

E1-2022-21

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ON THE POSSIBILITY OF PION CONDENSATION  
FROM  $\pi^-$ -C INTERACTIONS AT 40 GeV/ $c$

Submitted to “Physics of Atomic Nuclei”

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E1-2022-21

О возможности конденсации пионов из  $\pi^-$ -С-взаимодействий при 40 ГэВ/с

В этой статье мы рассматриваем возможное наблюдение конденсации  $\pi^-$ -мезонов на основе зависимости температуры ( $T$ ) от массы мишени ( $m_t$ ) и химического потенциала ( $\mu_{\text{sec}}(\mu_B, T)$ ) из взаимодействий  $\pi^- + \text{C}$  при 40 ГэВ/с.

Работа выполнена в Лаборатории физики высоких энергий им. В. И. Векслера и А. М. Балдина ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 2022

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On the Possibility of Pion Condensation from  $\pi^-$ -C Interactions at 40 GeV/c

In this paper we consider an opportunity of observing the  $\pi^-$ -meson condensation on the basis of the dependence of temperature ( $T$ ) on the target mass ( $m_t$ ) and chemical potential ( $\mu_{\text{sec}}(\mu_B, T)$ ) obtained for  $\pi^- + \text{C}$  interactions at 40 GeV/c.

The investigation has been performed at the Veksler and Balдин Laboratory of High Energy Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 2022

## INTRODUCTION

Condensation is the change of the state of matter from the gas state into the liquid phase. Being a quasi-statistical process, the condensation occurs under equilibrium conditions of coexisting phases and is the phase transition of the first order. The equilibrium state of any system is achieved at minimum values of the potential energy. The condensation is the phenomenon related with bosons, the particles with integer spins. It is well known that behaviour of elementary particles essentially changes at phase transitions.

The Bose condensate has a remarkable property of resisting the removal of individual quanta from it, and bosons also seek to drag as many other bosons as possible into the state which they occupy. It is the property that leads to the superconducting behaviour of the system.

The condensation of the pion field was proposed in 1971, first by A. B. Migdal [1]. He saw the incidence of a condensed pion field in the nuclear matter as instability of the Bose field. Since this pioneering work, different theoretical studies have been proposed to explain this instability. But practically without experimental result.

This paper is devoted to the experimental study of the condensation of  $\pi^-$  mesons from the following reaction:

$$\pi^- + C \rightarrow \pi^- + X. \quad (1)$$

### 1. EXPERIMENTAL METHOD

The experimental material was obtained using the Dubna 2-meter propane ( $C_3H_8$ ) bubble chamber exposed to  $\pi^-$  mesons with a momentum of 40 GeV/c from the Serpukhov accelerator. The advantage of the bubble chamber experiment in this paper is that the distributions are obtained under the condition of  $4\pi$  geometry of secondary  $\pi^-$  mesons.

The average error of the momentum measurements is  $\sim 12\%$ , and the average error of the angular measurements is  $\sim 0.6\%$ .

The average boundary momentum from which  $\pi^-$  mesons are detected in the propane bubble chamber is  $\sim 70$  MeV/c. So,  $\pi^-$  mesons with a momentum from  $\sim 0.070$  to 40 GeV/c are used in our distributions. In this analysis, 30145  $\pi^-$  mesons produced in reaction (1) have been used.

## 2. VARIABLES USED IN THE ANALYSIS

**2.1. The Cumulative Number  $n_c$  and Target Mass.** The relativistic invariant variable  $n_c$  called the cumulative number is determined by the following formula:

$$n_c = \frac{P_a P_c}{P_a P_b} = \frac{E_c - \beta_a p_c^{\parallel}}{m_p}, \quad (2)$$

where  $P_a$ ,  $P_b$  and  $P_c$  are the four-dimensional momenta of the projectile, target and the secondary particles under consideration;  $E_c$  and  $p_c^{\parallel}$  are the energy and the longitudinal momentum of the secondary particles;  $\beta_a$  is the velocity of the incident particle;  $m_p$  is the proton mass. From formula (2) we have determined the value of the target mass  $m_t$  by the following formula:

$$m_t = m_p n_c. \quad (3)$$

The target mass  $m_t$  distribution of  $\pi^-$  mesons from  $\pi^-C$  interactions is presented in Fig. 1.

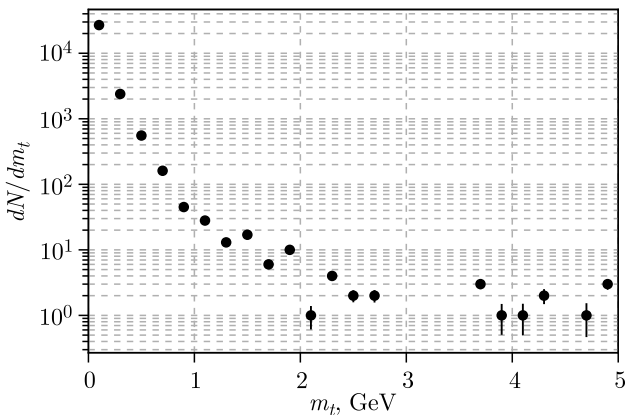


Fig. 1. Target mass  $m_t$  distribution of  $\pi^-$  mesons from  $\pi^-C$  interactions

**2.2. The Effective Temperature  $T$ .** In our previous paper [2], the transverse mass distributions ( $dN/E_t dE_t = ae^{-bE_t}$ , where  $E_t = \sqrt{p_t^2 + m^2}$ ) are obtained at different intervals of the variable  $n_c$  and these distributions are described by exponential functions. The effective temperature  $T$  is determined as an inverse slope parameter:

$$T = \frac{1}{b}. \quad (4)$$

**2.3. The Baryon Chemical Potential  $\mu_B$ .** The baryon chemical potential  $\mu_B$  is determined by the following formula [3, 5]:

$$\mu_B = M_N - B \approx 0.923 \text{ GeV},$$

where  $M_N$  is the nucleon mass and  $B$  the binding energy per nucleon of nuclear matter.

In the case of the baryonic chemical potential  $\mu_B$ , the cumulative number  $n_c$  is redefined in the form

$$n'_c = \frac{E_c - \beta_a p_c^{\parallel}}{\mu_B} = \frac{E_c - \beta_a p_c^{\parallel}}{0.923}. \quad (5)$$

Similarly to the target mass  $m_t$ , the baryonic chemical potential  $\mu_B$  in every  $n'_c$  interval is determined by the following formula:

$$\mu_B(n'_c) = n'_c \cdot 0.923 \text{ GeV}. \quad (6)$$

**2.4. The Chemical Potential  $\mu_{\text{sec}}(\mu_B, T)$ .** In paper [4], according to the chiral perturbation theory the temperature dependence of the second-order phase transition has been obtained:

$$\mu_{\text{sec}}(T) = \frac{m_\pi}{2} + \frac{1}{32 f_\pi^2} (N_f - 1) \sqrt{\frac{m_\pi^3 T^3}{2\pi^3}} \zeta\left(\frac{3}{2}\right). \quad (7)$$

To take into account in this formula the dependence on the baryon chemical potential  $\mu_B$ , we changed the meson mass  $m_\pi$  to the baryon chemical potential  $\mu_B(n'_c)$  determined by formula (6):

$$\mu_{\text{sec}}(\mu_B(n'_c), T) = \mu_B(n'_c) + \frac{1}{32 f_\pi^2} (N_f - 1) \sqrt{\frac{\mu_B^3(n'_c) T^3}{2\pi^3}} \zeta\left(\frac{3}{2}\right), \quad (8)$$

where  $\zeta(3/2) = 2.6123$  is the Riemann zeta function,  $f_\pi = 0.184 \text{ GeV}^{-1}$  is the pion decay constant, and  $N_f$  is the flavor number.

We note that the chemical potential  $\mu_{\text{sec}}(\mu_B, T)$  calculated by formula (8) is in good agreement in the region  $m_t \leq 0.303 \text{ GeV}$ , but at large values of the variable  $m_t$  we have observed the disagreement (see the table).

Figure 2 shows the dependence of the effective temperature  $T$  on the target mass  $m_t$ . The target mass value which is required to produce the secondary particle under consideration is determined by formula (3). This dependence shows that with increasing of the target mass  $m_t$  the effective temperature  $T$  increases and then from the value  $m_t \simeq 0.070 \text{ GeV} \simeq m_\pi/2$  the parameter  $T$  remains practically constant on the level  $T_c \simeq 0.233 \text{ GeV}$  and after that from the point  $m_t \geq 0.303 \text{ GeV}$  increases again.

Figure 3 shows the dependence of average energies  $\langle E_{\pi^-} \rangle$  of  $\pi^-$  mesons in every  $n_c$  interval produced in  $\pi^- C$  interactions at  $40 \text{ GeV}/c$  on the target mass  $m_t$ . With increasing of the variable  $m_t$  the average energies  $\langle E_{\pi^-} \rangle$  decrease and reach the minimum value at  $m_t \simeq 0.303 \text{ GeV}$  and then essentially increase.

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<sup>1</sup> Steven Weinberg. The Quantum Theory of Fields. Volume II.

### Average values of the different characteristics in every $n_c$ interval

No.	$N$	$\langle n_c \rangle$	$\langle m_t \rangle$ , GeV	$\mu_B$ , GeV	$\mu_{\text{sec}}(\mu_B(n'_c), T)$ , GeV	$T$ , GeV
1	4864	0.0053	0.0051	0.0050	0.0050	$0.097 \pm 0.008$
2	4488	0.0147	0.0138	0.0135	0.0136	$0.135 \pm 0.002$
3	3297	0.0247	0.0232	0.0227	0.0228	$0.157 \pm 0.002$
4	2482	0.0347	0.0326	0.0319	0.0320	$0.165 \pm 0.003$
5	1844	0.0448	0.0421	0.0412	0.0414	$0.176 \pm 0.005$
6	1477	0.0548	0.0514	0.0504	0.0507	$0.181 \pm 0.006$
7	1248	0.0651	0.0610	0.0598	0.0602	$0.199 \pm 0.012$
<b>8</b>	<b>1102</b>	<b>0.0749</b>	<b>0.0703</b>	<b>0.0688</b>	<b>0.0694</b>	<b><math>0.240 \pm 0.005</math></b>
9	837	0.0850	0.0797	0.0780	0.0788	$0.231 \pm 0.007$
10	766	0.0947	0.0888	0.0871	0.0880	$0.231 \pm 0.007$
11	2642	0.1225	0.1149	0.1127	0.1140	$0.229 \pm 0.007$
12	1555	0.1720	0.1641	0.1584	0.1606	$0.236 \pm 0.009$
13	1005	0.2234	0.2096	0.2058	0.2091	$0.237 \pm 0.012$
14	699	0.2725	0.2555	0.2511	0.2553	$0.227 \pm 0.014$
<b>15</b>	<b>501</b>	<b>0.3234</b>	<b>0.3033</b>	<b>0.2977</b>	<b>0.3033</b>	<b><math>0.233 \pm 0.017</math></b>
16	333	0.3737	0.3505	0.3437	0.3520	$0.262 \pm 0.021$
17	441	0.4435	0.4159	0.4081	0.4185	$0.257 \pm 0.022$
18	215	0.5461	0.5121	0.5017	0.5174	$0.275 \pm 0.034$
19	118	0.6493	0.6089	0.5975	0.6194	$0.288 \pm 0.040$
20	73	0.7504	0.7038	0.6920	0.7235	$0.317 \pm 0.047$
21	58	0.8744	0.8200	0.8075	0.8757	$0.455 \pm 0.081$
22	52	1.2020	1.1272	1.1060	1.3022	$0.672 \pm 0.151$
23	48	2.3570	2.2104	2.1090	3.4719	$1.283 \pm 0.330$

Figure 3 shows that the part with increasing  $m_t$  and energy begins at  $m_t = 0.303 \text{ GeV} \simeq 2.15 m_\pi$ . We would like to note that, in the case of the secondary protons from this interaction, the increase in the average value of proton energy begins at  $m_t = m_p = 0.938 \text{ GeV}$  (at  $n_c = 1$ ). So, we have observed the experimental fact that the cumulative production of  $\pi^-$  mesons and protons begins at different values of the target mass. We also note that as in the case of Fig. 2 we have observed the tendency of the breaking points at  $m_t \simeq 0.070 \text{ GeV} \simeq m_\pi/2$  (the first one) and at  $m_t = 0.303 \text{ GeV}$  (the second one).

Figures 2 and 3 also show the dependences of the effective temperature  $T$  and of the average energies  $\langle E_{\pi^-} \rangle$  on the chemical potential  $\mu_{\text{sec}}$ , respectively.

From Fig. 3 we see that the point at minimum  $m_t = 0.303 \text{ GeV}$  separates the decreasing and increasing parts of the average energies  $\langle E_\pi \rangle$  of  $\pi^-$  mesons.

The first breaking point at  $m_t \simeq 0.070 \text{ GeV} \simeq m_\pi/2$  separates the highly excited hadronic state from the equilibrium state (mixed phase) and the second

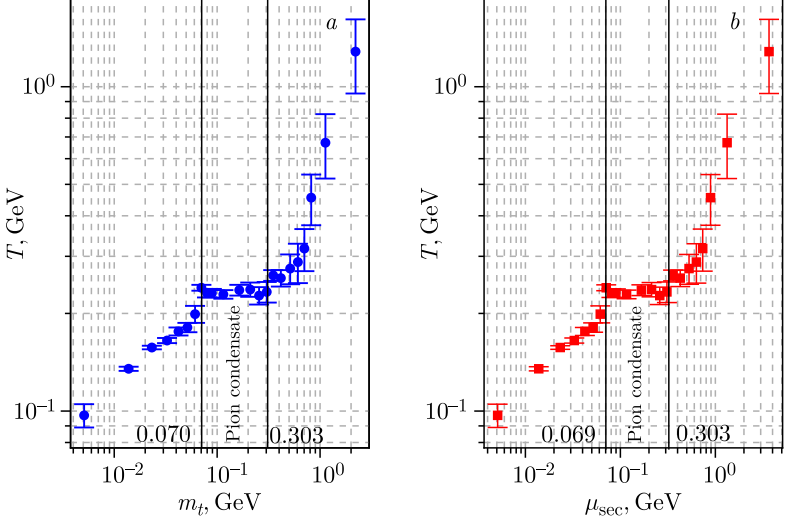


Fig. 2. Dependences of the effective temperature  $T$  on (a) the target mass  $m_t$  and (b) the chemical potential  $\mu_{\text{sec}}$

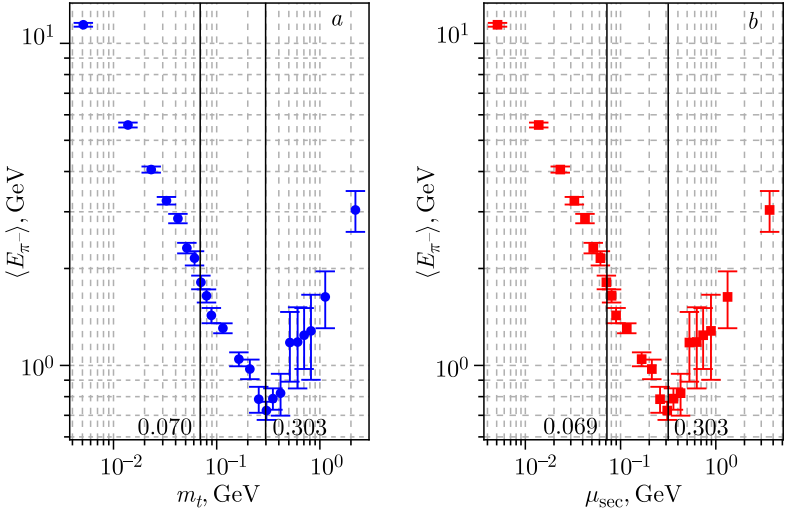


Fig. 3. Dependences of the average energies  $\langle E_{\pi^-} \rangle$  on (a) the target mass  $m_t$  and (b) the chemical potential  $\mu_{\text{sec}}$

breaking point at  $m_t = 0.303$  GeV separates the equilibrium state from the QGP state.

We would like to note that these two dependences (Figs.2 and 3) complement each other; for example, Fig. 2 gives the first breaking point more clearly at  $m_t \simeq 0.070$  GeV  $\simeq m_\pi/2$ , but Fig. 3 gives the second breaking point exactly at  $m_t = 0.303$  GeV  $\simeq 2.15 m_\pi$ .

Note that the region  $0.070 \text{ GeV} < m_t \leq 0.303 \text{ GeV}$  with practically constant temperature  $T \simeq 0.233 \text{ GeV}$  corresponds to the equilibrium state (the mixed phase).

## CONCLUSIONS

The obtained experimental results of the onset of the phase transition at  $m_t = 0.070 \text{ GeV} = m_\pi/2$  and its coincidence with the theoretical prediction of the onset of the pion condensation at  $\mu_B = m_\pi/2$ , and the exits to plateaus of temperature dependences from this point in the both cases have indicated possible observation of the condensation of  $\pi^-$  mesons from  $\pi^-$ C interactions at  $40 \text{ GeV}/c$ .

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Received on April 26, 2022.



Редактор *Е. И. Кравченко*

Подписано в печать 2.06.2022.

Формат 60 × 90/16. Бумага офсетная. Печать офсетная.

Усл. печ. л. 0,5. Уч.-изд. л. 0,43. Тираж 220 экз. Заказ № 60441.

Издательский отдел Объединенного института ядерных исследований  
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