a factor of 10^4

This picture is in good agreement with that of the paper [1] calculated for ^{120,128}Sn. We have exactly the same characteristic structure with three pikes, the highest one being disposed at the point $R = r = 0$. The second in height pike is on the nuclear surface and the lowest one is in between. The width of these pikes in s direction, which is associated with the size of Cooper pairs, is 2–3 fm, that agrees with the results of [1]. It is interesting to look on the angular (angle θ between $\mathbf{R}$ and $\mathbf{s}$) dependence of κ^2. To this end we have calculated κ^2 at $\theta = 0°$ and $\theta = 66°$ (Fig. 3), $\theta = 78°$ and $\theta = 90°$ (Fig. 4).

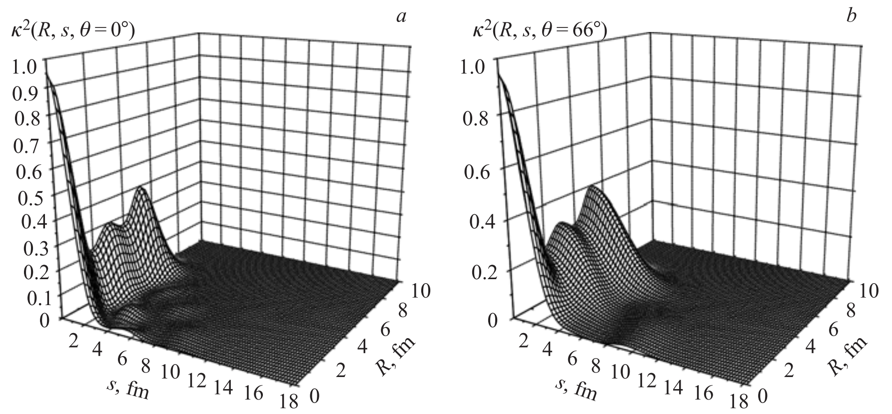

Fig. 3. $\kappa^2(R, s, \theta)$ for $\theta = 0°$ (*a*) and $\theta = 66°$ (*b*) for ^{134}Ba; scale has been multiplied by a factor of 10^4

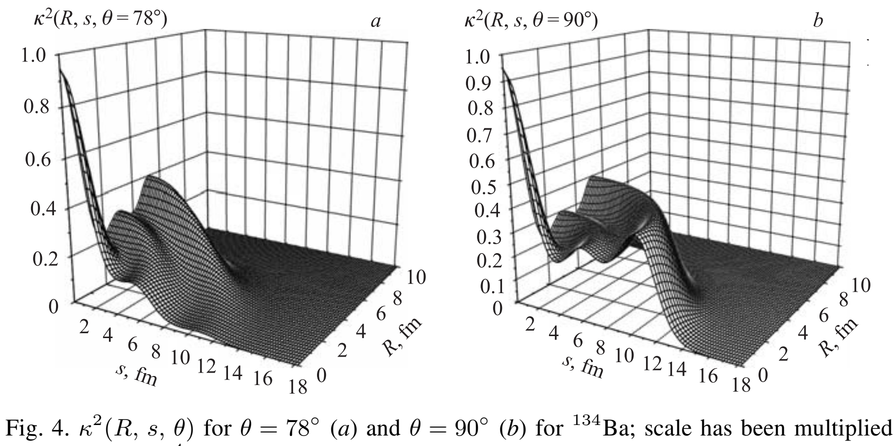

Fig. 4. $\kappa^2(R, s, \theta)$ for $\theta = 78°$ (*a*) and $\theta = 90°$ (*b*) for ^{134}Ba; scale has been multiplied by a factor of 10^4

The structure of the function $\kappa^2(R,\, s,\, \theta = 0°)$ (Fig. 3) is quite similar to that of the angle-averaged function $\kappa^2(R,\, s)$ (Fig. 2): the same three-pike structure in the vicinity of R axis ($0 \leqslant s < 3$) and small deviation at large s. It is interesting that practically the same picture is observed at $\theta = 66°$ and $\theta = 78°$ — only at $\theta = 78°$ the width of pikes increased a little bit. Principally another picture is observed at $\theta = 90°$ (Fig. 4). Here there is no pronounced concentration of κ^2 along the R axis, that says about strong anisotropy of the abnormal density.

So, in the area $0 \leqslant \theta \leqslant 78°$ the function $|\kappa(R, s, \theta)|^2$ is changed very slowly, conserving approximately the same shape as it has at $\theta = 0°$. Beginning from $\theta \sim 80°$ it is changed very quickly, receiving finally the shape shown in Fig. 4 ($\theta = 90°$).

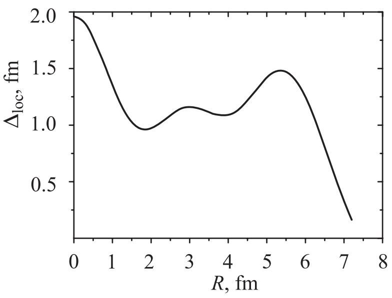

Fig. 5. The pairing field Δ_{loc} calculated in the local approximation for ^{134}Ba

Therefore, it is not surprising, that after averaging over θ one gets the picture reminding very much that of at $\theta = 0°$.

It turns out that three-pike structure, observed in the $\kappa^2(R,\, s)$ behaviour is repeated in the behaviour of the pairing field calculated in the local approximation (Fig. 5):

$$\Delta_{\mathrm{loc}}(R) \equiv \Delta(R, p_F(R)), \quad p_F^2 = 2m(\lambda - V(R)).$$

Such a behaviour of Δ_{loc} is in excellent qualitative agreement with the result of [2] calculated with Gogny force and with the results of [8] calculated with Argonne v_{14}, low-momentum interaction $V_{\mathrm{low}\,k}$ and density-dependent delta interaction. The pike on the surface was predicted by semiclassical calculation [6]. However, the appearance of the very high pike in the center of nucleus is rather unexpected. Obviously, it is the shell effect explained by the strong influence of $3\mathrm{s}\frac{1}{2}$ state (see analogous remark in [1] concerning ^{120,128}Sn).

Finally, we calculated the so-called coherence length

$$\xi(R) = \frac{(\int s^4 |\kappa(R,\, s)|^2 ds)^{1/2}}{(\int s^2 |\kappa(R,\, s)|^2 ds)^{1/2}}, \tag{8}$$

which is shown in Fig. 6.

We obtained the typical curve with minimum at the nuclear surface, which agrees very well with the results of [1, 2, 8] calculated with various realistic interactions. The only substantial difference is seen in the absolute value of ξ at the point of minimum on the nuclear surface. Our result is approximately two times bigger than that of other authors. Inside of nucleus all results are similar.

We suspect that such a difference is connected with different mean fields: Woods–Saxon in our case and the self-consistent well obtained from various two-body forces in other cases. Being more or less similar inside of nucleus these mean fields can differ substantially in the surface area that can lead to big differences for coherence length. It would be interesting to study this problem performing the detailed comparison of abnormal densities obtained with various mean fields. It will be done in the forthcoming papers. We want to discuss the coherence length behaviour outside of nucleus. The growing curve far outside of nuclear surface, practically in the empty space, looks suspicious. To understand the situation we have shown the numerator and denominator of expression (8) in the same figure.

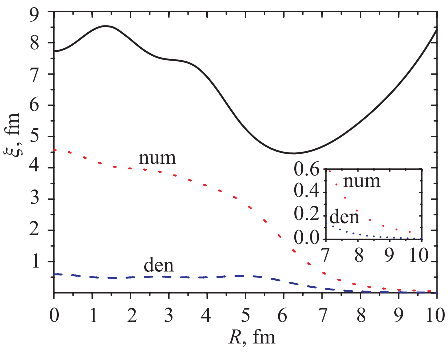

Fig. 6. Coherence length $\xi(R)$ for ^{134}Ba (num — the numerator of expression (8), den — the denominator of (8))

Both curves fall quickly with R increasing. Their behaviour in the vicinity of the nuclear surface ($7 < R < 8$ fm) is determined by the tails of single-particle wave functions. However, at $R > 8$ fm their values are determined mainly by numerical errors and there is no sense to speak about any physics in this area.

CONCLUSION

We have demonstrated that calculations of the abnormal density and the pair field (gap) with the Woods–Saxon mean field potential are able to reproduce very well the results obtained in the self-consistent calculations with realistic interactions. As we understand, the lack of self-consistency is completely compensated by the proper choice of Woods–Saxon parameters, which are fitted to reproduce the nuclear single-particle levels near the Fermi surface. These results shall be used in the calculation of scissors mode.

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