

E2-2012-19

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SELF-SIMILARITY OF JET PRODUCTION IN  $pp$  AND  $\bar{p}p$   
COLLISIONS AT RHIC, TEVATRON, AND LHC

Submitted to «International Journal of Modern Physics A»

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Самоподобие рождения струй в  $pp$ - и  $\bar{p}p$ -столкновениях на RHIC, тэватроне и LHC

Самоподобие рождения струй в  $pp$ - и  $\bar{p}p$ -столкновениях изучается в рамках теории  $z$ -скейлинга. Анализируются инклюзивные спектры рождения струй, измеренные коллаборациями STAR на RHIC, CDF и DØ на тэватроне и CMS и ATLAS на LHC. Экспериментальные спектры сравниваются с результатами КХД-расчетов. Показано, что свойство самоподобия, диктуемое  $z$ -скейлингом, накладывает жесткие ограничения на асимптотическое поведение скейлинговой функции  $\psi(z)$  при больших  $z$  и отличается от предсказываемого пертурбативной КХД. Обсуждаются новые результаты об энергетической и угловой независимости и асимптотическом поведении скейлинговой функции. Полученные результаты рассматриваются как подтверждение свойств локальности, самоподобия и фрактальности рождения струй, отражающих особенности структуры адронов, взаимодействия их конstituентов и процесса адронизации.

Работа выполнена в Лаборатории физики высоких энергий им. В. И. Векслера и А. М. Балдина ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 2012

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Self-Similarity of Jet Production in  $pp$  and  $\bar{p}p$  Collisions at RHIC, Tevatron, and LHC

Self-similarity of jet production in  $pp$  and  $\bar{p}p$  collisions is studied in the framework of  $z$  scaling. Inclusive jet transverse momentum distributions measured by the STAR Collaboration at RHIC, the CDF and DØ Collaborations at Tevatron, and the CMS and ATLAS Collaborations at LHC are analyzed. The experimental spectra are compared with next-to-leading order QCD calculations in  $p_T$  and  $z$  presentations. It is shown that self-similar features of jet cross sections manifested by the  $z$  scaling give strong restriction on the scaling function  $\psi(z)$  at high  $z$ . New results on energy and angular independence and asymptotic behavior of  $\psi(z)$  are discussed. The obtained results are considered as confirmation of self-similarity of jet production, fractality of hadron structure, and locality of constituent interactions at small scales.

The investigation has been performed at the Veksler and Balдин Laboratory of High Energy Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 2012

## 1. INTRODUCTION

Search for scaling regularities in high-energy collisions is always a subject of intense investigations [1–10]. Such regularities could give experimental indications on new physics phenomena and additional insight into theory. Production of particles with high transverse momenta from the collisions of hadrons and nuclei at high energies has relevance to constituent interactions at small scales. Universal approach to the description of the processes could provide more detailed understanding of the underlying physical phenomena and impose restrictions on phenomenological ingredients used in comparison of experimental data with theory. Jets are traditionally considered as best probes of the constituent interactions and most suitable objects for precise test of the perturbative QCD [11–17]. Study of jets is of interest both for jet production itself and search for new particles identified by the jets.

In this paper we analyse data on inclusive cross sections of jet production in  $\bar{p}p$  and  $pp$  collisions at Tevatron [18, 19], RHIC [20], and LHC [21, 22] in the framework of  $z$  scaling. The method was developed for phenomenological description of differential cross sections of particles in the inclusive reactions at high energies (see [23] and references therein). It is based on the principles of locality, self-similarity, and fractality reflecting properties of hadron structure, constituent interactions, and processes of particle formation. The  $z$  scaling is treated as a manifestation of the fractality of the structure of the colliding objects (hadrons, nuclei), locality of the interactions of their constituents, and self-similarity of the particle production process.

For the first time, jet production in the  $z$  presentation was analyzed in [24]. The analysis was based on the experimental data on jet cross sections obtained by the UA1, UA2, CDF, and DØ Collaborations. New data on inclusive production of jets at the LHC allow us to verify the  $z$  scaling in the multi-TeV energy region. Here we report on study of  $z$  presentation of the jet transverse momentum distributions measured by the CMS [21] and ATLAS [22] Collaborations in  $pp$  collisions at  $\sqrt{s} = 7$  TeV. The obtained results are compared with the data on jet production in  $\bar{p}p$  collisions at the Tevatron [25–28]. The LHC measurements confirmed the energy independence of the  $z$  scaling for jet production in the new energy domain. The power behavior of the scaling function  $\psi(z)$  at high  $z$  is verified. The transverse momentum spectra of jets are calculated in the next-to-leading order of QCD and are compared with data in  $p_T$  and  $z$  presentation.

## 2. $z$ SCALING

In the present paper we use the  $z$  scaling in the form considered in [24]. At sufficiently high energies, the collision of extended objects like hadrons and nuclei is considered as an ensemble of individual interactions of their constituents. The constituents are partons in the parton model or quarks and gluons in the theory of QCD. Multiple interactions are assumed to be similar. This property represents a self-similarity of the hadron interactions at a constituent level. The structures of the colliding objects with masses  $M_1$  and  $M_2$  are characterized by parameters  $\delta_1$  and  $\delta_2$ . The interacting constituents carry the fractions  $x_1$  and  $x_2$  of the incoming momenta  $P_1$  and  $P_2$ . The inclusive particle with mass  $m_1$  carries the momentum  $p$ . The produced recoil with mass  $m_2$  ensures conservation of the additive quantum numbers.

The elementary subprocess is considered as a binary collision of the constituents with masses  $x_1 M_1$  and  $x_2 M_2$  resulting in the scattered and recoil objects with masses  $m_1$  and  $M_X = x_1 M_1 + x_2 M_2 + m_2$ , respectively. The momentum conservation law of the subprocess is connected with the recoil mass  $M_X$  which is written as follows

$$(x_1 P_1 + x_2 P_2 - p)^2 = M_X^2. \quad (1)$$

This equation is expression of the locality of the hadron interaction at a constituent level. It represents a constraint on the fractions  $x_1$  and  $x_2$ . The structural parameters  $\delta_1$  and  $\delta_2$  are connected with the corresponding momentum fractions by the function

$$\Omega(x_1, x_2) = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2}. \quad (2)$$

The quantity  $\Omega$  is proportional to relative number of all such constituent configurations which contain the configuration defined by the fractions  $x_1$  and  $x_2$ . The parameters  $\delta_1$  and  $\delta_2$  are interpreted as fractal dimensions in the space of the momentum fractions  $\{x_1, x_2\}$ . The fractions  $x_1$  and  $x_2$  are determined in a way to maximize the function  $\Omega$  under the condition (1). The maximal value of  $\Omega$  is used in the definition of the scaling variable  $z$  which has the form

$$z = z_0 \Omega^{-1}. \quad (3)$$

The quantity  $z_0 = s_{\perp}^{1/2} / (dN/d\eta|_0) m_N$  is proportional to the transverse kinetic energy  $s_{\perp}^{1/2}$  of the subprocess consumed on the production of the inclusive particle with mass  $m_1$  and its counterpart with mass  $m_2$ . The multiplicity density  $dN/d\eta|_0$  is taken in the center-of-mass  $N - N$  system at pseudorapidity  $\eta = 0$ . The constant  $m_N$  is a nucleon mass. We set  $\delta_1 = \delta_2 \equiv \delta$ ,  $M_1 = M_2 \equiv m_N$ , and  $m_2 = m_1 \equiv 0$  for jet production in proton-(anti)proton interactions. Here we use the value  $\delta = 1$  of the nucleon fractal dimension which was found in the analysis

of jet production at lower energies [24]. The scaling function

$$\psi(z) = -\frac{\pi s}{(dN/d\eta)\sigma_{in}} J^{-1} E \frac{d^3\sigma}{dp^3} \quad (4)$$

is expressed in terms of the inclusive cross section, multiplicity density  $dN/d\eta$ , and total inelastic cross section  $\sigma_{in}$ . The symbols  $s$  and  $J$  stand for the square of the center-of-mass energy and the corresponding Jacobian, respectively.

As known from numerous analyses of jet production, different procedures of jet reconstruction are correctly defined only at a relatively high transverse energy [29]. At low  $E_T$ , the main problem is connected with jet overlapping and energy redistribution among multiple jets. It follows that jet multiplicity density cannot be determined directly from the experimental data on jet transverse momentum distributions. Therefore, instead of  $dN/d\eta$  we use the scaled multiplicity density  $\rho(s)/\rho_0$  normalized to unity at the energy  $\sqrt{s} = 1800$  GeV. Here we exploit the parameterization  $\rho(s) \sim s^\Delta$  with  $\Delta = 0.163$  which corresponds to the results found in [24]. As a consequence, the absolute normalization of the scaling function  $\psi(z)$  is obtained up to a constant.

### 3. INCLUSIVE JET PRODUCTION

The inclusive jet cross section is a measure of probability of observing a hadron jet with a given transverse energy  $E_T$  and pseudorapidity  $\eta$ . In terms of these variables, the cross section is expressed as follows  $d^2\sigma/dE_T d\eta$ . The measured variables are the transverse energy and pseudorapidity, and the jets are usually assumed to be massless. For most measurements, the cross section is averaged over some range of pseudorapidity. The invariant cross section is written as follows:

$$E \frac{d^3\sigma}{dp^3} = \frac{1}{2\pi E_T} \frac{d^2\sigma}{dE_T d\eta}. \quad (5)$$

Jets are experimentally defined as the amount of energy deposited in the cone of radius  $R = [(\Delta\eta)^2 + (\Delta\phi)^2]^{1/2}$  in the space  $\{\eta, \phi\}$ , where  $\Delta\eta$  and  $\Delta\phi$  specify the extent of the cone in the pseudorapidity and azimuth. The pseudorapidity  $\eta$  is determined via the center-of-mass angle  $\theta$  by the formula  $\eta = -\ln(\tan \theta/2)$ . Different algorithms (cone,  $k_T$ , anti- $k_T$ ) of jet reconstruction are developed (see [30] and references therein) and applied for RHIC, Tevatron, and LHC data analysis.

**3.1. Jets at RHIC.** The STAR Collaboration measured the inclusive cross sections of jet production in  $pp$  collisions at  $\sqrt{s} = 200$  GeV at RHIC [20]. The data cover the kinematic range of the pseudorapidity  $|\eta| < 0.8$  and the transverse momentum  $p_T = 13\text{--}57$  GeV/ $c$ . Figure 1 shows the dependence of the spectra on  $p_T$ . As seen from Fig. 1, the NLO QCD calculations with the

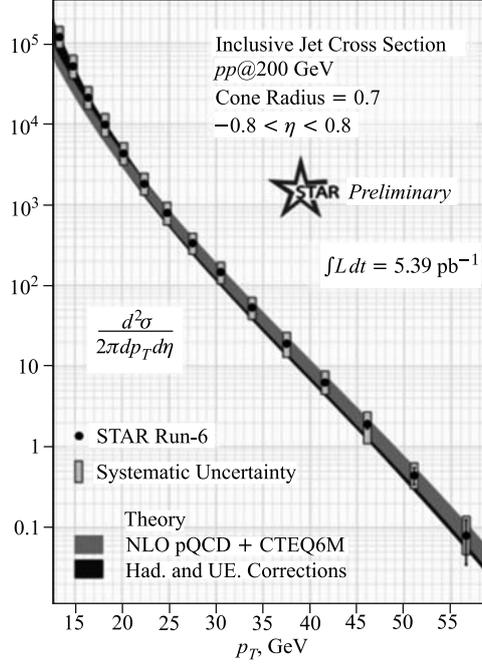


Fig. 1. Inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 200$  GeV and  $|\eta| < 0.8$  measured by the STAR Collaboration [20] at RHIC

CTEQ6M parton distribution functions at the factorization and renormalization scales equal to the transverse momentum ( $\mu_R = \mu_F = p_T$ ) demonstrate a satisfactory agreement with the data. Figure 2, *a* shows comparison of the STAR inclusive cross sections with the FNAL [31] and ISR [32] data obtained at lower energies,  $\sqrt{s} = 38.8, 45, 63$  GeV. As seen, the strong dependence of the cross sections on the collision energy  $\sqrt{s}$  increases with  $p_T$ . The STAR data are in good agreement with the UA1 data [33] obtained for jet production in  $\bar{p}p$  collision at the energy  $\sqrt{s} = 200$  GeV. The same data are shown in  $z$  presentation in Fig. 2, *b*. The energy independence and power law (shown by the dashed line) of the scaling function  $\psi(z)$  are observed.

Thus we conclude that the obtained results confirm  $z$  scaling of jet production in  $pp$  collisions at RHIC energy. Some deviation of UA1 data from the asymptotic behaviour of  $\psi(z)$  is seen at  $z < 10^2$ . The value of the slope parameter  $\beta$  is in agreement with  $\beta = 5.95 \pm 0.21$  found in [24]. Further measurements of the jet spectra in  $pp$  collisions at  $\sqrt{s} = 200$  and 500 GeV at higher  $p_T$  are of interest for precise test of QCD and for verification of the asymptotic behavior of  $\psi(z)$  predicted by the  $z$  scaling in  $pp$  and  $\bar{p}p$  interactions.

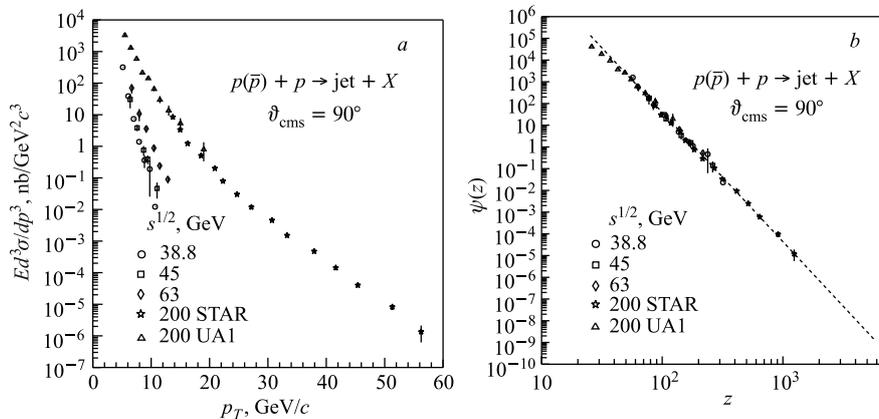


Fig. 2. Inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 38.8, 45, 63$  and 200 GeV in middle rapidity range in (a)  $p_T$  and (b)  $z$  presentation. Experimental data are measured by the E557 [31], AFS [32], and STAR [20] Collaborations. The UA1 data [33] for  $p\bar{p}$  collisions are shown for comparison

**3.2. Jets at Tevatron.** The collision energy reached at the Tevatron is much larger than maximal energy at the Sp $\bar{p}$ S and also essentially exceeds the energy of  $pp$  interactions at the ISR, SPS, and RHIC. Jet production has been successfully measured at the proton-antiproton collider at FNAL over a range  $\sqrt{s} = 630 - 1960$  GeV in Run I and Run II by the CDF and D $\bar{\phi}$  Collaborations [34]. The experimental data on inclusive cross sections of jet production at different collision energies allow us to study energy independence of the scaling function and compare it with similar analysis in  $pp$  collisions. The angular independence of  $\psi(z)$  is tested with the CDF [18] and D $\bar{\phi}$  [19] data measured in a wide range of pseudorapidity  $|\eta| < 2.4$ .

Figure 3 shows  $z$  presentation of the jet spectra at  $\sqrt{s} = 630, 1800$ , and 1960 GeV. All data demonstrate the energy independence of the scaling function. As seen from Fig. 3,  $\psi(z)$  has power behavior over a wide range of  $z = (0.1 - 7) \cdot 10^3$ . The scaling function changes more than twelve orders of magnitude in this range. The obtained result means that mechanism of jet production manifests property of self-similarity. This feature is most distinctly visible in the  $z$  representation in contrast to the  $p_T$  presentation of inclusive spectra.

The angular dependence of the transverse momentum spectra of jet production in  $p\bar{p}$  collisions was studied experimentally by the CDF Collaboration [18]. The differential cross sections obtained at  $\sqrt{s} = 1960$  GeV are shown in Fig. 4, a. The data cover the pseudorapidity range  $|\eta| < 2.1$ . The highest measured transverse energy carried by one jet was about 600 GeV. As seen from Fig. 4, a, the

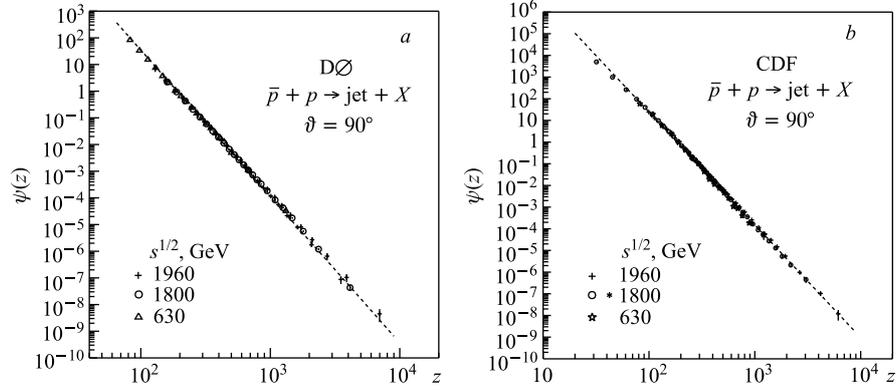


Fig. 3. The  $z$  presentation of inclusive spectra of jet production measured by the DØ [25,26] and CDF [27,28] Collaborations in  $p\bar{p}$  collisions at  $\sqrt{s} = 630, 1800,$  and  $1960$  GeV and  $\theta \simeq 90^\circ$

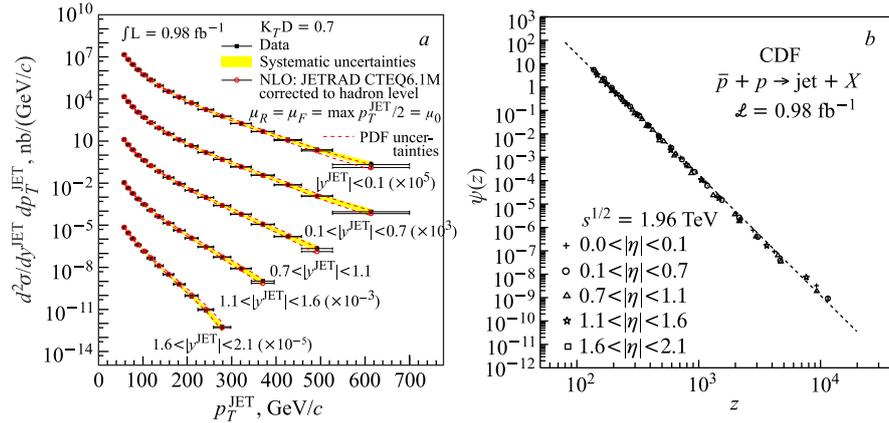


Fig. 4. Inclusive spectra of jet production in  $p\bar{p}$  collisions at  $\sqrt{s} = 1960$  GeV and different pseudorapidity intervals in (a)  $p_T$  and (b)  $z$  presentation. Experimental data obtained by the CDF Collaboration are taken from [18]

transverse momentum spectra demonstrate strong dependence on the pseudorapidity of the produced jet. The  $z$  presentation of the same data is shown in Fig. 4, b. Points correspond to the mean values of the transverse momentum in the respective  $p_T$  bins. The data demonstrate angular independence and power behavior of the scaling function  $\psi(z)$  over the range  $z \simeq (0.15-10) \cdot 10^3$ . We would like to emphasize that this result is a new confirmation of the properties of the  $z$  scaling (energy and angular independence of  $\psi(z)$ ) found at lower energy [24].

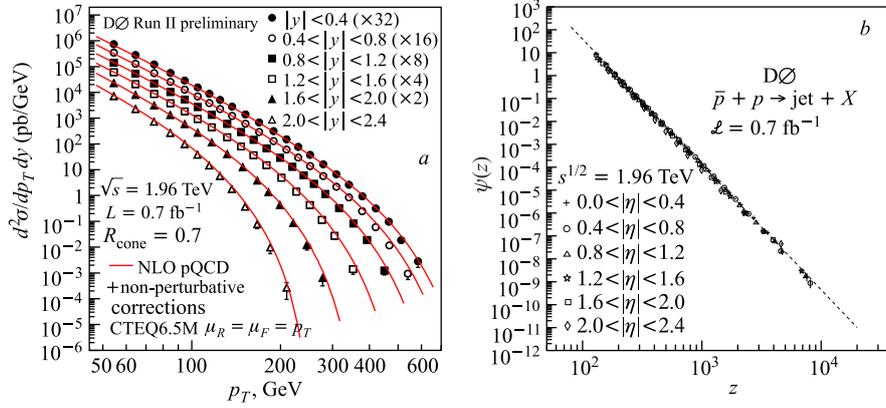


Fig. 5. Inclusive spectra of jet production in  $p\bar{p}$  collisions at  $\sqrt{s} = 1960$  GeV and different pseudorapidity intervals in (a)  $p_T$  and (b)  $z$  presentation. Experimental data obtained by the DØ Collaboration are taken from [19]

Figure 5 shows the inclusive spectra [19] of jet production in  $p\bar{p}$  collisions at the energy  $\sqrt{s} = 1960$  GeV over the transverse momentum  $p_T = 50 - 600$  GeV/c and pseudorapidity  $|\eta| < 2.4$  ranges in (a)  $p_T$  and (b)  $z$  presentation. The experimental data collected by the DØ Collaboration correspond to the integrated luminosity of  $0.7$  fb $^{-1}$ . As seen from Fig. 5, a, the spectra measured for different pseudorapidity intervals demonstrate strong angular dependence. The difference in the behavior of the cross sections is enhanced with the increasing transverse momentum. The jet yields decrease with  $p_T$  in the measured momentum range more than ten orders of magnitude. The theoretical calculations of the jet spectra in the NLO QCD approximation (see [19] and references therein) shown by the solid lines are in good agreement with the data. Figure 5, b demonstrates the angular independence of  $\psi(z)$  over a wide kinematic range. The scaling function is described by a power law,  $\psi(z) \sim z^{-\beta}$ , with constant value of the slope parameter  $\beta$ . The value of  $\beta$  is in agreement with  $\beta = 5.48 \pm 0.02$  found in [24].

Based on the obtained results we conclude that the Tevatron data on jet production measured by the CDF and DØ Collaborations in Run II confirm  $z$  scaling. Jet production is usually considered as a signature of the hard collisions of hadron constituents (quarks and gluons). The obtained results mean that the interactions of the constituents, their substructure and mechanism of jet formation reveal properties of self-similarity over a wide scale range (up to  $10^{-4}$  Fm).

**3.3. Jets at LHC.** The highest energy reached in the proton–proton collisions at the LHC is  $\sqrt{s} = 7000$  GeV till now. It is expected to be increased up to  $\sqrt{s} = 14000$  GeV soon. The collider gives us new possibility for investigation of

jet physics [15–17]. It concerns study of the interactions of hadron constituents in processes with high transverse momenta up to  $p_T \simeq 5000$  GeV and search for new physics in the reactions where jets are used as triggers.

We analyzed new LHC data [21,22] on inclusive cross sections of jet production in  $pp$  collisions at  $\sqrt{s} = 7000$  GeV in the framework of  $z$  scaling and compared the results with the Tevatron data [25–28] at lower energies. The LHC data allow us to study the energy and angular independence of the scaling function  $\psi(z)$  over a wide range of collision energy  $\sqrt{s} = 630 - 7000$  GeV and pseudorapidity  $|\eta| < 4.4$ . Figure 6 shows the spectra of jets produced in  $pp$  collisions at  $\sqrt{s} = 7000$  GeV and  $|\eta| < 3$  measured by the CMS Collaboration [21].

Figure 7, *a* demonstrates the  $z$  presentation of the jet spectra obtained by the CMS Collaboration in various pseudorapidity intervals. A comparison of the  $z$  presentation of the jet distribution in the most central pseudorapidity region with jet spectra measured by the DØ Collaboration at the Tevatron is shown in Fig. 7, *b*.

The results manifest the angular and energy independence of the scaling function. The corresponding values of  $\psi(z)$  change more than twelve orders of magnitude. As seen from Fig. 7, the power behavior of  $\psi(z)$  is observed over a wide range of  $z = (0.1 - 7) \cdot 10^3$ . The dashed line indicates the asymptotic behavior of  $\psi(z)$  at high  $z$ . Some deviation of data from the power asymptotic for  $z < 10^2$  is seen.

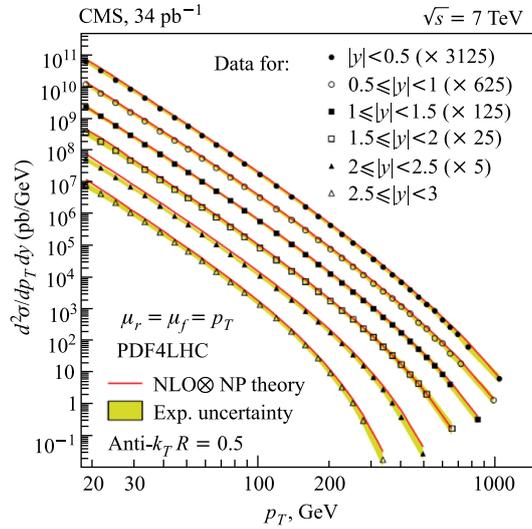


Fig. 6. Inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 7000$  GeV over a pseudorapidity range  $|\eta| < 3$  measured by the CMS Collaboration [21] at the LHC

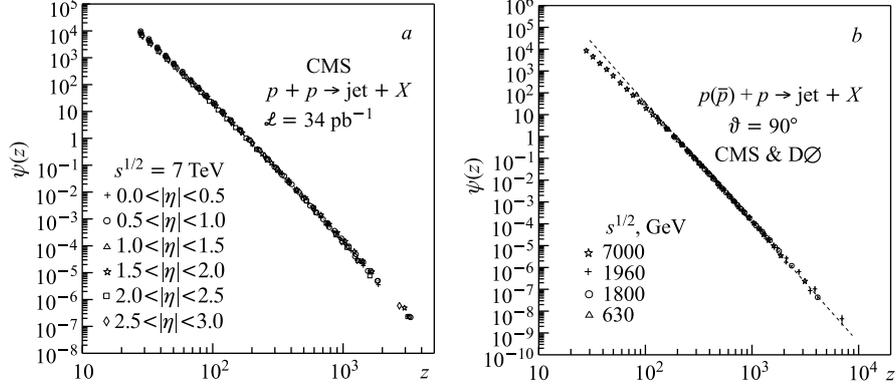


Fig. 7. *a*) Inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 7000$  GeV over a pseudorapidity range  $|\eta| < 3$  measured by the CMS Collaboration [21] in  $z$  presentation. *b*) Inclusive spectra at different energies  $\sqrt{s} = 630, 1800, 1960,$  and  $7000$  GeV and central pseudorapidity range in  $z$  presentation. Data are taken from [21,25,26]

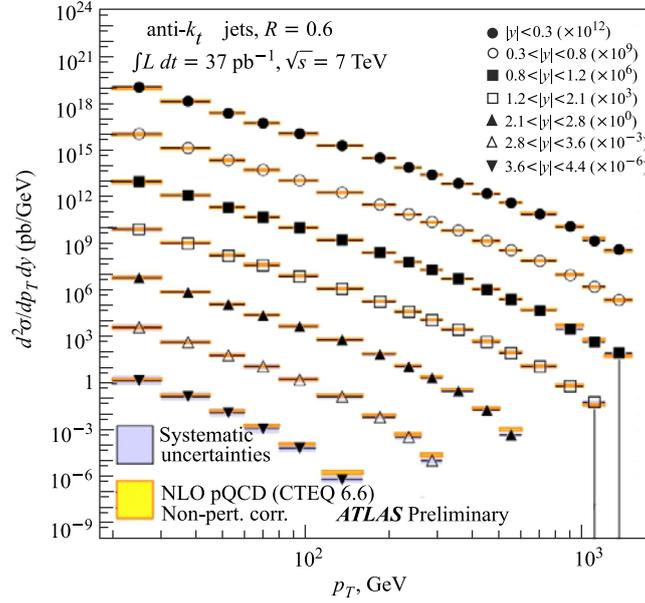


Fig. 8. Inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 7000$  GeV over a pseudorapidity range  $|\eta| < 4.4$  measured by the ATLAS Collaboration [22] at LHC

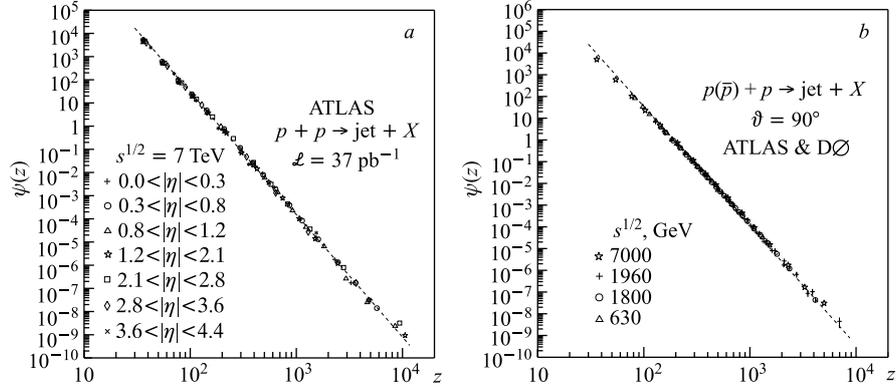


Fig. 9. *a*) Inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 7000$  GeV over a pseudorapidity range  $|\eta| < 4.4$  measured by the ATLAS Collaboration [22] at LHC in  $z$  presentation. *b*) Inclusive spectra in  $z$  presentation at different energies  $\sqrt{s} = 630, 1800, 1960,$  and  $7000$  GeV and central pseudorapidity range. Data are taken from [25, 26] and [22]

Figure 8 shows the inclusive jet spectra in  $pp$  collisions at the center-of-mass energy  $\sqrt{s} = 7000$  GeV over the transverse momentum  $p_T = 20 - 1350$  GeV/c and pseudorapidity  $|\eta| < 4.4$  ranges measured by the ATLAS Collaboration [22] at the LHC. Figure 9, *a* demonstrates  $z$  presentation of the same data. The points correspond to the mean values of the transverse momenta in the respective  $p_T$  bins. The angular independence and power law of the scaling function is observed up to  $z \simeq 10^4$ . The energy independence of  $\psi(z)$  over a range  $\sqrt{s} = 630, 1800, 1960,$  and  $7000$  GeV is demonstrated in Fig. 9, *b*. The dashed line corresponds to asymptotic behavior of  $\psi(z)$ .

As seen from Figs. 7, *a* and 9, *a*, the CMS and ATLAS data manifest the angular independence of  $\psi(z)$ . The performed comparison of the Tevatron and LHC data shown in Figs. 7, *b* and 9, *b* demonstrates the energy independence of the scaling function. The function  $\psi(z)$  is described by the power law,  $\psi(z) \sim z^{-\beta}$ , at high  $z$  with a constant value of the slope parameter  $\beta$ . The power behavior of  $\psi(z)$  is observed over a wide range of  $z$ . The obtained results give us new confirmation of self-similarity of jet production in  $pp$  collisions at the LHC.

#### 4. UNIVERSALITY OF JET PRODUCTION AT TEVATRON AND LHC

We would like to remind one more that a jet is usually considered as a direct evidence of hard interaction of hadron constituents (quarks and gluons). The spectra of jets can be connected with constituent interactions in terms of  $z$

scaling. We analysed new LHC data [21,22] on the inclusive cross sections of jet production in  $pp$  collisions at  $\sqrt{s} = 7000$  GeV in the framework of the  $z$  scaling and compared the results with Tevatron data [25–28] at lower collision energies. The data sets allowed us to study the energy and angular independence of the scaling function  $\psi(z)$  over a wide range of  $\sqrt{s} = 630$ –7000 GeV. The analysis was performed with the same values of the parameters  $\delta = 1$  and  $m_1 = m_2 = 0$  as used in [24].

Figure 10 shows  $p_T$  and  $z$  presentations of the jet spectra measured by the CMS and ATLAS Collaborations at the LHC and by the DØ and CDF Collaborations at the Tevatron. The data [21] collected by the CMS Collaboration at  $\sqrt{s} = 7000$  GeV correspond to the integrated luminosity of  $34 \text{ pb}^{-1}$  and cover the momentum and pseudorapidity range  $p_T = 18$ –1100 GeV/c and  $|\eta| < 3$ . The ATLAS Collaboration measured the inclusive jet spectra [22] in  $pp$  collisions at the energy  $\sqrt{s} = 7000$  GeV over the momentum and pseudorapidity range  $p_T = 20$ –1350 GeV/c and  $|\eta| < 4.4$ , respectively. The measurements were based on data sample with integrated luminosity of  $37 \text{ pb}^{-1}$ .

All these data confirm the energy independence of the scaling function. The function is described by a power law,  $\psi(z) \sim z^{-\beta}$ , at high  $z$  with a constant value of the slope parameter  $\beta$ . The power behavior of  $\psi(z)$  is observed over a wide range of  $z$ . As seen from Fig. 10, *b*, the maximal value of  $z \simeq 7 \cdot 10^3$  at  $|\eta| < 0.5$  was reached at the Tevatron energy  $\sqrt{s} = 1960$  GeV. This value is not exceeded by the LHC data at midrapidity yet. The CMS and ATLAS data on jet production demonstrate angular independence of  $\psi(z)$  as well. The

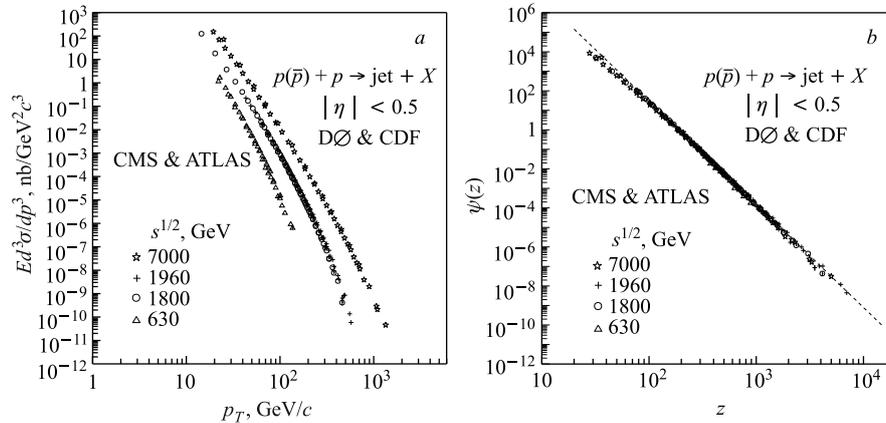


Fig. 10. Inclusive spectra of jet production in  $pp$  and  $p\bar{p}$  collisions at  $\sqrt{s} = 7000$  GeV and  $\sqrt{s} = 630, 1800, 1960$  GeV and central pseudorapidity range measured by the CMS [21], ATLAS [22] and the DØ [25, 26], CDF [27, 28] Collaborations in (a)  $p_T$  and (b)  $z$  presentation

result is in good agreement with the angular independence of  $z$  presentation of jet spectra measured by the CDF and DØ Collaborations at lower energies [35]. The comparison of the LHC and Tevatron data indicates universality of the scaling function in  $pp$  and  $p\bar{p}$  collisions. It gives new confirmation of the self-similarity of jet production in  $pp$  and  $p\bar{p}$  collisions at small scales.

## 5. QCD TEST OF $z$ SCALING

The  $z$  scaling is a phenomenological method of analysis of spectra of inclusive particles produced in the collisions of extended objects like hadrons and nuclei. It is based on the principles of locality and self-similarity of interactions of the hadron constituents at different scales. Validity of the scaling was obtained from many analyses of numerous experimental data on differential cross sections of the inclusive reactions. In that view one of the interesting problems is QCD test of the  $z$  scaling. The theory of strong interactions, QCD, is based on fundamental principles and was experimentally verified in many situations. Relation between QCD and  $z$  scaling could give more deep understanding of physical phenomena especially those which require nonperturbative approach or phenomenological ingredients for their description.

In this part we show  $z$  presentation of the inclusive cross sections of jet production in  $pp$  and  $p\bar{p}$  collisions in the next-to-leading order (NLO) QCD approximation and compare it with the scaling function  $\psi(z)$  obtained from analysis of experimental data. We used the code from Ellis–Kunszt–Soper [36] for calculations of the jet transverse momentum spectra. The calculations by EKS group are based on the matrix elements published in [37,38]. For the parton distribution functions (PDF) we use the CTEQ6M [39] and MRST01 [40] which incorporate a large amount of experimental information evaluated at the NLO level. The choice of the factorization ( $\mu_F$ ) and renormalization ( $\mu_R$ ) scales influences the NLO jet cross sections. The typical value of these parameters varies in the range  $(p_T/2, 2p_T)$  in standard calculations. In our calculations, the renormalization and factorization scales are taken to be  $\mu_R = \mu_F = p_T/2$ . The quantity  $p_T$  refers to the transverse momentum of the jet in jet-level expressions. The NLO results depend on the jet-cone radius  $R = [(\Delta\eta)^2 + (\Delta\phi)^2]^{1/2}$  in the pseudorapidity and azimuth  $\{\eta, \phi\}$  space and the jet-separation parameter  $R_{\text{sep}}$ . Both parameters are connected by the relation  $R_{\text{sep}} = 2R$ , and the «optimal value» of the cone radius  $R = 0.7$  is taken.

The inclusive differential cross section for hadron production with the transverse momentum  $p_T$  and the pseudorapidity  $\eta$  is written as follows:

$$E_C \frac{d^3\sigma}{dp_C^3} = \sum_{abc} \int dx_a dx_b \frac{dz}{z} f_{a/A}(x_a, \mu_F) \times \\ \times f_{b/B}(x_b, \mu_F) \epsilon_c \frac{d^3\sigma}{dk_c^3}(p_c/z\sqrt{s}, \mu_R) D_{C/c}(z, \mu_H). \quad (6)$$

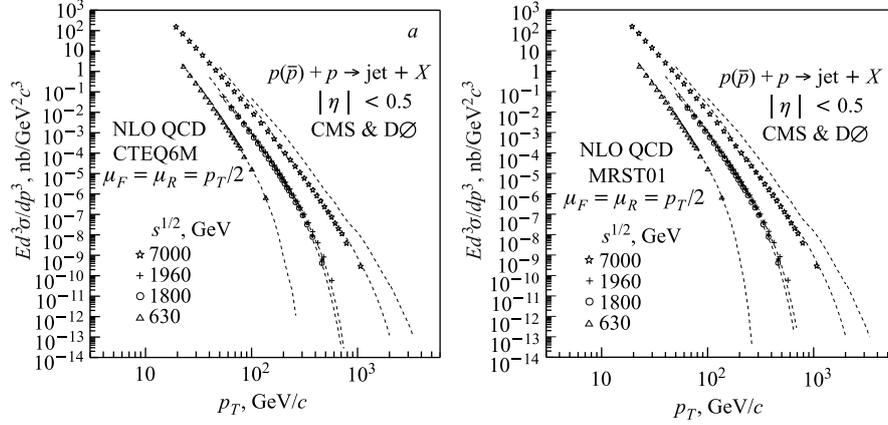


Fig. 11. The dependence of the inclusive cross sections of jet production in  $p\bar{p}$  and  $pp$  collisions on the transverse momentum  $p_T$ . Dashed lines are calculated results in NLO QCD with (a) CTEQ6M [39] and (b) MRST01 [40] PDFs at  $\sqrt{s} = 630, 1800, 1960, 7000$ , and  $14000$  GeV. Points are the experimental data obtained by the DØ [25,26] and CMS [21] Collaborations

The sum is taken over the various flavors of partons that can participate in the hard scattering process. The functions  $f_{a/A}(x_a, \mu_F)$  and  $f_{b/B}(x_b, \mu_F)$  describe the distributions of a parton  $a$  and  $b$  in the hadron  $A$  and  $B$  on the momentum fraction  $x_a$  and  $x_b$  at the factorization scale  $\mu_F$ , respectively. The fragmentation of a parton  $c$  into the hadron  $C$  is described by the function  $D_c^C(z, \mu_H)$ . The parton momentum fraction carried by the hadron  $C$  at the fragmentation scale  $\mu_H$  is equal to  $z$ . Inclusive cross section  $\epsilon_c d^3\sigma/dk_c^3$  of the parton subprocess  $a + b \rightarrow c + X$  is calculated perturbatively in the NLO QCD. In our case we have  $c \equiv \text{jet}$ . It is assumed that fragmentation does not destroy the transverse distribution of jet and  $z = 1$ . The  $\overline{\text{MS}}$  scheme is used to subtract final state collinear singularities. The strong coupling  $\alpha_S(\mu_R)$  is defined in the  $\overline{\text{MS}}$  renormalization scheme at the scale  $\mu_R$ .

The invariant cross sections of jet production as a function of the collision energy  $\sqrt{s}$  and transverse momentum  $p_T$  calculated in the NLO QCD are shown in Fig. 11. The calculations depicted by the dashed lines were performed with the parton distribution functions CTEQ6M [39] and MRST01 [40]. The renormalization, factorization and fragmentation scales were set to be equal each other  $\mu_R = \mu_F = \zeta p_T$ , where  $\zeta$  is the scale factor usually varied in the range  $\zeta = 0.5, 2$ . The spectra were calculated over a wide range of the transverse momentum  $p_T = 20\text{--}3000$  GeV/ $c$  and the energy  $\sqrt{s} = 630\text{--}14000$  GeV at  $\theta \simeq 90^\circ$ . As seen from Fig. 11, the strong dependence of the spectra on the collision energy

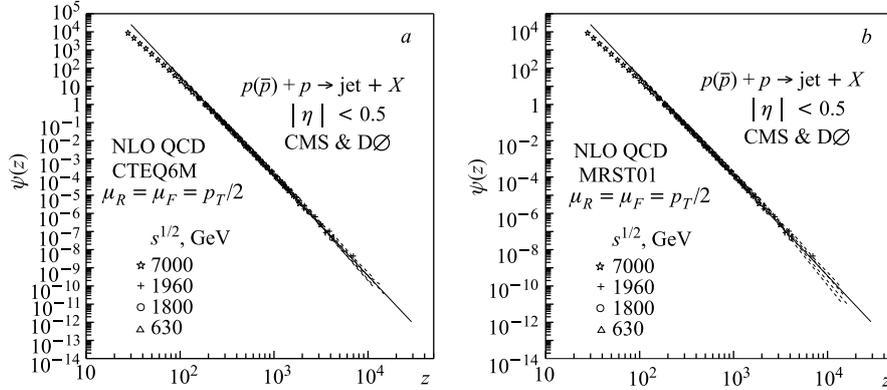


Fig. 12. The scaling function  $\psi(z)$  of jet production in  $pp$  and  $p\bar{p}$  collisions versus  $z$ . Dashed lines are calculated results in the NLO QCD with (a) CTEQ6M [39] and (b) MRST01 [40] PDFs at  $\sqrt{s} = 630, 1800, 1960, 7000$ , and  $14000$  GeV. Points ( $\Delta, \circ, +, \star$ ) are obtained by using the experimental data [21, 25, 26], and  $[]$  at  $\sqrt{s} = 630, 1800, 1960$ , and  $7000$ , respectively

increases with  $p_T$ . Experimental data on cross sections at  $\sqrt{s} = 630, 1800, 1960$ , and  $7000$  GeV obtained by the  $D\phi$  [25, 26], and CMS [21] Collaborations are shown by the symbols ( $\Delta, \circ, +, \star$ ).

The same data are plotted in the dependence on the variable  $z$  in Figs. 12, a and 12, b. The solid line represents the asymptotic behavior of the scaling function  $\psi(z)$  obtained from analysis of experimental data. The value of the slope parameter  $\beta$  corresponds to  $5.48 \pm 0.02$ . The NLO QCD calculation results of the jet spectra in  $z$  presentation are shown by the dashed lines.

Note, that a good agreement between the experimental data and the corresponding NLO QCD calculations is observed. One can see, however, that the NLO QCD predictions demonstrate some deviation from the asymptotic behavior of  $\psi(z)$  predicted by the  $z$  scaling as the collision energy and transverse momentum increase.

The deviation is clearly visible in the region of high- $p_T$  where experimental measurements of inclusive jet transverse momentum spectra are not performed yet. Similar comparison of the NLO QCD calculations with the ATLAS and  $D\phi$  data is shown in Figs. 13 and 14.

Some factors can modify the asymptotic behavior of  $\psi(z)$ . They are quark and gluon distribution functions. The fragmentation of a quark and gluon into jet is rather complex process described by a fragmentation function. We consider that it should be taken into account in QCD calculations as well. We assume that large uncertainties of the gluon distribution function could be essentially

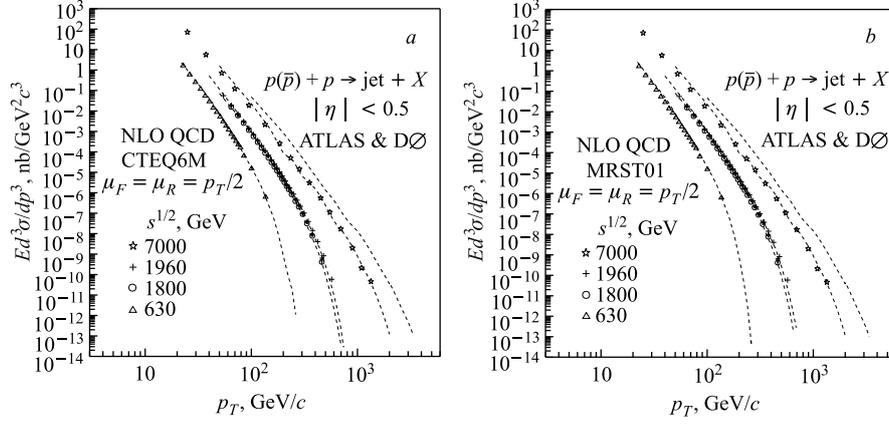


Fig. 13. The dependence of the inclusive cross sections of jet production in  $pp$  and  $p\bar{p}$  collisions on the transverse momentum  $p_T$ . Dashed lines are calculated results in NLO QCD with (a) CTEQ6M [39] and (b) MRST01 [40] PDFs at  $\sqrt{s} = 630, 1800, 1960, 7000$  and  $14000$  GeV. Points represent the experimental data obtained by the DØ [25, 26] and ATLAS [22] Collaborations

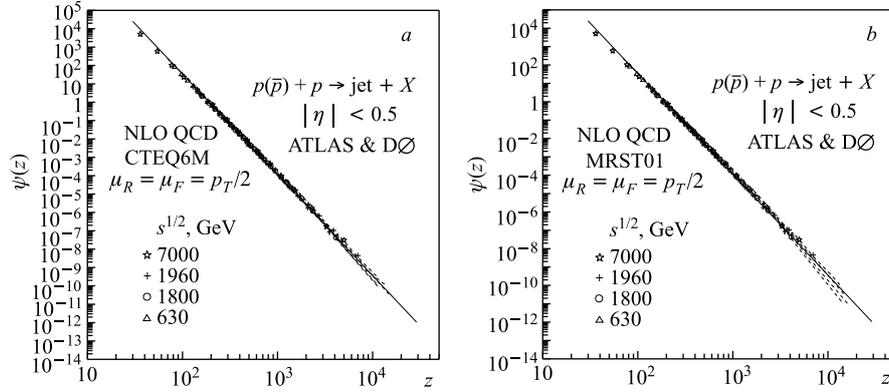


Fig. 14. The scaling function  $\psi(z)$  of jet production in  $pp$  and  $p\bar{p}$  collisions versus  $z$ . Dashed lines are calculated results in the NLO QCD with (a) CTEQ6M [39] and (b) MRST01 [40] PDFs at  $\sqrt{s} = 630, 1800, 1960, 7000,$  and  $14000$  GeV. Points ( $\Delta, \circ, +, \star$ ) are obtained by using the experimental data [25], [26] and [22] at  $\sqrt{s} = 630, 1800, 1960,$  and  $7000,$  respectively

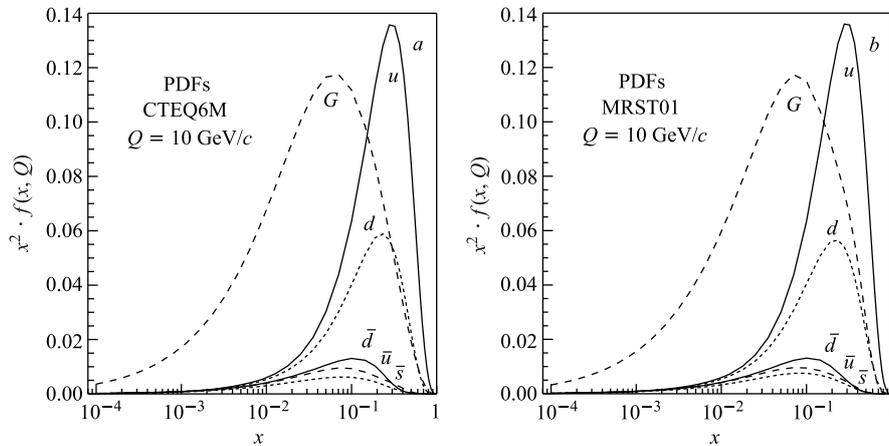


Fig. 15. The CTEQ6M [39] (a) and MRST01 [40] (b) quark ( $u, d, \bar{u}, \bar{d}, \bar{s}$ ) and gluon ( $G$ ) distribution functions versus  $x$  at  $Q = 10 \text{ GeV}/c$

restricted by the power behavior of  $\psi(z)$  established over a wide range of  $\sqrt{s}$  and high- $p_T$ . It could give us the new additional constraint on PDFs and FFs which are phenomenological quantities weakly controlled by the perturbative QCD.

Here we illustrate the dependence of the quark and gluon distribution functions on the fraction  $x$  and momentum  $Q$ . Figure 15 shows the distribution functions CTEQ6M (a) and MRST01 (b) over a range  $x = 10^{-4} - 1$  at  $Q = 10 \text{ GeV}/c$ . As seen from Fig. 15 the shape of the corresponding distributions is very similar. Some difference between CTEQ6M and MRST01 gluon distributions is observed in the range  $x = 0.2 - 0.6$ .

Based on the obtained results we conclude that self-similar features of jet production dictated by the  $z$  scaling give strong restriction on the asymptotic behavior of the scaling function  $\psi(z)$  for  $\bar{p}p$  and  $pp$  collisions. The behavior of  $\psi(z)$  is reasonably reproduced at Tevatron and LHC energies by the NLO QCD evolution of the cross sections with the phenomenological parton distribution functions used in the present analysis.

## 6. $z$ - $p_T$ PLOT

Systematic analysis of experimental data on inclusive jet production obtained at RHIC, Tevatron and LHC showed that data on jet cross sections are described by the power function,  $\psi(z) \sim z^{-\beta}$ , over a wide kinematic range (up to the highest collision energy  $\sqrt{s} = 7000 \text{ GeV}$  and transverse momentum  $p_T \simeq 1500 \text{ GeV}/c$ ). We interpreted the results as a strong confirmation that self-similarity is a general

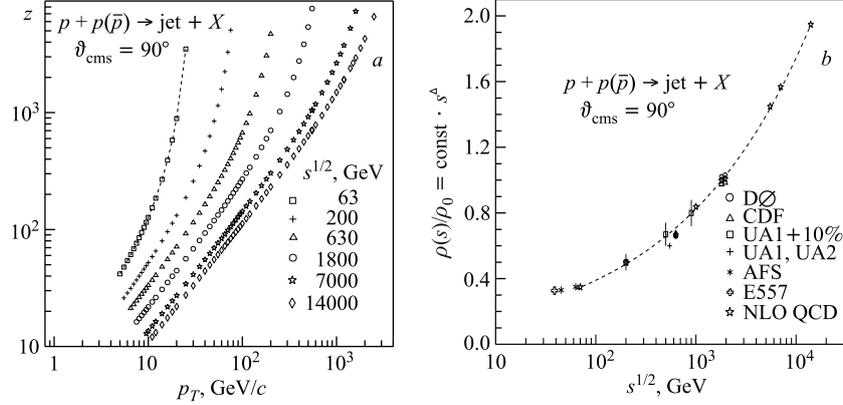


Fig. 16. a)  $z - p_T$  plot for jet production in  $pp$  and  $p\bar{p}$  collisions at  $\sqrt{s} = 63 - 14000$  GeV and angle  $\theta_{\text{cms}} = 90^\circ$ . b) The energy dependence of the normalized jet multiplicity density used in the analysis of  $pp$  and  $p\bar{p}$  collisions

property of hadron structure, interactions of their constituents and mechanism of jet formation. More precise study of properties of the  $z$  scaling is an interesting task in the experimentally accessible region. Search for new phenomena in processes with jet production requires a knowledge of the kinematic conditions which are preferable for such investigations. The dependence of the variable  $z$  on the transverse momentum  $p_T$  (the  $z - p_T$  plot) shown in Fig. 16, a allows us to select such region.

As seen from Figs. 2, b, 3, 4, b, 5, b, 7, 9, and 10, b the power law is valid up to  $z \simeq 10^4$ . It means that new effects could be expected for  $p_T > 60, 500, 1000$ , and  $2000$  GeV/c at  $\sqrt{s} = 200, 1800, 7000$ , and  $14000$  GeV, respectively. The scaled multiplicity density taken in the form  $\rho(s)/\rho_0 = \text{const} \cdot s^\Delta$  [24] is shown in Fig. 16, b. The estimated values of  $\rho(s)/\rho_0$  at  $\sqrt{s} = 7000$  GeV and  $14000$  GeV are found to be 1.56 and 1.95, respectively.

## CONCLUSIONS

Experimental data on inclusive spectra of jet production in  $pp$  and  $p\bar{p}$  collisions obtained at RHIC, Tevatron, and LHC were analyzed in the framework of  $z$  scaling. We found confirmation of the energy and angular independence of the  $z$  presentation of these data. The power behavior of the scaling function,  $\psi(z) \sim z^{-\beta}$ , is observed up to the highest jet transverse momentum  $p_T \simeq 1500$  GeV/c. The obtained results show that properties of jet production reflect the self-similarity, locality, and fractality of hadron interactions at a constituent level.

A QCD test of the  $z$  scaling of jet production in  $p\bar{p}$  and  $pp$  collisions was performed. The inclusive cross sections were calculated in the NLO QCD with CTEQ6M and MRST01 parton distribution functions and compared in the  $z$  presentation with the scaling function obtained from analysis of experimental data. It was shown that self-similar features of particle production dictated by the  $z$  scaling give strong restriction on the asymptotic behavior of the scaling function and inclusive spectra in high- $p_T$  region. The result is important for further tests of the QCD and more precise specification of the phenomenological ingredients (such as PDFs and FFs) of the theory. This can be used as an additional constraint on the gluon distribution function in the global QCD analysis of experimental data.

More precise study of the  $z$  scaling in jet production will be possible in  $pp$  and  $p\bar{p}$  collisions at RHIC, Tevatron, and LHC. A violation of the scaling is suggested to be considered as a signature of new phenomena which may be expected in the region  $z > 10^4$ .

**Acknowledgements.** The investigations have been partially supported by the IRP AVOZ10480505 and by the Ministry of Education of the Czech Republic, grants LA08002, LA08015.

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Received on February 28, 2012.

Редактор *Э. В. Ивашкевич*

Подписано в печать 26.06.2012.

Формат 60 × 90/16. Бумага офсетная. Печать офсетная.

Усл. печ. л. 1,44. Уч.-изд. л. 1,93. Тираж 375 экз. Заказ № 57687.

Издательский отдел Объединенного института ядерных исследований  
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