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# MANIFESTATIONS OF THE INTERMEDIATE STRUCTURE IN THE $\gamma$ DECAY OF NON-ANALOG RESONANCES

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Проявления промежуточной структуры при  $\gamma$ -распаде неаналоговых резонансов

Анализируются угловые распределения  $\gamma$ -излучения при распаде неаналоговых резонансов в реакциях  ${}^{58,60,62}$ Ni $(p,\gamma)^{59,61,63}$ Cu. Проанализированы коэффициенты разложения угловых распределений по полиномам Лежандра, корреляции ширин и функции распределения смеси мультипольностей  $\delta$  для M1 + E2  $\gamma$ -распадов неаналоговых резонансов. Получены однозначные свидетельства нестатистического характера исследованных резонансов. Обсуждается возможная причина проявления нестатистических эффектов в  $(p, \gamma)$ -реакциях.

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Manifestations of the Intermediate Structure in the  $\gamma$  Decay of Non-Analog Resonances

The angular distributions of  $\gamma$  radiation during the decay of non-analog resonances in reactions  $^{58,60,62}\mathrm{Ni}(p,\gamma)^{59,61,63}\mathrm{Cu}$  are analyzed. The coefficients of decomposition of angular distributions by Legendre polynomials, the correlation of widths and the distribution function of a mixture of multipoles  $\delta$  for M1+E2  $\gamma$  decays of non-analog resonances are analyzed. Unambiguous evidence of the non-statistical nature of the studied resonances has been obtained. A possible reason for the manifestation of non-statistical effects in  $(p,\gamma)$  reactions is discussed.

The investigation has been performed at the Flerov Laboratory of Nuclear Reactions, JINR.

#### **INTRODUCTION**

According to [1], an intermediate structure is a violation of the statistical model in a localized energy region. In the statistical model, the wave function of the state  $\Psi_{St}$  is written as follows:

$$\Psi_{\rm St} = \sum_{k}^{n} C_k \varphi_k, \quad \sum_{k}^{n} |C_k|^2 = 1, \tag{1}$$

where  $\varphi_k$  is the wave function of a "simple" configuration,  $C_k$  are random numbers, and  $n \gg 1$ . When non-statistically considered [2], one or a small number of simple configurations  $\varphi_0$  are dominated in the wave function of the non-statistical state  $\Psi_{\rm NSt}$ :

$$\Psi_{\rm NSt} = C_0 \,\varphi_0 + \sum_k^n C_k \,\varphi_k, \quad |C_0| \gg |C_k|, \quad |C_0|^2 + \sum_k^n |C_k|^2 = 1.$$
(2)

The study of the decay characteristics of nuclear states and resonances at excitation energies exceeding 3-5 MeV in medium and heavy nuclei plays an important role in understanding the structure of atomic nuclei [2-9]. With an increase in the excitation energy, the density of levels in the nucleus increases rapidly and the wave function of such excited states can have a rather complex structure, since even a small residual interaction can lead to mixing of closely spaced states. Therefore, as a rule, it is assumed that the structure of states at significant excitation energies of the nucleus is quite complex and the coefficients of decomposition (1) of the wave function in simple configurations obey statistical patterns. The characteristics of various types of atomic nucleus decay are guite simply calculated in such statistical models. In particular, the gamma-decay width distributions obey the Porter-Thomas distribution [4, 5, 10], the beta-decay strength function  $S_{\beta}(E)$  is proportional to the density of the daughter nucleus levels [11], there are no correlations of partial decay widths along various channels [2, 4], and the amplitude ratios along various spin channels obey the Cauchy distribution [2, 4, 5]. Therefore, the question naturally arises of identifying and studying non-statistical effects during the excitation and decay of nuclear states and resonances, determining the degree of mixing of the "simple" component responsible for non-statistical effects with the levels of the

compound nuclei and interpreting the structure of the studied non-statistical states and resonances at the microscopic level [2, 4, 5, 11].

Non-statistical effects are closely related to the symmetry of interaction in atomic nuclei [2]. A number of the first and most distinct manifestations of non-statistical effects in the decay of highly excited states and resonances in atomic nuclei were found for isobar analog resonances (IAR). The manifestation of the discovered non-statistical properties of the IAR was due to the isospin symmetry of the nuclear forces [9]. Since the value of the isospin quantum number corresponding to this symmetry for the IAR exceeds the isospin value for neighboring levels of the compound nuclei by one, there is no strong mixing of the IAR with nearby states of the compound nuclei. At excitation energies in medium and heavy atomic nuclei exceeding 3-5 MeV, a large number of other (non-analog) states and resonances are observed. There are two possible approaches to the interpretation of their properties: statistical and non-statistical [2, 4, 5]. In the first case, it is assumed that the wave function of states and resonances has a statistical character (1), whereas in the second case it is assumed that the non-statistical component dominates in the wave function (2), and the energy distribution of this non-statistical component manifests itself in the form of a giant resonance type structure (intermediate structure, Fig. 1). In the second case, non-statistical approaches should be applied when interpreting experimental data [2, 4].



Fig. 1. The manifestation of the gross structure, intermediate structure and fine structure in the excitation function of resonances in the  $(p, \gamma)$  reaction [13]

In addition to analog resonances, a large number of non-analog resonances are manifested in the excitation functions of nuclear reactions [3-8]. The intensity of these resonances is often comparable to the intensity of analogues. A direct proof of the non-statistical nature of non-analog resonances would be the discovery of an intermediate structure [9] in the cross sections of nuclear

reactions. The simplest manifestation of the intermediate structure is the presence of a maximum in the distribution of squares of the reduced resonance widths depending on the excitation energy. Attempts to find such maxima in the widths characteristics for non-analog resonances were unsuccessful [12]. Only the fine structure of analog states are clearly visible in the reduced proton widths distribution. The same situation is observed for  $\gamma$  widths. When distributing the strength of a resonance other than analog (for example, the Gamow–Teller resonance), an intermediate structure must also be formed, just as it is formed in the case of analogues. The difference is that this structure should be more extended in energy (Fig. 1) than the structure of analogues, and therefore it is more difficult to detect experimentally [5].

A more effective method for detecting intermediate structures turned out to be the study of the amplitude ratios of the reduced widths for various channels of the resonant nuclear reaction. In the statistical model, the ratios of the reduced amplitudes are described by the Cauchy distribution [2, 4, 5]. The main assumption of the statistical model is the random distribution of the reduced amplitudes. It follows from this that, in general, the sign of the ratio of the given amplitudes also has a random character and both positive and negative values of such a ratio are equally common. In this case, the average value of the ratio of the reduced amplitudes of the reaction channels will have zero value according to the statistical nature of the studied resonances. Using this method, non-statistical effects were found in the study of proton decay of non-analog resonances through various spin channels [4]. Further, similar effects were found both in the analysis of the distributions of the multipole mixture ratios  $\delta$  for  $E2 + M1 \gamma$  decays of non-analog resonances, and in the analysis of the coefficients  $A_2$  of the angular distributions of  $\gamma$  transitions [2, 5, 14–18]. In addition, correlations of the reduced probabilities of B(M1) and  $B(E2) \gamma$  transitions for  $\gamma$  decay of non-analog resonances were observed [2, 16, 18]. From the analysis of the distributions of the values of the multipole mixture ratios  $\delta$  for  $E2 + M1 \gamma$  decays, the fractions of a simple non-statistical component in the wave functions of a number of non-analog resonances were determined [2, 14].

In this paper, the angular distributions of  $\gamma$  radiation in the  ${}^{58,60,62}$ Ni $(p,\gamma)^{59,61,63}$ Cu reactions for the  $\gamma$ -decay of non-analog resonances into the ground and a number of excited states of  ${}^{59,61,63}$ Cu nuclei are investigated. The intermediate structure manifests itself in  $\gamma$  decay only into a number of excited states of the final nucleus. Such a manifestation of the intermediate structure is interpreted as a selective distribution of the strength of non-statistical configurations (for example, satellites of a more complex structure. Also the specific selection rules for  $\gamma$  decay of non-statistical configurations (for example, solve for  $\gamma$  decay of non-statistical configurations (for example, solve for  $\gamma$  decay of non-statistical configurations (for example, connected with the Wigner spin-isospin SU(4) symmetry) may be important. The manifestation of non-statistical effects for non-analog resonances excited in reactions with protons is associated with the presence of neutron excess in the nuclei under consideration. In reactions

with neutrons, such non-statistical effects are not observed in nuclei with an excess of neutrons [19, 20].

# 1. THE METHOD OF THE EXPERIMENT. THE ANGULAR DISTRIBUTION OF GAMMA RADIATION IN $(p, \gamma)$ REACTIONS

The study of the angular distributions of  $\gamma$  transitions from resonances in the reaction  $(p, \gamma)$  on even-even isotopes of nickel (<sup>58,60,62</sup>Ni) was carried out at the Frank Laboratory of Neutron Physics of JINR. The protons were accelerated using an electrostatic generator EG-5 with a maximum proton energy of  $E_n = 5$  MeV. The currents were about 10  $\mu$ A. The search for resonances [2, 5, 14] was carried out using a scintillation  $\gamma$ -ray detector based on a NaI(Tl) crystal with a size of  $100 \times 100$  mm. The NaI(Tl) detector operated in the integral counting mode. The discrimination threshold was chosen in such a way that the recorded  $\gamma$ -radiation energy was approximately half of the excitation energy in a given nucleus  $E^*/2$ . In this case, the registration of some low-energy background lines was excluded and, at the same time, significant information about the decay of the resonance was preserved. If the resonance decay occurred at the lower levels of the nucleus, then the detector registered these  $\gamma$  transitions having energy  $E > E^*/2$ . When the resonance decays into highly excited states of the nucleus,  $\gamma$  transitions from these states to low-lying and ground states of the nucleus were recorded.

To study resonances, it is necessary to choose the appropriate thickness of the target. The energy losses in it should be less than the energy distance between the resonances. The lower limit for the thickness of the target was determined by the energy resolution of the accelerator. For nuclei with  $A \sim 60$  and  $E_p \sim 3$  MeV, when using a proton beam with a resolution of 1–2 keV, targets with a thickness of 10–20  $\mu$ g/cm<sup>2</sup> turned out to be convenient.

After the resonance was detected, the spectra of its  $\gamma$  decay were measured using a Ge(Li) detector with a volume of  $40 \text{ cm}^3$  and an energy resolution of 7 keV for  $\gamma$  radiation with an energy of about 7 MeV. The  $\gamma$ -radiation energies in the range up to 2.6 MeV were graded according to many internal benchmarks. In the region of high excitation energies, the photopics and peaks of the single and double escape of the  $\gamma$  transition with an energy of 6.129 MeV, which occurs in the reaction  ${}^{19}F(p,\alpha){}^{16}O$ , served as convenient reference points. The calibration also used the well-known  $\gamma$  transitions during the decay of IAR. The Ge(Li) detector was positioned at angles of  $0^{\circ}$ ,  $30^{\circ}$ ,  $60^{\circ}$  and  $90^{\circ}$  to the direction of the incident proton beam. When measuring the angular distributions of gamma quanta, the intensity of the proton beam was determined using a current integrator and the integral intensity of the  $\gamma$  quanta recorded by a NaI(Tl) crystal located at 90° to the beam. This made it possible to normalize the intensity of  $\gamma$  radiation detected by the Ge(Li) detector by both the number of protons passing through the target and the number of  $\gamma$  quanta emitted from the target. A semiconductor detector

registering elastically scattered protons was used as an additional monitor. The probability of  $\gamma$  radiation being emitted in a nuclear reaction at an angle  $\theta$  to the direction of the incident particles is represented [21] in the form of a Legendre polynomial decomposition:

$$W(\theta) = \sum_{k} A_k P_k(\cos \theta).$$
(3)

If a zero-spin target is used in the reaction  $(p, \gamma)$  and the excited state is an isolated resonance with a certain spin value, then the  $A_k$  coefficients for direct  $\gamma$  radiation depend only on the spin of the resonance  $J_{\text{res}}$ , the spin of the final state  $J_{\text{lev}}$  and on multipole mixture ratios  $\delta$  [2, 5, 21]. Expressions for the coefficients  $A_k$  for the case of Legendre polynomial expansion of degree no higher than 4 are given below.

> $J_{res} = 1/2$ . All angular distributions are isotropic.  $J_{res} = 3/2$ .  $A_4 = 0$  for all distributions.

$$J_{\text{lev}} = 1/2, \qquad A_2 = \frac{-0.5 - 1.732\delta + 0.5\delta^2}{1 + \delta^2};$$
  

$$J_{\text{lev}} = 3/2, \qquad A_2 = \frac{0.4 - 1.55\delta}{1 + \delta^2};$$
  

$$J_{\text{lev}} = 5/2, \qquad A_2 = \frac{-0.10 + 1.18\delta - 0.357\delta^2}{1 + \delta^2};$$
  

$$J_{\text{lev}} = 7/2, \qquad A_2 = 0.1428.$$

$$J_{
m res}=5/2.$$

$$J_{\text{lev}} = 1/2, \qquad A_2 = 0.571, \qquad A_4 = -0.571;$$
  

$$J_{\text{lev}} = 3/2, \qquad A_2 = \frac{-0.4 - 2.03\delta + 0.204\delta^2}{1 + \delta^2}, \qquad A_4 = \frac{0.65\delta^2}{1 + \delta^2};$$
  

$$J_{\text{lev}} = 5/2, \qquad A_2 = \frac{-0.457 - 2.084\delta - 0.204\delta^2}{1 + \delta^2}, \qquad A_4 = \frac{-0.376\delta^2}{1 + \delta^2};$$
  

$$(4)$$

$$J_{\text{lev}} = 7/2, \qquad A_2 = \frac{-0.143 + 1.485\delta - 0.347\delta^2}{1 + \delta^2}, \qquad A_4 = \frac{0.109\delta^2}{1 + \delta^2}.$$

$$J_{
m res}=7/2.$$

$$\begin{aligned} J_{\text{lev}} &= 3/2, & A_2 = 0.51, & A_4 = -0.367; \\ J_{\text{lev}} &= 5/2, & A_2 = \frac{-0.357 - 2.06\delta + 0.085\delta^2}{1 + \delta^2}, & A_4 = \frac{0.653\delta^2}{1 + \delta^2}; \\ J_{\text{lev}} &= 7/2, & A_2 = \frac{0.476 - 0.825\delta - 0.272\delta^2}{1 + \delta^2}, & A_4 = \frac{-0.49\delta^2}{1 + \delta^2}. \end{aligned}$$

To compare the experimentally obtained angular distributions for  $\gamma$  transitions from a given resonance, the spin of which is not known, with

theoretical distributions under various assumptions about the spin of the resonance, we can construct (Fig. 2) the function:

$$\chi^{2}(\delta) = \sum_{i=1}^{N} \frac{(Y_{i} - W_{i})^{2}}{\sigma_{i}^{2}},$$
(5)

where  $Y_i$  is the relative intensity of the  $\gamma$  transition at an angle  $\theta$ ;  $W_i$  is the theoretical value of the probability of a  $\gamma$  transition at this angle;  $\sigma_i$  is the error in determining the relative intensity.



Fig. 2. Analysis of the angular distribution (*a*) of the  $\gamma$  transition from the non-analog resonance to the level with the excitation energy  $E_{\rm lev} = 475$  keV,  $J_{\rm lev} = 1/2^-$ . The proton energy in the  ${}^{60}{\rm Ni}(p,\gamma){}^{61}{\rm Cu}$  reaction was  $E_p = 2442$  keV, the resonance excitation energy in the  ${}^{61}{\rm Cu}$  nucleus was  $E^* = 7193$  keV. In addition to the main minimum  $\chi^2$  corresponding (*b*) to the spin of the resonance  $J_{\rm res} = 3/2$ , the figure also shows the minima  $\chi^2$  corresponding to the spins of the resonance  $J_{\rm res} = 1/2$  and  $J_{\rm res} = 5/2$ . The analysis allows us to attribute to the resonance the value of the spin  $J_{\rm res} = 3/2$ 

The minimum value of  $\chi^2$  corresponds [22–24] to the best set of values of the resonance spin and a mixture of multipoles  $\delta$ . The angular distribution for a single transition sometimes does not allow us to uniquely determine the spin of the resonance. A joint analysis of the angular distributions of several transitions from a given resonance can significantly facilitate the determination of the resonance spin. The value of  $\delta$  is related to the coefficients  $A_2$  by a quadratic equation, the solution of which yields two values of  $\delta$ . One of them is small and corresponds to the main contribution of M1-multipole to the intensity of the  $\gamma$  transition. The second value is large and refers mainly to the E2 multipole. Meanwhile, it has been experimentally established [5,8] that strong M1 transitions with an admixture of E2, exceeding 10% only in rare cases, are characteristic of the gamma decay of resonances in the nuclei of the fp shell. A large value of  $\delta$  can be discarded [5], since its use leads to unreasonably large values of the reduced probabilities B(E2) for the studied region of nuclei. Thus, from the analysis of the angular distributions of  $\gamma$  radiation, we obtain not only the value of the resonance spin, but also the value of  $\delta$  with a certain sign (Fig. 2). It is not only the sign of  $\delta$  itself that makes sense, but also the relative sign of  $\delta$  for the  $\gamma$  decays of various resonances to different levels of the nuclei under study.

# 2. DISTRIBUTION OF A<sub>2</sub> COEFFICIENTS AND NON-STATISTICAL EFFECTS

The values of the  $A_2$  coefficients and the corresponding distribution functions are shown in Tables 1–3 and Figures 3–12.

Table 1. Angular distribution coefficients  $A_2$  in  ${}^{58}\text{Ni}(p,\gamma){}^{59}\text{Cu}$  reaction,  $E_p = 2120-3460$  keV,  $E_{\text{res}} = 5520-6815$  keV,  $J_{\text{res}}^{\pi} = 3/2^-$  [5,25]

<sup>59</sup> Cu	$E_{ m lev}$ , keV, $J_{ m lev}$ 0, 3/2 <sup>-</sup>	2265, 3/2 <sup></sup>	2324, 3/2 <sup></sup>	491, 1/2 <sup></sup>	912, 5/2 <sup></sup>	1987, 5/2 <sup></sup>	2707, 5/2 <sup></sup>
$E_p$ , keV, $J_{\rm res} 3/2^-$	$A_2$	$A_2$	$A_2$	$A_2$	$A_2$	$A_2$	$A_2$
2161	-0.13(10)						
2210	-0.52(10)						
2338	-0.20(13)		-0.44(11)				-0.55(12)
2512	-0.25(9)		-0.31(40)				
2574	-0.49(6)	-0.40(23)					
2668	-0.41(5)	-0.45(34)		-0.69(11)		0.02(38)	
2704				-0.66(11)			
2721	-0.49(5)			-0.39(20)			
2756	0.05(13)			-0.39(18)			
2831	0.22(12)			-0.64(7)		-0.08(11)	
2869	-0.63(9)			-0.46(7)			
2938	0.10(8)	-0.13(25)		-0.64(10)		-0.41(11)	-0.39(10)
2960	-0.54(10)	-0.43(19)		-0.55(14)			
2978	-0.51(26)	-0.17(19)		-0.60(26)	-0.48(12)		
2999	0.07(8)			-0.73(5)	-0.30(14)		
3051	-0.39(7)					-0.33(15)	
3062		-0.33(20)	-0.24(23)	-0.47(14)	0.03(20)		-0.32(25)
3106				-0.74(11)	-0.58(17)		
3453	0.38(24)			-0.89(12)		-0.69(21)	

We obtain an expression for the distribution of  $A_2$  coefficients in the statistical model. We know the relationship (4) between the coefficient  $A_2$  and the value of the mixture of multipoles  $\delta$ . In the statistical model, the matrix elements of the  $\gamma$  transitions have a normal distribution with an average value of zero and a standard deviation of  $\sigma$ . In this case, the values of the mixture of multipoles  $\delta$  for the  $\gamma$  decay of the studied resonances to a fixed level are



Fig. 3. Distribution of the angular correlation coefficients  $A_2$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the g.s.  $(3/2^-)$  of  ${}^{59}$ Cu.  ${}^{58}$ Ni $(p, \gamma)$   ${}^{59}$ Cu reaction,  $E_p = 2120-3460$  keV,  $E_{res} = 5520-6815$  keV. Fit-normal distribution



Fig. 4. Distribution of the  $A_2$  coefficients for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the 491 keV  $(1/2^-)$  level in <sup>59</sup>Cu. <sup>58</sup>Ni $(p, \gamma)^{59}$ Cu reaction,  $E_p = 2120-3460$  keV,  $E_{\rm res} = 5520-6815$  keV. Fit-normal distribution with parameters  $\langle A_2 \rangle = -0.60$ ,  $\sigma = -0.15$  and I = 1.3

described by the Cauchy distribution [2, 5, 15]:

$$P\left(\delta\right) = \frac{a}{\pi(a^2 + \delta^2)},\tag{6}$$

where  $a = \sigma(E2)/\sigma(M1)$ . Estimates [5] values of 1/a for the  $\gamma$  transitions  $J_{\rm res} = 3/2^- \rightarrow {\rm g.s.}~(3/2^-)$  gave values of 1/a = 1.4 for  ${}^{59}{\rm Cu}$ , 1/a = 1.9 for  ${}^{61}{\rm Cu}$  and 1/a = 1.8 for  ${}^{63}{\rm Cu}$ .

<sup>61</sup> Cu	$E_{\rm lev}$ , keV, $L = 0.3/2^{-1}$	$1663, 3/2^{-}$	2357, $3/2^{-}$	2473, $3/2^{-}$	476, $1/2^{-}$	$2089, 5/2^{-}$	970, 5/2 <sup></sup>	1395, 5/2 <sup></sup>
$E_p$ , keV,	$A_2$	$A_2$	A2	A2	A2	A2	A2	A2
J <sub>res</sub> 3/2	0.20(15)							
1491	0.39(15)		ļ					
1515	0.22(11)	0.07(10)	ļ		0.44(0)			
15/7	-0.35(9)	-0.37(12)			-0.44(8)			
$p_{3/2}$	0.18(5)		-0.15(21)	0.12(9)		-0.59(22)	-0.26(14)	-0.31(7)
1599, IAR, p <sub>3/2</sub>	0.10(6)			-0.29(9)	-0.80(13)	-0.50(8)	-0.30(8)	-0.25(6)
$1605, IAR, p_{3/2}$	0.22(6)		「	0.09(18)		-0.45(2)	-0.13(11)	-0.12(6)
1620, IAR, p <sub>3/2</sub>	0.14(2)		0.64(13)			-0.50(16)	-0.09(26)	-0.12(14)
1649	-0.36(15)							
1694	0.29(4)	0.14(14)			-0.52(7)		-0.19(10)	
1698	0.26(13)							
1734	-0.31(6)	-0.13(22)			-0.58(17)		0.06(17)	-0.04(19)
1764	0.09(5)	0.26(11)			0.23(13)			-0.27(10)
1770	0.37(4)	-0.09(9)			-0.62(13)			
1793	0.05(12)				-0.25(13)			
1815			-0.14(11)					
1835	0.10(8)		0.22(13)					
1850	-0.30(3)							
1931	0.12(10)	-0.1(17)	0.1(26)		-0.39(9)	0.04(32)	-0.11(17)	
1944	-0.20(17)	-0.64(35)			-0.14(70)			0.42(19)
1972	-0.52(10)			-0.12(26)			0.16(13)	0.54(22)
2150	-0.45(11)				-0.37(19)			
2173	-0.15(10)				-0.59(14)	-0.5(2)	0.05(14)	
2242	-0.21(10)	-0.28(15)			-0.38(17)		0.16(13)	
2253	0.12(11)				-0.55(15)			-0.20(12)
2270	-0.45(11)						-0.07(13)	
2283	-0.23(9)	-0.29(12)				-0.29(18)		
2299	-0.34(10)	-0.29(4)					0.05(10)	-0.39(16)
2359	-0.30(10)				-0.19(15)			
2442	-0.42(10)	-0.48(13)			-0.64(14)		0.17(13)	-0.42(16)
2455	0.03(9)				-0.71(12)		-0.04(13)	-0.63(12)

Table 2. Angular distribution coefficients  $A_2$  in  ${}^{60}$ Ni $(p, \gamma)^{61}$ Cu reaction,  $E_p = 1451-2455$  keV,  $E_{res} = 6200-7205$  keV [5, 15, 25]

The type of Cauchy distribution for different values of parameter *a* is shown in Fig. 13. From formula (6) it follows that the average value  $\langle \delta \rangle = 0$  and the distribution  $P(\delta)$  is symmetric with respect to  $\delta = 0$ ; i.e. positive and negative values of  $\delta$  are equally common (Fig. 13).

<sup>63</sup> Cu	$E_{\text{lev}}$ , keV, $J_{\text{lev}}$ 0, 3/2 <sup>-</sup>	1547, 3/2-	668, 1/2-	962, 5/2-	1410, 5/2-
$E_p$ , keV, $J_{\rm res} 3/2^-$	$A_2$	$A_2$	$A_2$	$A_2$	$A_2$
1943	-0.54(18)	0.73(18)	-0.10(10)		
1953	-0.38(13)	0.22(20)	-0.24(15)		0.06(12)
1958	0.003(90)			0.13(30)	
1976	0.014(70)	-0.38(15)		0.58(20)	
2022	0.31(13)			0.34(17)	
2085	-0.11(8)	0.22(17)	0.38(19)	0.22(13)	0.08(11)
2113	-0.21(10)	0.08(3)	-0.64(14)	0.17(34)	0.25(15)
2169	-0.59(15)	0.16(15)	-0.46(13)	-0.11(9)	
2231	-0.47(9)			0.045(144)	
2238	-0.25(10)	-0.34(12)	-0.38(17)	0.04(19)	0.11(19)
2251	-0.43(10)				
2268	-0.34(9)				
2275	0.063(102)			-0.10(16)	
2285	-0.29(19)				
2512	-0.37(11)	-0.25(18)	-0.24(21)		
2584	-0.34(16)	-0.16(32)	0.01(22)	-0.01(29)	
2613	-0.54(11)	-0.10(26)			-0.041(21)
2620	-0.42(9)	0.07(22)			-0.23(18)
2635	-0.29(11)	0.07(20)			
2642	-0.36(10)		-0.70(15)		
2675	-0.49(13)	-0.71(18)	-0.45(12)		
2682	-0.53(9)				
2690	-0.21(19)		-0.44(18)		
2696	-0.35(12)		-0.66(18)	-0.47(14)	-0.15(18)
2710	-0.63(12)		-0.38(15)		
2722	-0.70(10)				-0.53(25)
2730	-0.67(10)		-0.44(16)		
2765	-0.16(9)		-0.27(12)		
2783	0.13(11)		0.16(23)		
2811	-0.28(10)	-0.24(31)		0.19(16)	
2818	-0.22(10)	0.57(35)		0.00(16)	
2833	-0.54(13)		-0.55(14)		
2839	-0.41(9)	-0.20(29)	-0.24(26)		
2865	-0.60(12)	-0.59(16)			
2880	0.05(10)	-0.13(23)			
2933		-0.76(30)	-0.39(24)	-0.29(18)	
2951	0.10(29)				
3154	-0.76(12)				
3185	-0.04(23)				

Table 3. Angular distribution coefficients  $A_2$  in  ${}^{62}Ni(p,\gamma){}^{63}Cu$  reaction,  $E_p = 1943-3175$  keV,  $E_{res} = 8040-9250$  keV [5,25]



Fig. 5. Distribution of the  $A_2$  coefficients for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the g.s.  $(3/2^-)$  of  ${}^{61}$ Cu.  ${}^{60}$ Ni $(p, \gamma)^{61}$ Cu reaction,  $E_p = 1451-2455$  keV,  $E_{\rm res} = 6200-7205$  keV. Fit-normal distribution with parameters  $\langle A_2 \rangle = -0.10$ ,  $\sigma = 0.28$  and I = 2.72



Fig. 6. Distribution of the angular correlation coefficients  $A_2$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the 1663 keV  $(3/2^-)$  level in  ${}^{61}$ Cu.  ${}^{60}$ Ni $(p, \gamma)$  ${}^{61}$ Cu reaction,  $E_p = 1451-2455$  keV,  $E_{\rm res} = 6200-7205$  keV. Fit-normal distribution with parameters  $\langle A_2 \rangle = -0.21$ ,  $\sigma = 0.26$  and I = 1.20

The density of the distribution  $R(A_2)$  of the  $A_2$  coefficients is related to the distribution (6)  $P(\delta)$  as follows [22-24]:

$$R(A_{2}) = P(\delta_{1}(A_{2})) \left| \frac{d(\delta_{1})}{d(A_{2})} \right| + P(\delta_{2}(A_{2})) \left| \frac{d(\delta_{2})}{d(A_{2})} \right|,$$
(7)

where  $\delta_1(A_2)$  and  $\delta_2(A_2)$  are two solutions of a quadratic equation of type (4) connecting the quantities  $A_2$  and  $\delta$ . For the case  $J_{\text{res}} = J_{\text{lev}} = 3/2$ , the graph of the function  $A_2(\arctan(\delta))$  is shown in Fig. 14. In this case, the values of



Fig. 7. Distribution of the angular correlation coefficients  $A_2$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the 476 keV  $(1/2^-)$  level in  ${}^{61}$ Cu.  ${}^{60}$ Ni $(p, \gamma)^{61}$ Cu reaction,  $E_p = 1451-2455$  keV,  $E_{\rm res} = 6200-7205$  keV. Fit-normal distribution with parameters  $\langle A_2 \rangle = -0.41$ ,  $\sigma = 0.24$  and I = 1.50



Fig. 8. Distribution of the angular correlation coefficients  $A_2$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the g.s.  $(3/2^-)$  of  ${}^{63}$ Cu.  ${}^{62}$ Ni $(p, \gamma)^{63}$ Cu reaction,  $E_p = 1943 - 3175$  keV,  $E_{\rm res} = 8040 - 9250$  keV. Fit-normal distribution with parameters  $\langle A_2 \rangle = -0.31$ ,  $\sigma = 0.25$  and I = 4.09

 $A_2$  have, in accordance with (4), both upper (0.99) and lower (-0.6) limits. The specified limits correspond, taking into account measurement errors, to the experimental data given in Tables 1–3.

Taking into account the type of distribution function  $\Phi = \arctan(\delta)$ (Fig. 13) and knowing the type (6) of the distribution  $P(\delta)$  for the statistical model, it is possible to determine the characteristic features of the function  $R(A_2)$ . For all values of parameter *a*, the distribution function of the value  $\Phi = \arctan(\delta)$  in the statistical model is symmetric with respect to  $\Phi = 0$ .



Fig. 9. Distribution of the angular correlation coefficients  $A_2$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the 1547 keV  $(3/2^-)$  level in  ${}^{63}$ Cu.  ${}^{62}$ Ni $(p, \gamma)$  ${}^{63}$ Cu reaction,  $E_p = 1943 - 3175$  keV,  $E_{\rm res} = 8040 - 9250$  keV. Fit-normal distribution with parameters  $\langle A_2 \rangle = -0.091$ ,  $\sigma = 0.39$  and I = 1.9



Fig. 10. Distribution of the angular correlation coefficients  $A_2$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the 668 keV  $(1/2^-)$  level in  ${}^{63}$ Cu.  ${}^{62}$ Ni $(p, \gamma)$  ${}^{63}$ Cu reaction,  $E_p = 1943 - 3175$  keV,  $E_{\rm res} = 8040 - 9250$  keV. Fit-normal distribution with parameters  $\langle A_2 \rangle = -0.32$ ,  $\sigma = 0.28$  and I = 2.05

In the case a = 1, the distribution  $R(A_2)$  in the statistical model has the following form [2, 17]:

$$R(A_2) \sim \frac{1}{2\pi\sqrt{0.6 - A_2(A_2 - 0.4)}};$$
 (8)

that is, the function  $R(A_2)$  has a minimum at  $A_2 \sim 0.2$  and further increases with both decreasing and increasing (Fig. 15) value of  $A_2$ .

At the  $\gamma$  decay of the studied non-analog resonances in the nuclei of  $^{59,61,63}$ Cu, the mentioned growth is not observed and the type of the  $R(A_2)$ 



Fig. 11. Distribution of the angular correlation coefficients  $A_2$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the 962 keV (5/2<sup>-</sup>) level in  ${}^{63}$ Cu.  ${}^{62}$ Ni( $p, \gamma$ ) ${}^{63}$ Cu reaction,  $E_p = 1943 - 3175$  keV,  $E_{\rm res} = 8040 - 9250$  keV. Fit-normal distribution with parameters  $\langle A_2 \rangle = 0.05$ ,  $\sigma = 0.26$  and I = 1.4



Fig. 12. Distribution of the angular correlation coefficients  $A_2$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the 1410 keV (5/2<sup>-</sup>) level in  ${}^{63}$ Cu.  ${}^{62}$ Ni( $p, \gamma$ ) ${}^{63}$ Cu reaction,  $E_p = 1943 - 3175$  keV,  $E_{\rm res} = 8040 - 9250$  keV. Fit-normal distribution with parameters  $\langle A_2 \rangle = -0.056$ ,  $\sigma = 0.24$  and I = 0.8

function (Fig. 15) differs qualitatively from the experimental data (Figs. 4–12, Tables 1–3), which indicates a significant contribution of the non-statistical component to the wave functions of the studied resonances [2, 17, 18]. The difference from the statistical model is even more pronounced in the distribution of the values of the mixture of multipoles  $\delta$ , which will be discussed in Section 4. Nevertheless, it should be noted that in some cases, when the gamma decay of the non-statistical component to any level is inhibited, the admixture of the statistical component in the resonance wave function can play a significant role [2, 14].



Fig. 13. The distribution function of the value  $\Phi = \arctan(\delta)$ , the value  $\delta$ is distributed in accordance with (6) (Cauchy distribution). The numbers on the curves correspond to the value of parameter *a*. For *a* = 1, the corresponding distribution (or the distribution of the value  $\Phi = \arctan(\delta/a)$ ) is a straight line [4, 5]







Fig. 15. The distribution function  $R(A_2)$  of the coefficients of the angular distribution of  $\gamma$  radiation  $A_2$  at the decay of resonances  $J_{\rm res} = J_{\rm lev} = 3/2^-$  in the statistical model at the value of parameter (6) of the model a = 1

# 3. CORRELATIONS OF THE REDUCED PROBABILITIES B(E2)AND B(M1) OF GAMMA RADIATION

In the statistical model, there are no correlations during decay along various reaction channels, including various spin channels [2, 4]. In the case of  $(p, \gamma)$  reactions, the statistical model should lack [2, 16, 18] correlations for the reduced probabilities of B(E2) and  $B(M1) \gamma$  transitions ( $\gamma$  decay along various spin channels). Knowing the experimental values of the quantities of the mixture of multipoles  $\delta$ , the angular distributions of  $\gamma$  radiation  $W(\theta)$ , energy  $E_{\gamma}$  and intensity  $I_{\gamma}$  of gamma transitions, it is possible to determine the values  $x_i$  and  $y_i$ :

$$x_{i} = \frac{I_{\gamma i}\delta_{i}^{2}k_{p}^{2}}{(1+\delta_{i}^{2})E_{\gamma i}^{5}W_{i}(\theta)\varepsilon_{\gamma}(i)},$$

$$y_{i} = \frac{I_{\gamma i}k_{p}^{2}}{(1+\delta_{i}^{2})E_{\gamma i}^{3}W_{i}(\theta)\varepsilon_{\gamma}(i)},$$
(9)

where  $k_p$  is the proton wave vector and  $\varepsilon_{\gamma}(i)$  is the efficiency of registration of  $\gamma$  radiation with energy  $E_{\gamma i}$ . Next, the correlation coefficient is determined:

$$\rho(x,y) = \frac{\sum_{i} (x_i - \langle x \rangle)(y_i - \langle y \rangle)}{\left[\sum_{i} (x_i - \langle x \rangle)^2 \sum_{i} (y_i - \langle y \rangle)^2\right]^{1/2}} \epsilon,$$
(10)

where  $\langle x \rangle$  and  $\langle y \rangle$  are the corresponding average values, and  $\epsilon$  is the correction (10) associated with the errors in determining the values of  $x_i$  and  $y_i$ :

$$\epsilon \approx \left\{ 1 - \frac{1}{2} \left[ \frac{\sum_{i} (\Delta x_i)^2}{\sum_{i} (x_i - \langle x \rangle)^2} + \frac{\sum_{i} (\Delta y_i)^2}{\sum_{i} (y_i - \langle y \rangle)^2} \right] \right\}.$$
 (11)

Under certain experimental conditions [16], it turns out that the equality  $\rho(B(E2), B(M1)) = \rho(x, y)$  is fulfilled with good accuracy and it is possible to determine the correlation coefficient  $\rho(B(E2), B(M1))$  without measuring the values of B(E2) themselves and B(M1). The data analyzed in the work from experimental data for the  $\gamma$  decay of the *i* resonance were obtained under the following experimental conditions [16]:  $\Delta E_p \gg \Gamma$  and  $\Gamma_{\gamma} \ll \Gamma_p$ , where  $\Gamma$  is the full width of the resonance,  $\Gamma_p$  is the width along the input channel,  $\Gamma_{\gamma}$  is the width along the output channel of the reaction, and  $\Delta E_p$  is the energy resolution for the proton beam. During the experiments [5, 16], the above quantities had the following values:  $\Delta E_p \cong 2-3$  keV,  $\Gamma_{\gamma} \sim 10^{-2}$  eV,  $\Gamma_p \sim 10-100$  eV, which allows the equality  $\rho(B(E2), B(M1)) = \rho(x, y)$  to determine the desired correlation coefficient of the reduced probabilities of transitions along various spin channels (in this case E2 and  $M1 \gamma$  transitions). The method discussed above, proposed in [16], allows us to determine the correlation coefficients in experiments using a "thin" (10–20 µg/cm<sup>2</sup>) and the

data on relative efficiency [26] of detecting  $\gamma$  radiation. These advantages make it possible to identify and determine the values of the correlation coefficients more accurately, compared with the "thick" (up to 500  $\mu$ g/cm<sup>2</sup>) target method [13].

of  $\rho(B(E2), B(M1))$ The values were determined bv this [2, 16, 18] for the  $\gamma$  decay of non-analog resonances method with  $J_{\rm res} = 3/2$  to the ground states of Cu nuclei  $(J_{\rm lev} = 3/2^{-})$  in reactions  ${}^{58,60,62}{\rm Ni}(p,\gamma)^{59,61,63}{\rm Cu}$ . The excitation energies of non-analog resonances in the <sup>63</sup>Cu nucleus ranged from 8.04 to 9.25 MeV; in the <sup>61</sup>Cu nucleus, from 6.2 to 7.2 MeV; in the <sup>59</sup>Cu nucleus, from 5.5 to 6.8 MeV. In all cases, it turned out that the correlation coefficients were the same within the error limits and amounted to  $\rho(B(E2), B(M1)) = 0.7 \pm 0.1$ . The presence of a correlation coefficient obviously different from zero ( $\rho \neq 0$ ) confirms the non-statistical nature of the studied non-analog resonances excited in  $(p, \gamma)$ reactions.

### 4. DISTRIBUTIONS OF *E*2/*M*1 GAMMA RADIATION MULTIPOLE MIXTURES RATIOS AND NON-STATISTICAL EFFECTS

For  $\gamma$  radiation with mixed multipolarities (E2 + M1), the multipole mixing ratio  $\delta$  is defined as the ratio of the reduced matrix elements corresponding to E2 and  $M1 \gamma$  transitions between initial  $(I_i)$  and final  $(I_f)$  states [27]:



$$\delta = \frac{\langle I_f || O(E2) || I_i \rangle}{\langle I_f || O(M1) || I_i \rangle},\tag{12}$$

Fig. 16. E2/M1 multipole mixture  $\delta$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the g.s.  $(3/2^-)$  of  ${}^{59}$ Cu.  ${}^{58}$ Ni $(p, \gamma)^{59}$ Cu reaction,  $E_p = 2120-3460$  keV,  $E_{\rm res} = 5520-6815$  keV. Fit-normal distribution with  $\langle \delta \rangle = 0.45625$ ,  $\sigma = 0.4458$  and I = 3.20. For  ${}^{63}$ Cu, estimates [5] give the value of the parameter  $1/a = \sigma(M1)/\sigma(E2) = 1.4$ . The dotted line corresponds to the statistical model (Cauchy distribution)

<sup>59</sup> Cu	$E_{lev}$ , keV, $J_{lev}$ 0, $3/2^-$	2265, 3/2 <sup>-</sup>	2324, 3/2 <sup>-</sup>	491, 1/2 <sup></sup>	912, 5/2 <sup></sup>	1987, 5/2 <sup></sup>	2707, 5/2 <sup></sup>	2927, 5/2 <sup></sup>
$E_p$ , keV, $J_{\rm res} 3/2^-$	δ	δ	δ	δ	δ	δ	δ	δ
2161	0.35(8)							
2210	$0.81^{+44}_{-19}$							
2338	0.41(12)		$0.67^{+33}_{-11}$				$-0.40^{+12}_{-17}$	
2512	0.45(9)		$0.51\substack{+75 \\ -31}$					
2574	0.77(12)	$0.62^{+58}_{-24}$						
2668	0.63(6)	$0.70^{+59}_{-36}$		0.12(9)		$0.11^{+79}_{-34}$		
2704				0.10(8)				
2721	$0.76^{+13}_{-8}$			-0.06(11)				
2756	0.23(9)			-0.06(10)				
2831	0.11(8)			0.08(5)		$0.02^{+4}_{-9}$		
2869	$1.29^{+0}_{-39}$			-0.02(4)				
2938	0.19(5)	0.35(17)		0.09(7)		-0.28(11)	-0.25(10)	-0.18(9)
2960	$0.88^{+41}_{-20}$	$0.67^{+53}_{-22}$		0.03(9)				
2978	$0.80^{+49}_{-35}$	$0.38^{+32}_{-20}$		$0.06^{+21}_{-14}$	$-0.34^{+11}_{-14}$			
2999	0.21(5)			0.15(4)	-0.17(13)			
3051	-0.61(9)					-0.20(14)		
3062		$0.54\substack{+45 \\ -19}$	$0.45\substack{+27 \\ -19}$	-0.02(7)	$0.12^{+21}_{-18}$		-0.19(23)	
3106				0.16(9)	-0.45(9)			
3453	$0.02^{+14}_{-25}$			$0.30^{+25}_{-12}$		$0.61\substack{+27 \\ -44}$		$-0.55\substack{+20\\-9}$

Table 4. E2/M1 multipole mixture  $\delta$  in  ${}^{59}$ Cu,  $E_p = 2120-3460$  keV,  $E_{\rm res} = 5520-6815$  keV,  $J_{\rm res}^{\pi} = 3/2^{-}$  [5, 15, 25]



Fig. 17. E2/M1 multipole mixture  $\delta$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the 491 keV (1/2<sup>-</sup>) level in  ${}^{59}$ Cu.  ${}^{58}$ Ni( $p, \gamma$ ) ${}^{59}$ Cu reaction,  $E_p = 2120-3460$  keV,  $E_{\rm res} = 5520-6815$  keV. Fit-normal distribution with  $\langle \delta \rangle = 0.071$ ,  $\sigma = 0.10$  and I = 0.73

where E2 and M1 are the two mixed multipolarities, O(E2) and O(M1) are the electromagnetic operators for electric quadrupole (E2) and magnetic dipole (M1) type of  $\gamma$  transition. For  $\delta^2$ , one can write

$$\delta^2 = \frac{\Gamma_\gamma(E2)}{\Gamma_\gamma(M1)},\tag{13}$$

where  $\Gamma_{\gamma}(E2)$  and  $\Gamma_{\gamma}(M1)$  are the probabilities of  $\gamma$  decay via each of the multipolarities.



Fig. 18. E2/M1 multipole mixture  $\delta$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the g.s.  $(3/2^-)$  of  ${}^{61}$ Cu.  ${}^{60}$ Ni $(p, \gamma){}^{61}$ Cu reaction,  $E_p = 1451-2455$  keV,  $E_{\rm res} = 6200-7200$  keV. Fit-normal distribution with  $\langle \delta \rangle = 0.339$ ,  $\sigma = 0.232$  and I = 3.19



Fig. 19. E2/M1 multipole mixture  $\delta$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the 1395 keV (5/2<sup>-</sup>) level in  ${}^{61}$ Cu.  ${}^{60}$ Ni $(p, \gamma)^{61}$ Cu reaction,  $E_p = 1451-2455$  keV,  $E_{\rm res} = 6200-7200$  keV. Fit-normal distribution with  $\langle \delta \rangle = 0.00416$ ,  $\sigma = 0.40$  and I = 1.275

<sup>61</sup> Cu	$E_{\rm lev}$ , keV,	1663,	2357,	476,	2089,	970,	1395,
Em keV	$J_{\rm res}  0,  3/2$	3/2	3/2	1/2	5/2	5/2	5/2
$J_{\rm res} 3/2^{-}$	δ	δ	δ	δ	δ	δ	δ
1491	$0.00^{+10}_{-9}$						
1515	$0.12^{+7}_{-8}$						
1577	0.56(10)	$0.58^{+18}_{-12}$		$-0.03^{+4}_{-6}$			
1588, IAR, p <sub>3/2</sub>	0.14(3)	-12	$0.37\substack{+20 \\ -15}$		$0.05^{+17}_{-13}$	-0.13(11)	-0.17(7)
1599, IAR, p <sub>3/2</sub>	$0.41^{+3}_{-5}$			$0.20^{+14}_{-9}$	0.00(4)	-0.17(7)	-0.12(4)
1605, IAR, $p_{3/2}$	0.11(4)				-0.03(1)	-0.03(10)	-0.02(5)
1620, IAR, p <sub>3/2</sub>	0.16(1)		-0.17(10)		0.00(10)	0.01(25)	-0.02(10)
1649	$0.57^{+23}_{-15}$						
1694	0.07(2)	$0.16^{+10}_{-9}$		0.01(4)		-0.07(8)	
1698	0.09(9)						
1734	0.51(6)	$0.35^{+22}_{-15}$		$0.04^{+12}_{-9}$		$0.17^{+16}_{-16}$	0.05(19)
1764	0.19(4)	$0.08^{+8}_{-6}$		$-0.39^{+6}_{-9}$			-0.14(8)
1770	0.02(3)	0.32(7)		$0.07^{+9}_{-8}$			
1793	$0.23^{+8}_{-10}$			-0.14(7)			
1815	-10		0.36(9)				
1835	0.19(5)		0.12(8)				
1850	0.51(3)						
1931	0.18(7)	0.25(12)	$0.19^{+18}_{-10}$	-0.06(5)	-0.29(18)	-0.01(15)	
1944	$0.41^{+18}_{-13}$	$1.25^{+9}_{-73}$		-0.19(44)			$0.8^{+0}_{-5}$
1972	$0.83^{+16}_{-18}$					$0.25^{+19}_{-13}$	$0.8^{+0}_{-3}$
2150	$0.70^{+25}_{-15}$			-0.07(10)			
2173	0.37(9)			0.05(9)	0.00(12)	0.14(13)	
2242	$0.42^{+11}_{-8}$	$0.49^{+17}_{-14}$		-0.07(9)		$0.15^{+19}_{-13}$	
2253	0.18(7)	-14		0.03(9)		-15	-0.08(11)
2270	$0.69^{+26}_{-14}$					0.03(12)	
2283	0.44(9)	$0.50^{+14}_{-10}$			-0.11(10)		
2299	0.55(12)	10				0.14(10)	-0.25(15)
2359	0.51(11)			-0.17(8)			
2442	$0.65^{+21}_{-13}$	$0.75^{+54}_{-19}$		$0.08^{+11}_{-8}$		-0.07(11)	$-0.28^{+14}_{-17}$
2455	0.24(8)			0.14(10)		0.05(12)	$-0.52^{+15}_{-21}$

Table 5. E2/M1 multipole mixture  $\delta$  in  ${}^{61}$ Cu,  $E_p = 1451 - 2455$  keV,  $E_{\rm res} = 6200 - 7205$  keV [5, 15, 25]



Fig. 20. E2/M1 multipole mixture  $\delta$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the 1663 keV ( $3/2^-$ ) level in  ${}^{61}$ Cu.  ${}^{60}$ Ni( $p, \gamma$ ) ${}^{61}$ Cu reaction,  $E_p = 1451-2455$  keV,  $E_{\rm res} = 6200-7200$  keV. Fit-normal distribution with  $\langle \delta \rangle = 0.373$ ,  $\sigma = 0.22$  and I = 1.00



Fig. 21. E2/M1 multipole mixture  $\delta$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the 476 keV  $(1/2^-)$  level in  ${}^{61}$ Cu.  ${}^{60}$ Ni $(p, \gamma)^{61}$ Cu reaction,  $E_p = 1451-2455$  keV,  $E_{\rm res} = 6200-7200$  keV. Fit-normal distribution with  $\langle \delta \rangle = -0.031$ ,  $\sigma = 0.144$  and I = 1.6

Experimental data on the values of  $\delta$  obtained in the study of  $\gamma$  decay of non-analog resonances in  ${}^{58,60,62}$ Ni $(p,\gamma)^{59,61,63}$ Cu reaction are given in Tables 4–6. The distribution functions of the  $\delta$  values are shown in Figs. 16–26.

The Cauchy distribution is symmetric with respect to  $\delta = 0$  and is shown in Fig. 13 for different values of parameter *a*. For the value  $\Phi = \arctan(\delta/a)$ it is a straight line in the range from  $-90^{\circ}$  to  $+90^{\circ}$ . The radical difference between the experimental values of  $\delta$  and the Cauchy distribution is obvious.

For <sup>61</sup>Cu, estimates [5] give the value of the parameter  $\frac{1}{a} = \frac{\sigma(M1)}{\sigma(E2)} = 1.9$ . It is obvious that the distribution of experimental values  $\delta$  for the reaction <sup>60</sup>Ni( $p, \gamma$ )<sup>61</sup>Cu differs significantly from the statistical model.

1		1	1	1	1
<sup>63</sup> Cu	$E_{\text{lev}}, \text{ keV}, J_{\text{res}} 0, 3/2^-$	$1547, 3/2^-$	668, $1/2^-$	962, $5/2^-$	1410, 5/2-
$E_p$ , keV, $J_{\rm res} 3/2^-$	δ	δ	δ	δ	δ
1943	$0.89^{+40}_{-31}$	$-0.25^{+15}_{-10}$	-0.22(6)		
1953	$0.60^{+20}_{-14}$	$0.11^{+15}_{-13}$	-0.14(8)		0.15(12)
1958	0.26(6)			$0.22^{+58}_{-28}$	
1976	0.25(5)	$0.60^{+24}_{-16}$		0.80(19)	
2022	0.06(9)			$0.57^{+23}_{-31}$	
2085	0.34(6)	0.12(11)	$-0.49^{+12}_{-15}$	$0.32^{+28}_{-14}$	$0.16^{+13}_{-10}$
2113	0.42(9)	0.20(2)	0.09(10)	$0.26^{+54}_{-32}$	$0.37^{+43}_{-18}$
2169	0.98(30)	0.15(11)	-0.03(7)	-0.01(8)	
2231	$0.72^{+22}_{-13}$			$0.13^{+16}_{-13}$	
2238	0.46(10)	$0.55^{+16}_{-10}$	-0.07(10)	$0.13^{+21}_{-17}$	$0.19^{+27}_{-20}$
2251	$0.69^{+18}_{-9}$				
2268	0.55(10)				
2275	0.21(7)			0.00(14)	
2285	$0.50^{+15}_{-12}$				
2512	$0.50^{+15}_{-12}$	$0.25\substack{+20 \\ -14}$	-0.14(11)		
2584	$0.55^{+23}_{-16}$	$0.37^{+37}_{-22}$	-0.28(12)	$0.03\substack{+35 \\ -20}$	
2613	$0.89^{+40}_{-19}$	$0.33^{+24}_{-18}$			$0.05^{+21}_{-18}$
2620	$0.65^{+15}_{-11}$	0.21(14)			-0.11(16)
2635	$0.49^{+13}_{-10}$	0.21(14)			
2642	$0.57^{+14}_{-10}$		0.13(11)		
2675	$0.76^{+59}_{-19}$	1.15(17)	-0.03(7)		
2682	$0.85^{+40}_{-17}$				
2690	$0.42^{+10}_{-8}$		-0.04(8)		
2696	$0.56^{+17}_{-12}$		0.10(12)	$-0.33^{+12}_{-16}$	-0.04(15)
2710	1.05(20)		-0.07(8)	10	
2722	1.29				$-0.39^{+24}_{-41}$
2730	1.25(5)		-0.04(9)		
2765	0.38(10)		$-0.13^{+5}_{-8}$		
2783	0.17(9)		-0.35(15)		
2811	0.48(9)	$0.40^{+50}_{-20}$		$0.30^{+30}_{-15}$	

Table 6. E2/M1 multipole mixture  $\delta$  in  ${}^{63}$ Cu,  $E_p = 1943 - 3175$  keV,  $E_{\rm res} = 8040 - 9250$  keV [5, 25]

Table 6 (continued)

1.5						
	<sup>63</sup> Cu	$E_{\rm lev}, \ {\rm keV}, \ J_{\rm res} \ 0, \ 3/2^-$	1547, 3/2-	668, 1/2-	962, $5/2^-$	1410, 5/2-
	$E_p$ , keV, $J_{\rm res}$ 3/2 <sup>-</sup>	δ	δ	δ	δ	δ
	2818	0.43(9)	0.80(37)		0.09(10)	
	2833	$0.87\substack{+42 \\ -20}$		0.03(12)		
	2839	0.63(13)	$0.41\substack{+35 \\ -20}$	-0.14(14)		
	2865	1.07(23)	0.80(36)			
	2880	0.30(10)	0.45(20)			
	2933		0.87(38)	$-0.06^{+20}_{-14}$	$-0.16^{+16}_{-20}$	
	2951	$0.17^{+23}_{-17}$				
	3154	1.29				
	3185	$0.29^{+19}_{-16}$				



Fig. 22. E2/M1 multipole mixture  $\delta$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the 970 keV (5/2<sup>-</sup>) level in  ${}^{61}$ Cu.  ${}^{60}$ Ni $(p, \gamma)^{61}$ Cu reaction,  $E_p = 1451-2455$  keV,  $E_{\rm res} = 6200-7200$  keV. Fit-normal distribution with  $\langle \delta \rangle = 0.032$ ,  $\sigma = 0.12$  and I = 0.7

#### 5. DISCUSSION

At nuclear excitation energies above 3–5 MeV, a large number of other, non-analog states and resonances are observed. They have two possible interpretations: statistical or non-statistical. In the first case it is assumed that they are statistical states, while in the second it is assumed that they are structures like the giant resonance, related to a distribution of simple excitations. In the second case, the physical interpretation of the experiments must differ from that of the statistical approach. In all cases, for  $\gamma$  decay of non-analog resonances from  $I^{\pi} = 3/2^{-}$  to the ground state of <sup>59,61,63</sup>Cu ( $I^{\pi} = 3/2^{-}$ ) nuclei, a pronounced asymmetry with respect to the zero value in the distributions of  $\delta$  values was observed (Figs. 16, 18, 23). This fact clearly indicates a significant contribution of the non-statistical component to the wave function of the considered non-analog resonances. However, since



Fig. 23. E2/M1 multipole mixture  $\delta$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the g.s.  $(3/2^-)$  of  ${}^{63}$ Cu.  ${}^{62}$ Ni $(p, \gamma)^{63}$ Cu reaction,  $E_p = 1943-3175$  keV,  $E_{\rm res} = 8040-9250$  keV. Fit-normal distribution with  $\langle \delta \rangle = 0.603$ ,  $\sigma = 0.32$  and I = 4.0533. The type of Cauchy distribution is shown in Fig. 13. For  ${}^{63}$ Cu, estimates [5] give the value of the parameter  $1/a = \sigma(M1)/\sigma(E2) = 1.8$ . The distribution of experimental values of  $\delta$  for the reaction  ${}^{62}$ Ni $(p, \gamma)^{63}$ Cu is radically different from the Cauchy distribution



Fig. 24. E2/M1 multipole mixture  $\delta$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the level with excitation energy 1547 keV ( $3/2^-$ ) in  $^{63}$ Cu.  $^{62}$ Ni( $p, \gamma$ ) $^{63}$ Cu reaction,  $E_p = 1943-3175$  keV,  $E_{\rm res} = 8040-9250$  keV. Fit-normal distribution with  $\langle \delta \rangle = 0.41$ ,  $\sigma = 0.32$  and I = 2.01

in the second case the resonance wave function may contain an admixture of the statistical component (2), this admixture leads to fluctuations in the distribution of both the angular distribution coefficients and the values of the mixture of multipoles  $\delta$  (Fig. 27). These fluctuations are especially clearly manifested in the graphs of the dependence of the experimental values of  $\delta$ 



Fig. 25. E2/M1 multipole mixture  $\delta$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the level with excitation energy 668 keV  $(1/2^-)$  in  $^{63}$ Cu. Fit-normal distribution with  $\langle \delta \rangle = -0.098, \ \sigma = 0.15$  and I = 1.9



Fig. 26. E2/M1 multipole mixture  $\delta$  for  $\gamma$  decay of  $3/2^-$  non-analog resonances to the level with excitation energy 962 keV (5/2<sup>-</sup>) in  ${}^{63}$ Cu.  ${}^{62}$ Ni $(p, \gamma)^{63}$ Cu reaction,  $E_p = 1943-3175$  keV,  $E_{\rm res} = 8040-9250$  keV. Fit-normal distribution with  $\langle \delta \rangle = 0.16$ ,  $\sigma = 0.28$  and I = 1.4

on the resonance energy (or the energy of the incoming proton, Fig. 27). By data on the magnitude of these fluctuations, it is possible [2, 14] to estimate the fraction of the non-statistical component in the resonance wave function. It turned out that for the considered non-analog resonances in the  $^{59,61,63}$ Cu, the fraction of the non-statistical component is about several tens of percent (from 20 to 50% [2, 14, 18]).

The experiments investigated the strongest  $\gamma$  transitions in the decay of non-analog resonances. In cases where  $\gamma$  decay is inhibited due to a non-statistical component [2, 14], the statistical component may make a dominant contribution to the probability of a  $\gamma$  transition. Such  $\gamma$  transitions for the



Fig. 27. Experimental dependence of the multipole mixing ratio  $\delta$  on the proton energy  $E_p$  in the  ${}^{62}\text{Ni}(p,\gamma){}^{63}\text{Cu}$  reaction [18]

studied resonances should be weaker in intensity, and for weak  $\gamma$  transitions, the statistical nature of the distribution of angular correlation coefficients and the values of a multipole mixing ratio  $\delta$  is quite possible.

At the same time, resonances excited in reactions with neutrons, as a rule, are well described by the statistical model [19, 20]. Such a difference between the properties of neutron and proton resonances can be explained by the existence of an excess of neutrons. Indeed, by irradiating nuclei with protons with (N - Z) > 0, the simplest configurations of the  $[(\pi p) \otimes (\nu h)]_{1+}$ type can be excited. These configurations in the studied energy range can be excited with a noticeable cross section only in the presence of an excess of neutrons in the nucleus, i.e., at (N-Z) > 0. On the other hand, nonstatistical effects indicate the presence of a certain type of symmetry of the nuclear interaction. Non-statistical effects caused by proton-particle and neutron hole configurations coupled into spin 1<sup>+</sup>, i.e.,  $[(\pi p) \otimes (\nu h)]_{1+}$ , can be caused by [2, 28] spin-isospin SU(4) symmetry of the nuclear interaction, and with an increase in the neutron excess, the effects of SU(4) symmetries can increase [2]. In reactions of  ${}^{58,60,62}$ Ni $(p, \gamma)$ <sup>59,61,63</sup>Cu, non-statistical effects can be caused by configurations of the type  $\left\{ [(\pi p) \otimes (\nu h)]_{1+} \otimes (\pi p)_J \right\}_{3/2}$ in wave functions of resonances [2, 28]. Thus, the analysis of the  $\gamma$  decay of non-analog resonances in  ${}^{58,60,62}$ Ni $(p,\gamma)$  ${}^{59,61,63}$ Cu reactions indicates the presence of partial SU(4) symmetry of the nuclear interaction. Since the non-statistical effects for non-analog resonances are less pronounced than for analog resonances, the spin-isospin SU(4) symmetry is, as expected, more approximate than isospin symmetry of the nuclear interaction.

# CONCLUSIONS

Experimental data are presented that clearly indicate the manifestation of non-statistical effects in the  $\gamma$  decay of non-analog resonances of a compound nucleus in reactions with protons. Non-statistical effects are associated with elementary modes of nuclear excitations, for example, a proton particle and a neutron hole coupled into spin 1<sup>+</sup>, i.e.,  $[(\pi p) \otimes (\nu h)]_{1^+}$  and their corresponding configurations of the type  $\{[(\pi p) \otimes (\nu h)]_{1^+} \otimes (\pi p)_J\}_{3/2}$ . Spreading of the elementary modes of nuclear excitations by the levels of the compound nuclei is responsible for non-statistical effects. If the fraction of such elementary excitations is enough high in the wave function of non-analog resonance, it will lead to the pronounced non-statistical effects. In this case, the physical interpretation of the experiments should differ from the statistical approach.

The considered set of experimental data allows us to conclude that the observed non-statistical effects are associated with the presence of an intermediate structure due to the distribution of the simple excitation among the levels of the compound nucleus [2,5]. In non-analog proton resonances, which are observed in the  $(p, \gamma)$  reaction in medium-mass nuclei  $(A \sim 60)$  at proton energies of 1–5 MeV, the intermediate structure is manifested in the following experimental facts:

1. The signs of multipole mixing ratio  $\delta$  of  $\gamma$  transitions from a number of resonances to the same nuclei levels turn out to be the same. The distributions of the multipole mixing ratio  $\delta$  values are different from the predictions of the statistical model.

2. The values of the reduced probabilities B(E2) and B(M1) of  $\gamma$  transitions are correlated.

3. The distribution of the values of the angular correlations coefficients  $A_2$  is different from the predictions of the statistical model.

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