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ABOUT OSCILLATIONS IN THE SYSTEM OF K^0 MESONS

Бештоев Х. М. E2-2011-48
Об осцилляциях в системе K^0-мезонов

Рассматриваются смешивания и осцилляции K^0-, $\bar{K}^0$-мезонов через K_1^0-, K_2^0-мезонные состояния при нарушении странности в слабых взаимодействиях, а также смешивания и осцилляции K_1^0-, K_2^0-мезонов через K_S-, K_L-мезонные состояния при нарушении CP-четности в слабых взаимодействиях без учета и с учетом ширин распадов. Мы работаем в рамках схемы массовых смешиваний. Показано, что на больших расстояниях от источника K^0-мезонов K_1^0-мезонные состояния после их распада ($\tau_L \gg \tau_S$ ($\tau_2 \gg \tau_1$)) появляются за счет осцилляций оставшихся K_2^0-мезонов, и тогда мы можем увидеть короткоживущие K_1^0-мезоны по их распадам на два π-мезона. Предполагается, что осцилляции мезонов $K_L \leftrightarrow K_S$ отсутствуют. Также рассматривается случай, когда при CP-нарушении унитарность нарушается, но ортогональность K_S-, K_L-состояний сохраняется. Получены общие выражения для вероятностей мезонных осцилляций (переходов).

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About Oscillations in the System of K^0 Mesons

This work considers K^0-, $\bar{K}^0$-meson mixings and oscillations via K_1^0-, K_2^0-meson states at strangeness violation by the weak interactions and K_1^0-, K_2^0-meson mixings and oscillations via K_S-, K_L-meson states at CP violation by the weak interactions without and with taking into account decay widths. We work in the framework of the masses mixing scheme. It is shown that K_1^0-(K_S-)meson states appear at big distances from the K^0-mesons source after their decays ($\tau_L \gg \tau_S$ ($\tau_2 \gg \tau_1$)) due to oscillations of residual K_2^0 (K_L) mesons and then again we see short-living K_1^0 (K_S) mesons. It is implied that $K_L \leftrightarrow K_S$ meson oscillations are absent. The case is also considered when at CP violation unitarity is violated, but orthogonality of K_S, K_L states remains. The general expressions for probabilities of meson oscillations (transitions) are given.

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1. INTRODUCTION

Oscillations of K^0 mesons (i.e., $K^0 \leftrightarrow \bar{K}^0$) were theoretically [1] and experimentally [2] investigated in the 1950s and 1960s. Recently there has been achieved an understanding that these processes go as a double-stadium process [3–6]. A detailed study of K^0 meson mixing and oscillations is very important since the theory of neutrino oscillations is built in analogy with the theory of K^0 meson oscillations.

Previously it was supposed that P parity is a well number; however, after theoretical [7] and experimental [8] works it has become clear that in weak interactions P parity is violated. Then in work [9] there was an advanced supposition that in the weak interactions CP parity is conserved but not P parity. Work [10] has reported that in K_L decays with a probability of about 0.2% there is two-π decay mode that is a detection of CP-parity violation.

Usually it is supposed that at big distances from K^0-meson sources only K_L-meson states remain. Since this meson is a superposition of K_1^0, K_2^0 mesons $K_L \simeq \alpha K_1^0 + \beta K_2^0$ $(\alpha^2 + \beta^2 = 1, \beta \gg \alpha)$ and

$$K_L(t) \simeq \alpha K_1^0(0)\ \mathrm{e}^{(-im_S - \Gamma_S/2)t} + \beta K_2^0(0)\ \mathrm{e}^{(-im_L - \Gamma_L/2)t},$$

at time $t \gg 1/\Gamma_S$ almost all K_S mesons have time to decay and $K_L \to K_2^0$ mesons will remain. Then there is the only possibility to generate K_1^0 mesons $K_2^0 \leftrightarrow K_1^0$ meson oscillations via K_S, K_L mesons; i.e., in reality at big distances K_2^0 are responsible for generation of K_1^0 mesons but not K_L mesons since $K_L \leftrightarrow K_S$ oscillations are absent.

It is necessary to remark that the literature devoted to this subject seldom mentions K_1^0, K_2^0 mesons which appear at violation of strangeness S. However, taking into account these states is very important since the weak interaction process with S violation is faster than the weak interaction process with CP violation; i.e., first K_1^0, K_2^0 mesons are produced and then the K_S-, K_L-meson states are produced. It is well seen from a very small probability of CP violation in the system of K^0 mesons. We cannot correctly understand the K^0 processes if we do not take into account the presence of K_1^0-, K_2^0-meson states.

A phenomenological analysis of K^0-meson processes was done in [11] (see also [12]). In this work another approach is used to consider K^0-meson processes.

This work is based on the principles of the quantum field theory or particle physics. It is supposed that particles (K^0 mesons) during production have no widths for decomposition; i.e., they can only decay in a usual way, as is the case in particle physics. This remark is important since in this case particles cannot form wave packets and the wave packets can then be formed only from a big number of identical particles (mesons). The supposition that K^0 mesons can be considered as wave packets is a hypothesis and has at present neither experimental nor theoretical confirmation. But at the same time, from our experience in particle physics we can draw a conclusion that elementary particles have no widths in order to consider them as wave packets.

In the literature [11, 12] a nonunitary transformation is used at obtaining of K_S, K_L states. It is supposed that these states arise at CP violation. The expression for these states has the following form:

$$\begin{aligned} K_S &= (K_1^0 + \varepsilon_1 K_2^0)/\sqrt{1+|\varepsilon_1|^2}, \\ K_L &= (K_2^0 + \varepsilon_1 K_1^0)/\sqrt{1+|\varepsilon_1|^2}, \end{aligned} \tag{1}$$

and on the contrary

$$\begin{aligned} K_1^0 &= (K_S - \varepsilon_1 K_L)\frac{\sqrt{1+|\varepsilon_1|^2}}{1-\varepsilon_1^2}, \\ K_2^0 &= (K_L - \varepsilon_1 K_S)\frac{\sqrt{1+|\varepsilon_1|^2}}{1-\varepsilon_1^2}. \end{aligned} \tag{2}$$

Writing the wave function of K_L, K_S mesons in the form

$$\begin{aligned} K_S &= \frac{1-\varepsilon_1}{\sqrt{2(1+|\varepsilon_1|^2)}} \ \mathrm{e}^{-im_S t - \frac{\Gamma_S t}{2}}, \\ K_L &= \frac{1-\varepsilon_1}{\sqrt{2(1+|\varepsilon_1|^2)}} \ \mathrm{e}^{-im_L t - \frac{\Gamma_L t}{2}}, \end{aligned} \tag{3}$$

putting expression (3) into expression (2) and then taking the first term of (2) in the quadratic form on the absolute value, we obtain ($\hbar = 1$)

$$|K_1^0|^2 = \frac{|1-\varepsilon_1|^2}{2(1-|\varepsilon_1|^2)} \times \\ \times \left(\mathrm{e}^{-\Gamma_S t} + |\epsilon|^2 \ \mathrm{e}^{-\Gamma_L t} - 2|\epsilon| \mathrm{e}^{-\frac{(\Gamma_S+\Gamma_L)t}{2}} \cos\left((m_L - m_S)t\right) \right). \tag{4}$$

In expression (4) a cross term appears which is responsible for oscillations. This term can be interpreted as oscillations between K_S, K_L states; i.e., these states are nonorthogonal ones.

In the framework of quantum mechanics, if the states are wave vectors, then expression (3) has to be written in the following form:

$$K_S(t) = \frac{1-\varepsilon_1}{\sqrt{2(1+|\varepsilon_1|^2)}}\ \mathrm{e}^{-im_S t - \frac{\Gamma_S t}{2}} K_S(0),$$
$$K_L(t) = \frac{1-\varepsilon_1}{\sqrt{2(1+|\varepsilon_1|^2)}}\ \mathrm{e}^{-im_L t - \frac{\Gamma_L t}{2}} K_L(0), \tag{5}$$

then after taking it in the quadratic form on the absolute value we get

$$|K_1^0|^2 = \frac{|1-\varepsilon_1|^2}{2(1-|\varepsilon_1|^2)}\left(\mathrm{e}^{-\Gamma_S t} + |\epsilon|^2\ \mathrm{e}^{-\Gamma_L t}\right). \tag{6}$$

In expression (6) the interference term is absent; i.e., the oscillations are absent.

Now we have to solve the problem: how do oscillations arise in the quantum mechanics approach and how do short-living mesons appear at long distances from K^0 source? Come to the solution of this problem.

At first we consider mixings of $K^0, \bar{K}^0$ mesons at violation of strangeness S, and $K^0, \bar{K}^0$ oscillations without and with taking into account the decay widths of K_1^0, K_2^0 mesons. Then we turn to the consideration of K_1^0, K_2^0 meson mixings at CP-parity violation when K_S, K_L mesons are produced. Further we consider K_1^0-, K_2^0-meson oscillations via K_S, K_L mesons without and with taking into account decay widths of K_S^0, K_L^0 mesons. In conclusion, we discuss the problem: what is the source of K_S (or rather K_2^0, K_S) mesons at large distances from the K^0-meson source. Taking into account the widths of meson decays, we will work in the framework of the commonly accepted approach [13]. It is necessary to note that the value for K_S-, K_L- (or more accurately K_1^0-, K_2^0-) meson masses difference was first measured in work [14] (for modern value for $m_{K_L} - m_{K_S}$ see in [15]).

2. VACUUM MIXINGS AND OSCILLATIONS OF $K^0, \bar{K}^0$ MESONS AT STRANGENESS VIOLATION BY THE WEAK INTERACTIONS WITHOUT AND WITH TAKING INTO ACCOUNT DECAY WIDTHS

2.1. K^0-, $\bar{K}^0$-Vacuum Mixings. K^0-, $\bar{K}^0$-meson states are produced in the strong interaction (i.e., they are eigenstates of these interactions), then the mass matrix of K^0 mesons will have a diagonal form [3–6]. Following the traditions, we will consider the K^0-meson mixings and oscillations by using the mass matrix, and for convenience the masses are used in the linear but not in the quadratic form. Then the mass matrix has the following form:

$$\begin{pmatrix} m_{K^0K^0} & 0 \\ 0 & m_{\bar{K}^0\bar{K}^0} \end{pmatrix}. \tag{7}$$

Because of the weak interactions violating strangeness ($s \leftrightarrow d$), this mass matrix (7) becomes a nondiagonal matrix:

$$\begin{pmatrix} m_{K^0K^0} & m_{K^0\bar{K}^0} \\ m_{\bar{K}^0K^0} & m_{\bar{K}^0\bar{K}^0} \end{pmatrix} \rightarrow U^-1 \begin{pmatrix} m_{K_1^0} & 0 \\ 0 & m_{K_2^0} \end{pmatrix} U, U = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}. \quad (8)$$

For obtaining the eigenstates of weak interactions which violate strangeness, we have to diagonalize this matrix by turning it through angle θ. By using this procedure, we get

$$\begin{aligned} \tan 2\theta &= \frac{2m_{K^0\bar{K}^0}}{\mid m_{K^0} - m_{\bar{K}^0} \mid}, \\ \sin 2\theta &= \frac{2m_{K^0\bar{K}^0}}{\sqrt{(m_{K^0} - m_{\bar{K}^0})^2 + (2m_{K^0\bar{K}^0})^2}}, \end{aligned} \quad (9)$$

$$m_{1,2} = m_{K_1,K_2} = \frac{1}{2}\Big[(m_{K^0} + m_{\bar{K}^0}) \pm \\ \pm \left((m_{K^0} - m_{\bar{K}^0})^2 + 4m^2_{K^0\bar{K}^0}\right)^{1/2}\Big], \quad (10)$$

where K_1^0 and K_2^0 states are eigenstates of the weak interactions violating strangeness. Now these states are superposition states of $K^0, \bar{K}^0$ mesons:

$$\begin{aligned} K_1^0 &= \cos\theta\, K^0 - \sin\theta\, \bar{K}^0, \\ K_2^0 &= \sin\theta\, K^0 + \cos\theta\, \bar{K}^0, \end{aligned} \quad (11)$$

and the inverse transformation gives

$$\begin{aligned} K^0 &= \cos\theta\, K_1^0 + \sin\theta\, K_2^0, \\ \bar{K}^0 &= -\sin\theta\, K_1^0 + \cos\theta\, \bar{K}_2^0, \end{aligned} \quad (12)$$

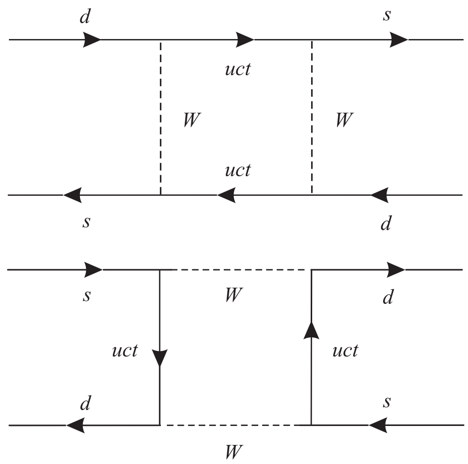
since $m_{K^0K^0} = m_{\bar{K}^0\bar{K}^0}$, for CPT invariance of the weak interactions this mixing angle θ will be equal to $\pi/4$. Then from expressions (11) and (12) we get

$$K_1^0 = \frac{K^0 - \bar{K}^0}{\sqrt{2}}, \quad K_2^0 = \frac{K^0 + \bar{K}^0}{\sqrt{2}}, \quad (13)$$

$$K^0 = \frac{K_1^0 + K_2^0}{\sqrt{2}}, \quad \bar{K}^0 = \frac{K_1^0 - K_2^0}{\sqrt{2}}. \quad (13')$$

It is necessary to remark that $CPK_1^0 = K_1^0$ and $CPK_2^0 = -K_2^0$; i.e., CP parity of K_1^0 meson is a positive value and it can decay into two π mesons, and CP parity of K_2^0 meson is a negative value and it can decay into three π mesons.

The computation of nondiagonal terms of the mass matrix (8)–(10) can be fulfilled by using the Feynman diagrams from the figure in the framework of the standard model of electroweak interactions [12, 16] with Kabibbo–Kobayashi–Maskawa matrices [17].



Diagrams for $d \leftrightarrow s$ quark transitions, i.e., for $K^0 \leftrightarrow \bar{K}^0$ transitions via W-boson exchanges by using Kobayashi–Maskawa matrix

2.2. Vacuum Oscillations of K^0 Mesons. Now we come to K^0-meson oscillations. The oscillation of D^0, B^0 mesons can be considered in an analogous way. K^0, $\bar{K}^0$ mesons besides masses have decay widths Γ_{K^0}, $\Gamma_{\bar{K}^0}$ and therefore they will decay into π mesons.

For example, we can consider oscillations of K^0 produced from the following reaction:

$$\pi^- + p \to K^0 + \Lambda. \tag{14}$$

At the moment $t = 0$ there are only K^0 mesons produced in the strong interactions, and if we take into account expression (12) at another moment $t > 0$, this state will be transformed into the following state:

$$K^0(t) = \frac{1}{2}\left[(K^0 + \bar{K}^0)\ \mathrm{e}^{-im_1 t - \frac{\Gamma_1}{2} t} + (K^0 - \bar{K}^0)\ \mathrm{e}^{-im_2 t - \frac{\Gamma_2}{2} t}\right], \tag{15}$$

where Γ_1, Γ_2, m_1, m_2 are widths and masses of K_1^0, K_2^0 mesons.

If Γ_1, Γ_2 are equal to zero, then $K^0, \bar{K}^0$ oscillations will continue without stopping and $K^0, \bar{K}^0$ will transform into each other with a periodicity of $t = \pi/(m_1 - m_2)$.

The length of K^0-meson oscillations at low velocities v is

$$L = vt = \frac{2\pi v}{\mid m_1 - m_2 \mid} = \frac{2\pi p_{K^0}}{2m_{K^0} 2m_{K^0\bar{K}^0}}. \tag{16}$$

In the standard approach [18, 19] to K^0-meson oscillations, it is supposed that K^0 mesons are produced at once in the form of superposition states (12). It means that at production of K^0, $\bar{K}^0$ mesons their mass matrix has a nondiagonal form. In order to find their eigenstates, we have to diagonalize this matrix. Then we see that their eigenstates are K_1^0, K_2^0 mesons; i.e., this case has to produce K_1^0, K_2^0 mesons but not K^0, $\bar{K}^0$ mesons.

As a matter of fact, since K^0 mesons are eigenstates of the strong interactions, they cannot be produced in superposition states of K_1^0, K_2^0 mesons. K^0 mesons become superposition states of K_1^0, K_2^0 mesons when weak interactions transform them into a superposition of eigenstates. It is important to note that, in contrast to the strong interactions, the weak interactions will produce K_1^0,- K_2^0-meson states. Now we come to a detailed consideration of K^0-meson oscillations in the framework of the mass mixing scheme.

The mass matrix of K^0 mesons has the form

$$\begin{pmatrix} m_{K^0} & 0 \\ 0 & m_{\bar{K}^0} \end{pmatrix}. \tag{17}$$

Strangeness is violated due to the weak interactions and nondiagonal terms appear in this masses matrix, then it gets the following form (CP is conserved):

$$\begin{pmatrix} m_{K^0} & m_{K^0\bar{K}^0} \\ m_{\bar{K}^0K^0} & m_{\bar{K}^0} \end{pmatrix}. \tag{18}$$

At diagonalization of this matrix we obtain K_1^0-, K_2^0-meson states and the states $K^0, \bar{K}^0$ are transformed into superposition of K_1^0, K_2^0 states (see expression (12)). Their mixing angle and masses are given by expressions (10)–(12).

The expression for $\sin^2 2\theta$ is given by (θ is the angle of mixing)

$$\sin^2 2\theta = \frac{(2m_{K^0\bar{K}^0})^2}{(m_{K^0} - m_{\bar{K}^0})^2 + (2m_{K^0\bar{K}^0})^2}, \quad \begin{pmatrix} m_{K_1^0} & 0 \\ 0 & m_{K_2^0} \end{pmatrix}. \tag{19}$$

The evolution of K_1^0-, K_2^0-meson states with masses m_1, m_2 will be given with the following expression:

$$K_1^0(t) = \mathrm{e}^{-iE_1t}K_1^0(0), \quad K_2^0(t) = \mathrm{e}^{-iE_2t}K_2^0(0), \tag{20}$$

where

$$E_k^2 = (p^2 + m_k^2), \quad k = 1, 2.$$

If these mesons are moving without interactions, then

$$\begin{aligned} K^0(t) &= \cos\theta\,\mathrm{e}^{-iE_1t}K_1^0(0) + \sin\theta\,\mathrm{e}^{-iE_2t}K_2^0(0), \\ \bar{K}^0(t) &= -\sin\theta\,\mathrm{e}^{-iE_1t}K_1^0(0) + \cos\theta\,\mathrm{e}^{-iE_2t}K_2^0(0). \end{aligned} \tag{21}$$

Using expression (11) for K_1^0 and K_2^0 and putting them into (21), we obtain

$$\begin{aligned} K^0(t) &= \left[\, \mathrm{e}^{-iE_1 t}\cos^2\theta \,+\, \mathrm{e}^{-iE_2 t}\sin^2\theta \,\right] K^0(0) + \\ &+ \left[\, \mathrm{e}^{-iE_1 t} - \mathrm{e}^{-iE_2 t}\right] \sin\theta\,\cos\theta\,\bar{K}^0(0), \\ \bar{K}^0(t) &= \left[\, \mathrm{e}^{-iE_1 t}\sin^2\theta \,+\, \mathrm{e}^{-iE_2 t}\cos^2\theta \,\right] \bar{K}^0(0) + \\ &+ \left[\, \mathrm{e}^{-iE_1 t} - \mathrm{e}^{-iE_2 t}\right] \sin\theta\,\cos\theta\,\bar{K}^0(0). \end{aligned} \tag{22}$$

The probability that meson K^0 produced at moment $t = 0$ will be at moment $t \neq 0$ in the state of $\bar{K}^0$ meson is given by a squared absolute value of the amplitude in (22); i.e.,

$$\begin{aligned} &P(K^0 \to \bar{K}^0) = |\,(\bar{K}^0(0)\cdot K^0(t))\,|^2 = \\ &= \frac{1}{2}\sin^2 2\theta\;[1 - \cos\left((E_2 - E_1)t\right)] \equiv \frac{1}{2}\left[1 - \cos\left((E_2 - E_1)t\right)\right], \end{aligned} \tag{23}$$

where $\theta = \pi/4$. Using expressions for masses of K_1^0, K_2^0 mesons, we obtain

$$m_{K_1^0} = m_{K^0} - \Delta, \quad m_{K_2^0} = m_{K^0} + \Delta m, \tag{24}$$

where $\Delta = 2m_{K^0\bar{K}^0}$. Since $m_{K^0} \gg \Delta$,

$$\begin{aligned} E_1 &= \sqrt{p^2 + m^2_{K_1^0}} \cong E_{K^0}\left(1 - \frac{m_{K^0}\Delta}{E^2_{K^0}}\right), \\ E_2 &= \sqrt{p^2 + m^2_{K_2^0}} \cong E_{K^0}\left(1 + \frac{m_{K^0}\Delta}{E^2_{K^0}}\right), \end{aligned} \tag{25}$$

$$E_2 - E_1 = \frac{2m_{K^0}\Delta}{E_{K^0}} = \frac{2\Delta}{\gamma}. \tag{26}$$

Then the length L_{12} of K^0-, $\bar{K}^0$-meson oscillations is

$$L_{12} = \frac{\gamma}{2\Delta} \equiv \frac{2\pi hc\gamma}{2\Delta}. \tag{27}$$

2.3. The Vacuum K^0-meson Oscillations with Taking into Account the Width of K_1^0-, K_2^0-meson Decays. Taking into account that K_1^0, K_2^0 decay and have decay widths Γ_1, Γ_2, we can rewrite expressions (20)–(26), and then K_1^0, K_2^0 mesons with masses m_1, m_2 evolve in dependence on time according to the following law:

$$K_1^0(t) = \mathrm{e}^{-iE_1 t - \frac{\Gamma_1 t}{2}} K_1^0(0), \quad K_2^0(t) = \mathrm{e}^{-iE_2 t - \frac{\Gamma_2 t}{2}} K_2^0(0), \tag{28}$$

$E_2 - E_1$ is given by (25) and it is equal to $2m_{K^0}\Delta/E_{K^0}$:

$$E_2 - E_1 \simeq \frac{2m_{K^0}\Delta}{E_{K^0}}. \tag{29}$$

In this work we suppose that $\Gamma_k = \gamma \Gamma_k^0$, where Γ_k^0 is K_k^0-meson width at rest and $\gamma = E_k/m_k$ is a usual relativistic factor ($k = 1, 2$).

If these mesons move without interaction, then

$$\begin{aligned} K^0(t) &= \cos\theta\, e^{-iE_1 t - \frac{\Gamma_1 t}{2}} K_1^0(0) + \sin\theta\; e^{-iE_2 t - \frac{\Gamma_2 t}{2}} K_2^0(0), \\ \bar{K}^0(t) &= -\sin\theta\, e^{-iE_1 t - \frac{\Gamma_1 t}{2}} K_1^0(0) + \cos\theta\, e^{-iE_2 t - \frac{\Gamma_2 t}{2}} K_2^0(0). \end{aligned} \tag{30}$$

Using expression (11) for K_1^0, K_2^0 and putting it into (30), we obtain

$$\begin{aligned} K^0(t) &= \left[\, e^{-iE_1 t - \frac{\Gamma_1 t}{2}} \cos^2\theta \,+\, e^{-iE_2 t - \frac{\Gamma_2 t}{2}} \sin^2\theta \right] K^0(0) + \\ &\quad + \left[\, e^{-iE_1 t - \Gamma_1/2} - e^{-iE_2 t - \frac{\Gamma_2 t}{2}} \right] \sin\theta\, \cos\theta\, \bar{K}^0(0), \\ \bar{K}^0(t) &= \left[\, e^{-iE_1 t - \frac{\Gamma_1 t}{2}} \sin^2\theta \,+\, e^{-iE_2 t - \frac{\Gamma_2 t}{2}} \cos^2\theta \right] \bar{K}^0(0) + \\ &\quad + \left[\, e^{-iE_1 t - \frac{\Gamma_1 t}{2}} - e^{-iE_2 t - \frac{\Gamma_2 t}{2}} \right] \sin\theta\, \cos\theta\, \bar{K}^0(0). \end{aligned} \tag{31}$$

The probability that meson K^0 produced at moment $t = 0$ will be at moment $t \neq 0$ in the state of $\bar{K}^0$ meson is given by a squared absolute value of the amplitude in (31); i.e.,

$$\begin{aligned} P(K^0 \to \bar{K}^0) &= \mid (\bar{K}^0(0) \cdot K^0(t)) \mid^2 = \cos^2\theta\, \sin^2\theta \times \\ &\times \left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2\, e^{-\frac{(\Gamma_1 + \Gamma_2) t}{2}} \cos\left((E_2 - E_1) t\right) \right], \end{aligned} \tag{32}$$

since $\cos^2\theta = \sin^2\theta = 1/2$,

$$P(K^0 \to \bar{K}^0) = \frac{1}{4} \left[\, e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2\, e^{-\frac{(\Gamma_1 + \Gamma_2) t}{2}} \cos\left((E_2 - E_1) t\right) \right], \tag{33}$$

where $E_2 - E_1$ is determined by expression (29).

Now together with K^0-meson oscillations, the K_1^0-, K_2^0-meson decays will take place. Since $\Gamma_1 \gg \Gamma_2$, after some time K_2^0 mesons will remain and K^0-meson oscillations will disappear. The above-considered case will be realized when CP violation is absent. Now we consider the case when CP violation takes place.

3. K_1^0-, K_2^0-MESON VACUUM MIXINGS AND OSCILLATIONS AT INDIRECT VIOLATION OF CP INVARIANCE WITHOUT AND WITH TAKING INTO ACCOUNT WIDTH DECAYS

At first we consider vacuum mixings of K_1^0, K_2^0 mesons, then come to the consideration of K_1^0-, K_2^0-meson oscillations in cases when width decays are not taken into account and when width decays are taken into account.

3.1. The Vacuum Mixings of K_1^0, K_2^0 Mesons. In the case of CP violation just as in the case of $K^0, \bar{K}^0$ mesons when they are transformed into superpositions of K_1^0, K_2^0 mesons, the K_1^0, K_2^0 mesons have to transform into superposition states of K_S and K_L mesons.

Following the traditions mentioned above, we will consider mixings and oscillations of K_1^0, K_2^0 mesons by using the mass matrix with masses in the linear form. Before CP violation the mass matrix of K_1^0, K_2^0 has a diagonal form:

$$\left(\begin{array}{cc} m_{K_1^0} & 0 \\ 0 & m_{K_2^0} \end{array} \right). \tag{34}$$

Then because of the presence of CP-parity violation in weak interactions the mass matrix becomes nondiagonal:

$$\begin{pmatrix} m_{K_1^0} & m_{12} \\ m_{21} & m_{\bar{K}_2^0} \end{pmatrix} \equiv U^-1 \begin{pmatrix} m_{K_S} & 0 \\ 0 & m_{K_L} \end{pmatrix} U, U = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix}. \tag{35}$$

Diagonalizing this matrix by turning it through angle β, we get

$$\tan 2\beta = \frac{2m_{12}}{\mid m_{K_1^0} - m_{K_2^0} \mid},$$

$$\sin\, 2\beta = \frac{2m_{12}}{\sqrt{(m_{K_1^0} - m_{K_2^0})^2 + (2m_{12})^2}}, \tag{36}$$

$$m_{K_S,K_L} = \frac{1}{2}\left[(m_{K_1^0} + m_{K_2^0}) \mp \left((m_{K_1^0} - m_{K_2^0})^2 + 4m_{12}^2\right)^{1/2}\right]. \tag{37}$$

This procedure leads to appearance of K_S, K_L states which consist of K_1^0, K_2^0 states:

$$\begin{aligned} K_S &= \cos\,\beta\, K_1^0 - \sin\,\beta\, K_2^0, \\ K_L &= \sin\,\beta\, K_1^0 + \cos\,\beta\, K_2^0. \end{aligned} \tag{38}$$

At inverse transformation we get

$$\begin{aligned} K_1^0 &= \cos\,\beta\, K_S + \sin\,\beta\, K_L, \\ K_2^0 &= -\sin\,\beta\, K_S + \cos\,\beta\, K_L. \end{aligned} \tag{39}$$

It is necessary to stress that in the above expression we have used unitary transformation, in contrast to nonunitary transformation which was applied in work [11].

Now come to computation of the value of K_S- and K_L-meson masses difference by using K_1^0-, K_2^0-meson mass values from expressions (24) and (37):

$$m_{S,L} = \frac{1}{2}\left(2m_{K^0} \mp \sqrt{(2\Delta)^2 + (2m_{12})^2}\right), \tag{40}$$

$$\Delta m_{LS} = m_L - m_S = \sqrt{(2\Delta)^2 + (2m_{12})^2}. \tag{41}$$

It is clear that term m_{12} is much bigger than Δ (see below); i.e.,

$$\Delta \gg m_{12}. \tag{42}$$

Then K_L- and K_S-masses difference is

$$\Delta m_{LS} = m_L - m_S \simeq 2\Delta. \tag{43}$$

Using expression (43) and value for $\sin^2 \beta$ obtained from experiments [10, 20] to determine the value of CP violation ($\sin^2 2\beta = 2.23 \cdot 10^{-3}$), we get

$$\sin^2 2\beta = \frac{(2m_{12})^2}{(m_{K_1^0} - m_{K_2^0})^2 + (2m_{12})^2} \equiv \frac{(2m_{12})^2}{(2\Delta)^2 + (2m_{12})^2} = 2.23 \cdot 10^{-3}. \tag{44}$$

Taking into account expression (44), we then get the estimation on $(2m_{12})^2$ ($1/2.23 \cdot 10^{-3} = 448.5$):

$$(2m_{12})^2 \simeq (2\Delta)^2 \cdot 2.23 \cdot 10^{-3}. \tag{45}$$

Oscillations of K_1^0, K_2^0 mesons will proceed on the background of K^0-, $\bar{K}^0$-meson oscillations, but since the mixing angle β is very small, it is difficult to detect such oscillations. What possibility does the Nature give to detect these oscillations (transitions)? The decay time of K_1^0 into two π mesons is much smaller than the decay time of K_2^0 on three π mesons and therefore at big distances from the source of K^0 mesons mainly $K_2^0 \approx K_L$ mesons remain. Then at the presence of $K_1^0 \to 2\pi$ mesons we can obtain information on K_1^0, K_S, and K_2^0, K_L mesons, i.e., about violation of CP parity.

Expressions (34)–(45) were used for obtaining the estimation on mass change at CP violation and we did not take into account the phase of CP violation. It is clear that we have to take into account this phase δ. We can do it by using the parametrization of Kobayashi–Maskawa matrix [17] proposed by L. Maiani [21]. The expressions for U, U^{-1} will then have the following form:

$$U = \begin{pmatrix} \cos\beta & -\sin\beta\, e^{-i\delta} \\ \sin\beta\, e^{i\delta} & \cos\beta \end{pmatrix} \quad U^{-1} = \begin{pmatrix} \cos\beta & \sin\beta\, e^{-i\delta} \\ -\sin\beta\, e^{i\delta} & \cos\beta \end{pmatrix}. \tag{46}$$

Now expressions (38) and (39) look like

$$\begin{aligned} K_S &= \cos\beta\, K_1^0 - \sin\beta\, K_2^0\, e^{-i\delta}, \\ K_L &= \sin\beta\, e^{i\delta} K_1^0 + \cos\beta\, K_2^0, \end{aligned} \tag{47}$$

$$\begin{aligned} K_1^0 &= \cos\beta\, K_S + \sin\beta\, e^{-i\delta} K_L, \\ K_2^0 &= -\sin\beta\, e^{i\delta} K_S + \cos\beta\, K_L. \end{aligned} \tag{48}$$

Now come to consider such oscillations. From exprsessions (24), (40), (43) and (45) we see that the mass difference between K_1^0, K_2^0 mesons and K_S, K_L mesons is very small; i.e., practically they are equal. In literature [22] it is already accepted; i.e., no distinction is made between them.

3.2. The Vacuum Oscillations of K_1^0, K_2^0 Mesons. K_S, K_L mesons with masses m_S and m_L evolve in dependence on time by the following expressions:

$$K_S(t) = \mathrm{e}^{-iE_S t} K_S(0), \qquad K_L(t) = \mathrm{e}^{-iE_L t} K_L(0), \tag{49}$$

where

$$E_k^2 = (p^2 + m_k^2), \quad k = S, L. \tag{50}$$

If these mesons move without interactions, then

$$\begin{aligned} K_1^0(t) &= \cos\beta\, \mathrm{e}^{-iE_S t} K_S(0) + \sin\beta\, \mathrm{e}^{-iE_L t}\, \mathrm{e}^{-i\delta} K_L(0), \\ K_2^0(t) &= -\sin\beta\, \mathrm{e}^{-iE_S t}\, \mathrm{e}^{i\delta} K_S(0) + \cos\beta\, \mathrm{e}^{-iE_L t} K_L(0). \end{aligned} \tag{51}$$

Using expressions for K_S and K_L from (47) and using them in (51), we obtain

$$\begin{aligned} K_1^0(t) &= \left[\, \mathrm{e}^{-iE_S t} \cos^2\beta \,+\, \mathrm{e}^{-iE_L t} \sin^2\beta \right] K_1^0(0) + \\ &\quad + \mathrm{e}^{-i\delta} \left[- \,\mathrm{e}^{-iE_S t} + \mathrm{e}^{-iE_L t}\right] \sin\beta \cos\beta\, K_2^0(0), \\ K_2^0(t) &= \left[\, \mathrm{e}^{-iE_S t} \sin^2\beta \,+\, \mathrm{e}^{-iE_L t} \cos^2\beta \right] K_1^0(0) + \\ &\quad + \mathrm{e}^{i\delta} \left[- \,\mathrm{e}^{-iE_S t} + \mathrm{e}^{-iE_L t}\right] \sin\beta \cos\beta\, K_2^0(0). \end{aligned} \tag{52}$$

The probability that meson K_1^0 produced at moment $t = 0$ will be at moment $t \neq 0$ in the state of K_2^0 meson is given by the squared absolute value of the amplitude in (52); i.e.,

$$\begin{aligned} P(K_1^0 \to K_2^0) = P(K_2^0 \to K_1^0) &= \mid (K_2^0(0) \cdot K_1^0(t)) \mid^2 = \\ &= \frac{1}{2} \sin^2 2\beta \left[1 - \cos\left((E_L - E_S)t\right)\right]. \end{aligned} \tag{53}$$

Using expression (40) for K_1^0-, K_2^0-meson masses, we get

$$\begin{aligned} m_S &= \frac{1}{2}\left(2m_{K^0} - \sqrt{(2\Delta)^2 + (2m_{12})^2}\right), \\ m_L &= \frac{1}{2}\left(2m_{K^0} + \sqrt{(2\Delta)^2 + (2m_{12})^2}\right), \end{aligned} \tag{54}$$

where $\Delta = 2m_{K^0 \bar{K}^0}$ (see expression (24)). Since $\Delta \gg 2m_{12}$,

$$m_S \simeq m_{K_1^0}, \quad m_L \simeq m_{K_2^0}, \tag{55}$$

further taking into account that $m_{K^0} \gg \Delta$, we obtain

$$E_S = \sqrt{p^2 + m^2_{K_S}} \cong \sqrt{p^2 + m^2_{K^0_1}} \cong E_{K^0}\left(1 - \frac{m_{K^0}\Delta}{E^2_{K^0}}\right),$$
$$E_L = \sqrt{p^2 + m^2_{K^0_2}} \cong \sqrt{p^2 + m^2_{K^0_2}} \cong E_{K^0}\left(1 + \frac{m_{K^0}\Delta}{E^2_{K^0}}\right),$$
$$E_L - E_S \cong \frac{2m_{K^0}\Delta}{E_{K^0}} = \frac{2\Delta}{\gamma}. \tag{56}$$

In this case the length of oscillations R_{LS} is

$$R_{LS} \cong \frac{\gamma}{2\Delta}. \tag{57}$$

From expressions (24), (40), (43) and (45) we see that the length of oscillations has to be of the order of the length of K^0-, $\bar{K}^0$-meson oscillations, right up they are nearly equal (by the way, it is usually presumed).

Now we consider K^0_1-, K^0_2-meson oscillations, taking into account the decay widths.

3.3. Vacuum Oscillations of K^0_1, K^0_2 Mesons with Taking into Account Decay Widths. If we take into account that K_S, K_L decay and have the decay widths Γ_S, Γ_L, we can rewrite expressions (49)–(52), and then K_S, K_L mesons with masses m_S and m_L evolve in dependence on time according to the following formula:

$$K_S(t) = e^{-iE_S t - \frac{\Gamma_S t}{2}} K_S(0), \qquad K_L(t) = e^{-iE_L t - \frac{\Gamma_L t}{2}} K_L(0), \tag{58}$$

where

$$E^2_k = (p^2 + m^2_k), \quad k = S, L.$$

If mesons are moving without interactions, then

$$\begin{aligned} K^0_1(t) &= \cos\beta\, e^{-iE_S t - \frac{\Gamma_S t}{2}} K_S(0) + \sin\beta\, e^{-i\delta}\, e^{-iE_L t - \frac{\Gamma_L t}{2}} K_L(0), \\ K^0_2(t) &= -\sin\beta\, e^{i\delta}\, e^{-iE_S t - \frac{\Gamma_S t}{2}} K_S(0) + \cos\beta\, e^{-iE_L t - \frac{\Gamma_L t}{2}} K_L(0). \end{aligned} \tag{59}$$

Using the expressions for K_S- and K_L-meson states from (38) and using them in expression (59), we get

$$\begin{aligned} K^0_1(t) &= \left[e^{-iE_S t - \frac{\Gamma_S t}{2}} \cos^2\beta + e^{-iE_L t - \frac{\Gamma_L t}{2}} \sin^2\beta \right] K^0_1(0) + \\ &\quad + e^{-i\delta} \left[e^{-iE_S t - \frac{\Gamma_S t}{2}} - e^{-iE_L t - \frac{\Gamma_L t}{2}} \right] \sin\beta\, \cos\beta\, K^0_2(0), \\ K^0_2(t) &= \left[e^{-iE_S t - \frac{\Gamma_S t}{2}} \sin^2\beta + e^{-iE_L t - \frac{\Gamma_L t}{2}} \cos^2\beta \right] K^0_1(0) + \\ &\quad + e^{i\delta} \left[e^{-iE_S t - \frac{\Gamma_S t}{2}} - e^{-iE_L t - \frac{\Gamma_L t}{2}} \right] \sin\beta \cos\beta\, K^0_2(0). \end{aligned} \tag{60}$$

Then the probability that meson K_1^0 produced at moment $t = 0$ will be at moment $t \neq 0$ in the state of K_2^0 meson is given by the squared absolute value of the amplitude in (60); i.e.,

$$P(K_2^0 \to K_1^0, t) =\mid (K_1^0(0) \cdot K_2^0(t)) \mid^2 =$$
$$= \frac{1}{4} \sin^2 2\beta \Big[\, e^{-\Gamma_S t} + \, e^{-\Gamma_L t} - 2\, e^{-\frac{(\Gamma_S + \Gamma_L)t}{2}} \cos\left((E_L - E_S)t\right)\Big] \simeq$$
$$\simeq \varepsilon \Big[\, e^{-\Gamma_S t} + \, e^{-\Gamma_L t} - 2\, e^{-\frac{(\Gamma_S + \Gamma_L)t}{2}} \cos\left((E_L - E_S)t\right)\Big] \quad (61)$$

and

$$P(K_2^0 \to K_1^0, t) = P(K_1^0 \to K_2^0, t). \quad (61')$$

How can we see oscillations at $K_2^0 \leftrightarrow K_1^0$ mesons transition? Since there is a big number of K_1^0 mesons, it is difficult to see these oscillations because they will be masked by their background. Then we have to see these oscillations at distances when the number of K_1^0 mesons $n_{K_1^0}$ is smaller than ε:

$$e^{-\Gamma_S t_1} < \varepsilon, \qquad t_1 > -\ln(\varepsilon)/\Gamma_S;$$

i.e., $t_1 > 6\tau_s$ where τ_s is the decay time of K_S mesons. If velocity v of K^0 $v \simeq c$, the distance L_1 is

$$L_1 > 6\tau_s c.$$

Then at $t > t_1$ expression (61) can be rewritten in the following form:

$$P(K_2^0 \to K_1^0, t) \simeq \varepsilon \Bigg[e^{-\Gamma_L t} +$$
$$+ 2\, e^{-s\frac{(\Gamma_S + \Gamma_L)t}{2}} \left(-1 + 2\sin^2\left((E_L - E_S)\frac{t}{2}\right)\right)\Bigg]. \quad (61'')$$

So, expression (61″) can be used to register the above oscillations and the length of such oscillations is determined by expression (64).

And $P(K_1^0 \to K_1^0)$ is

$$P(K_1^0 \to K_1^0) =\mid (K_1^0(0) \cdot K_1^0(t)) \mid^2 =$$
$$= \Big[\cos^4\beta\, e^{-\Gamma_S t} + \sin^4\beta\, e^{-\Gamma_L t} + 2\sin^2\beta\cos^2\beta\, e^{-\frac{(\Gamma_S + \Gamma_L)t}{2}} \cos\left((E_L - E_S)t\right)\Big] \simeq$$
$$\simeq \Big[\, e^{-\Gamma_S t} + \epsilon^2 \, e^{-\Gamma_L t} + 2\epsilon\, e^{-\frac{(\Gamma_S + \Gamma_L)t}{2}} \cos\left((E_L - E_S)t\right)\Big], \quad (62)$$

and $P(K_2^0 \to K_2^0)$ is

$$P(K_2^0 \to K_2^0) = \mid (K_2^0(0) \cdot K_2^0(t)) \mid^2 =$$
$$= \left[\sin^4\beta\, \mathrm{e}^{-\Gamma_S t} + \cos^4\beta\, \mathrm{e}^{-\Gamma_L t} + 2\sin^2\beta\cos^2\beta\, \mathrm{e}^{-\frac{(\Gamma_S+\Gamma_L)t}{2}} \cos\left((E_L - E_S)t\right)\right] \simeq$$
$$\simeq \left[\epsilon^2\, \mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} + 2\epsilon\, \mathrm{e}^{-\frac{(\Gamma_S+\Gamma_L)t}{2}} \cos\left((E_L - E_S)t\right)\right], \quad (62')$$

the above has taken into account that $\cos^2\beta \simeq 1$, $\sin^2\beta \simeq \epsilon$.

Using expressions (40) and (24) for K_1^0-, K_2^0-meson masses, we obtain the same expression as in (56):

$$E_L - E_S \cong \frac{2m_{K^0}\Delta}{E_{K^0}} = \frac{2\Delta}{\gamma}. \quad (63)$$

Then the length of R_{LS} of K_1^0, K_2^0 oscillations is

$$R_{LS} \cong \frac{\gamma}{2\Delta} \equiv \frac{2\pi hc\gamma}{2\Delta} = 0.352\gamma[m]. \quad (64)$$

Since the decay mode of K_L, K_S mesons slightly differs from the decay mode of K_1^0, K_2^0, we can suppose that $\Gamma_S \simeq \Gamma_1$ and $\Gamma_L \simeq \Gamma_2$. In this case expression (62) gets the following form:

$$P(K_1^0 \to K_2^0) \equiv P(K_2^0 \to K_1^0) = \mid (K_2^0(0) \cdot K_1^0(t)) \mid^2 =$$
$$= \frac{1}{4}\sin^2 2\beta \left[\mathrm{e}^{-\Gamma_1 t} + \mathrm{e}^{-\Gamma_2 t} - 2\,\mathrm{e}^{-\frac{(\Gamma_1+\Gamma_2)t}{2}} \cos\left((E_L - E_S)t\right)\right]. \quad (65)$$

3.4. Vacuum Oscillations of K_2^0, K_1^0 Mesons in the Case When Unitarity of Mixing Matrix Is Violated. In expression (46) matrix U is unitary; i.e., $UU^{-1} = 1$. In principle we can use the nonunitary matrix, i.e., use matrix U and for back transformation, use matrix U^T instead of U^{-1} ($\det U = \det U^T = 1$), then

$$U = \begin{pmatrix} \cos\beta & -\sin\beta\, \mathrm{e}^{-i\delta} \\ \sin\beta\, \mathrm{e}^{i\delta} & \cos\beta \end{pmatrix}, \quad U^T = \begin{pmatrix} \cos\beta & \sin\beta\, \mathrm{e}^{i\delta} \\ -\sin\beta\, \mathrm{e}^{-i\delta} & \cos\beta \end{pmatrix}. \quad (66)$$

Now instead of expressions (47) and (48) we get

$$\begin{aligned} K_S &= \cos\beta\, K_1^0 - \sin\beta\, K_2^0\, \mathrm{e}^{i\delta}, \\ K_L &= \sin\beta\, \mathrm{e}^{-i\delta} K_1^0 + \cos\beta\, K_2^0, \end{aligned} \quad (67)$$

$$\begin{aligned} K_1^0 &= \cos\beta\, K_S + \sin\beta\, \mathrm{e}^{-i\delta} K_L, \\ K_2^0 &= -\sin\beta\, \mathrm{e}^{i\delta} K_S + \cos\beta\, K_L. \end{aligned} \quad (68)$$

Now if mesons are moving without interactions, then

$$\begin{aligned} K_1^0(t) &= \cos\,\beta\,\mathrm{e}^{-iE_S t - \frac{\Gamma_S t}{2}} K_S(0) + \sin\,\beta\,\mathrm{e}^{-i\delta}\;\mathrm{e}^{-iE_L t - \frac{\Gamma_L t}{2}} K_L(0), \\ K_2^0(t) &= -\sin\,\beta\,\mathrm{e}^{i\delta}\,\mathrm{e}^{-iE_S t - \frac{\Gamma_S t}{2}} K_S(0) + \cos\,\beta\,\mathrm{e}^{-iE_L t - \frac{\Gamma_L t}{2}} K_L(0). \end{aligned} \tag{69}$$

Using the expressions for K_S- and K_L-meson states from (67) and putting them into expression (69), we get

$$\begin{aligned} K_1^0(t) = &\left[\;\mathrm{e}^{-iE_S t - \frac{\Gamma_S t}{2}} \cos^2\beta + \;\mathrm{e}^{-iE_L t - \frac{\Gamma_L t}{2}}\;\mathrm{e}^{-2i\delta}\sin^2\beta\right] K_1^0(0) + \\ &+ \left[-\mathrm{e}^{-iE_S t - \frac{\Gamma_S t}{2}}\;\mathrm{e}^{i\delta} + \;\mathrm{e}^{-iE_L t - \frac{\Gamma_L t}{2}}\;\mathrm{e}^{-i\delta}\right] \sin\,\beta \cos\,\beta\, K_2^0(0), \\ K_2^0(t) = &\left[\;\mathrm{e}^{-iE_S t - \frac{\Gamma_S t}{2}}\;\mathrm{e}^{2i\delta}\sin^2\beta\; + \;\mathrm{e}^{-iE_L t - \frac{\Gamma_L t}{2}}\;\cos^2\beta\right] K_1^0(0) + \\ &+ \left[-\mathrm{e}^{-iE_S t - \frac{\Gamma_S t}{2}}\;\mathrm{e}^{i\delta} + \;\mathrm{e}^{-iE_L t - \frac{\Gamma_L t}{2}}\;\mathrm{e}^{-i\delta}\right] \sin\,\beta \cos\,\beta\, K_2^0(0). \end{aligned} \tag{70}$$

The probability that meson K_1^0 produced at moment $t = 0$ will be at moment $t \neq 0$ in the state of K_1^0 meson is given by the squared absolute value of the amplitude in (71); i.e.,

$$\begin{aligned} P(K_1^0 \to K_1^0) =\mid (K_1^0(0) \cdot K_1^0(t)) \mid^2 = &\Big[\cos^4\beta\,\mathrm{e}^{-\Gamma_S t} + \sin^4\beta\,\mathrm{e}^{-\Gamma_L t} + \\ &. + 2\sin^2\beta\cos^2\beta\,\mathrm{e}^{-\frac{(\Gamma_S + \Gamma_L)t}{2}}\cos\left((E_L - E_S)t + 2\delta\right)\Big] \simeq \\ \simeq &\left[\mathrm{e}^{-\Gamma_S t} + \epsilon^2\;\mathrm{e}^{-\Gamma_L t} + 2\epsilon\,\mathrm{e}^{-\frac{(\Gamma_S + \Gamma_L)t}{2}}\cos\left((E_L - E_S)t + 2\delta\right)\right], \end{aligned} \tag{71}$$

and probability of $P(K_2^0 \to K_2^0)$ transition is

$$\begin{aligned} P(K_2^0 \to K_2^0) =\mid (K_2^0(0) \cdot K_2^0(t)) \mid^2 = &\Big[\sin^4\beta\,\mathrm{e}^{-\Gamma_S t} + \cos^4\beta\,\mathrm{e}^{-\Gamma_L t} + \\ &+ 2\sin^2\beta\cos^2\beta\,\mathrm{e}^{-\frac{(\Gamma_S + \Gamma_L)t}{2}}\cos\left((E_L - E_S)t + 2\delta\right)\Big] \simeq \\ \simeq &\left[\epsilon^2\,\mathrm{e}^{-\Gamma_S t} + \;\mathrm{e}^{-\Gamma_L t} + 2\epsilon\,\mathrm{e}^{-\frac{(\Gamma_S + \Gamma_L)t}{2}}\cos\left((E_L - E_S)t + 2\delta\right)\right]. \end{aligned} \tag{71$'$}$$

Then the probability that meson K_1^0 produced at moment $t = 0$ will be at moment $t \neq 0$ in the state of K_2^0 meson is given by the squared absolute value of the amplitude in (70); i.e.,

$$
\begin{aligned}
&P(K_2^0 \to K_1^0, t) = \mid (K_2^0(0) \cdot K_1^0(t)) \mid^2 = \\
&= \frac{1}{4}\sin^2 2\beta \left[\mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} - 2\,\mathrm{e}^{-\frac{(\Gamma_S+\Gamma_L)t}{2}} \cos\left((E_L - E_S)t + 2\delta\right) \right] \simeq \\
&\qquad \simeq \varepsilon \left[\mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} - 2\,\mathrm{e}^{-\frac{(\Gamma_S+\Gamma_L)t}{2}} \cos\left((E_L - E_S)t + 2\delta\right) \right], \quad (71'')
\end{aligned}
$$

and $P(K_2^0 \to K_1^0, t) = P(K_1^0 \to K_2^0, t)$ (the above has taken into account that $\cos^2\beta \simeq 1$, $\sin^2\beta \simeq \epsilon$).

The length of oscillations in this case is given by expressions (63), (64). Expression (71$''$) was obtained by using the standard technique of oscillations and it is analogous to the expression obtained in [11, 12] at violation of orthogonality of K_S, K_L states.

4. PROBABILITIES of $K^0 \leftrightarrow \bar{K}^0$ MESON TRANSITIONS (OSCILLATIONS) VIA K_S, K_L MESONS

In principle we can consider transition of $K^0, \bar{K}^0$ mesons into K_S, K_L mesons, then ($a = \cos\beta - \sin\beta, b = \sin\beta + \cos\beta$):

$$
\begin{aligned}
K^0 &= \frac{1}{\sqrt{2}}[(\cos\beta - \sin\beta)K_S + (\sin\beta + \cos\beta)K_L] = \frac{1}{\sqrt{2}}(aK_S + bK_L), \\
\bar{K}^0 &= \frac{1}{\sqrt{2}}[-(\sin\beta + \cos\beta)K_S + (\cos\beta - \sin\beta)K_L] = \frac{1}{\sqrt{2}}(-bK_S + aK_L),
\end{aligned} \quad (72)
$$

at the inverse transformation we get

$$
\begin{aligned}
K_S &= \frac{1}{\sqrt{2}}[(\cos\beta - \sin\beta)K^0 - (\cos\beta + \sin\beta)\bar{K}^0] = \frac{1}{\sqrt{2}}(aK^0 - b\bar{K}^0), \\
K_L &= \frac{1}{\sqrt{2}}[(\cos\beta + \sin\beta)K^0 + (\cos\beta - \sin\beta)\bar{K}^0] = \frac{1}{\sqrt{2}}(bK^0 + a\bar{K}^0).
\end{aligned} \quad (73)
$$

It is necessary to stress that in the above expression the normalization was not lost, while in [11] (see also [12]) there is a need to fulfil renormalization (there if the unitarity was lost, then it is necessary to restore it):

$$
\begin{aligned}
K^0 &= \frac{1}{\sqrt{2}(1+\epsilon)}[K_L + \sqrt{1+\mid\epsilon\mid^2}K_S], \\
\bar{K}^0 &= \frac{1}{\sqrt{2}(1-\epsilon)}[K_L - \sqrt{1+\mid\epsilon\mid^2}K_S].
\end{aligned} \quad (74)
$$

It is necessary especially to stress that a straight transition from $K^0, \bar{K}^0$ mesons to K_S, K_L mesons is not correct since K_1^0, K_2^0 mesons play an important role at CP violation.

Repeating the above procedure (30)–(39) for expressions (46) and (47) by using expression (58), we get

$$K^0(t) = \frac{1}{2}\left[\, e^{-iE_S t - \frac{\Gamma_S t}{2}} a^2 + \, e^{-iE_L t - \frac{\Gamma_L t}{2}} b^2\right] K^0(0) + \\ + \frac{1}{2}\left[e^{-iE_S t - \frac{\Gamma_S t}{2}} - \, e^{-iE_L t - \frac{\Gamma_L t}{2}}\right] ab\bar{K}^0(0), \quad (75)$$

$$\bar{K}^0(t) = \frac{1}{2}\left[\, e^{-iE_S t - \frac{\Gamma_S t}{2}} b^2 + \, e^{-iE_L t - \frac{\Gamma_L t}{2}} a^2\right] K^0(0) + \\ + \frac{1}{2}\left[\, e^{-iE_S t - \frac{\Gamma_S t}{2}} - \, e^{-iE_L t - \frac{\Gamma_L t}{2}}\right] ba\bar{K}^0(0). \quad (76)$$

The probability that meson K^0 produced at moment $t = 0$ will be at moment $t \neq 0$ in the state of $\bar{K}^0$ meson is given by the squared absolute value of the amplitude in (75), (76); i.e.,

$$P(K^0 \to \bar{K}^0, t) = P(\bar{K}^0 \to K^0, t) = \mid (\bar{K}^0(0) \cdot K^0(t)) \mid^2 = \\ = \frac{1}{4} a^2 b^2 \left[e^{-\Gamma_S t} + \, e^{-\Gamma_L t} - 2\, e^{-\frac{(\Gamma_S + \Gamma_L)t}{2}} \cos\left((E_L - E_S)t\right)\right]. \quad (77)$$

Then

$$P(K^0 \to K^0, t) = \mid (K^0(0) \cdot K^0(t)) \mid^2 = \\ = \frac{1}{4}\left[a^4\, e^{-\Gamma_S t} + b^4\, e^{-\Gamma_L t} + 2a^2 b^2\, e^{-\frac{(\Gamma_S + \Gamma_L)t}{2}} \cos\left((E_L - E_S)t\right)\right]. \quad (78)$$

and

$$P(\bar{K}^0 \to \bar{K}^0, t) = \mid (\bar{K}^0(0) \cdot \bar{K}^0(t)) \mid^2 = \\ = \frac{1}{4}\left[b^4\, e^{-\Gamma_S t} + a^4\, e^{-\Gamma_L t} + 2a^2 b^2\, e^{-\frac{(\Gamma_S + \Gamma_L)t}{2}} \cos\left((E_L - E_S)t\right)\right]. \quad (79)$$

From expressions (77)–(79) we see that these expressions have no sense since at transition of $K^0, \bar{K}^0$ mesons into superpositions of K_1^0, K_2^0 mesons the K_1^0 meson states decay very quickly and then K_2^0-meson states remain; i.e., further it is justified to consider only K_1^0-, K_2^0-meson states. It is necessary to remind that oscillations between K_S, K_L mesons are absent.

5. CONCLUSION

In the literature [11, 12] the nonunitary transformation is used at obtaining the K_S, K_L states. It is supposed that these states arise at CP violation. In expression (4) for $|K_1^0|^2$ cross term is present which is responsible for oscillations. This term can appear only at violation of orthogonality of K_S, K_L states. In the framework of the quantum approach we have to suppose that the K_S, K_L states are orthogonal. The problem we are solving in this work is: how do oscillations arise in the framework of quantum mechanics approach (without violation of unitarity and orthogonality) and how do short-living mesons appear at long distances from K^0 source? For this aim we have used the standard technique of oscillations.

This work has considered $K^0, \bar{K}^0$ mixings and oscillations via K_1^0-, K_2^0-meson states at strangeness violation by weak interactions and K_1^0-, K_2^0-meson mixings and oscillations via K_S-, K_L-meson states at CP violation by the weak interactions without and with taking into decay widths. We have worked in the framework of the mass mixing scheme while considering the oscillations. It has been shown that K_1^0-(K_S-)meson states appear at big distances from the K^0-meson source after their decays ($\tau_L \gg \tau_S$ ($\tau_2 \gg \tau_1$)) due to oscillations of residual K_2^0 (K_L) mesons, then we see again short-living K_1^0 (K_S) mesons. It is implied that $K_L \leftrightarrow K_S$ meson oscillations are absent. We have also considered the case when at CP violation the unitarity is violated but orthogonality of K_S, K_L states remains. The general expressions for probabilities of meson oscillations (transitions) have been given.

It is necessary to remark that usually it is supposed [22] that at long distances

$$K_L \simeq K_2 + \varepsilon K_1$$

mesons are presented and then the probability of CP violation is directly proportional to the parameter of CP violation ε:

$$P(K_L \to 2\pi, t) \sim \varepsilon.$$

But when we use the standard technique of oscillations at long distances, K_2 states remain and K_1 states appear as a result of oscillations, i.e., transition of K_2 mesons into K_1 mesons. Then

$$K_L = \sin\beta\, K_1^0 + \cos\beta\, K_2^0,$$

where $\sin\beta \simeq \varepsilon$. The probablity $P(K_2^0 \to K_1^0, t)$ of such transitions, i.e., CP violation, is proportional to ε^2; i.e.,

$$P(K_2^0 \to K_1^0, t) \sim \varepsilon^2$$

but not to ε, in contrast to [22].

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