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RESULTS OF JINR–SERBIA COOPERATION  
IN THE FIELD OF THEORETICAL STUDIES  
OF JOSEPHSON STRUCTURES

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Результаты сотрудничества ОИЯИ–Сербия в области теоретических исследований джозефсоновских структур

Представлен обзор результатов, полученных в рамках сотрудничества ОИЯИ–Сербия в области теоретического исследования джозефсоновских наноструктур. В частности, работа включает исследования широкого спектра нелинейных динамических эффектов в различных системах связанных джозефсоновских переходов, массивах джозефсоновских переходов и структурах сверхпроводник–ферромагнетик–сверхпроводник, которые в настоящее время интенсивно исследуются в связи с потенциальными областями применения в сверхпроводниковой электронике и спинтронике.

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Results of JINR–Serbia Cooperation in the Field of Theoretical Studies of Josephson Structures

An overview of the results obtained within the framework of the JINR–Serbia collaboration in the field of theoretical study of Josephson nanostructures is presented. In particular, the paper involves studies of a wide variety of nonlinear dynamic effects in various systems of coupled Josephson junctions, arrays of Josephson junctions, and superconductor–ferromagnet–superconductor structures, which are currently being investigated intensively due to potential applications in superconducting electronics and spintronics.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna, 2026

## INTRODUCTION

Scientific cooperation between JINR and the Vinča Institute of Nuclear Sciences (VINS) has a long history, starting in the early 1970s with the collaboration between the scientists from the JINR Bogoliubov Laboratory of Theoretical Physics (BLTP) and the Theoretical Physics Department of VINS. On 20 April 2007, a new stage of closer partnership between JINR and the Republic of Serbia began, when an Agreement was signed between the Joint Institute for Nuclear Research and the Ministry of Science and Environmental Protection of the Republic of Serbia (Fig. 1).

The Roadmap for the development of the cooperation was approved at the 4th session of the Coordinating Committee and officially presented in March 2017 during the events in Belgrade dedicated to the 10th anniversary

**The milestones of the cooperation**



**Protocol on stimulation of the joint work**  
**Dubna, 26 March, 2009**

- ❖ Nomination of JCC
- ❖ First contribution to JINR
- ❖ Future of TESLA project

Regular communication with Serbian embassy in Moscow started

**JCC-1 23 June 2010, Dubna**



**JCC-2 29 March 2011, Belgrade**



**JCC-3 22 April 2014, Belgrade**



Visit of the JINR Directorate to Belgrade, **May 2009**  
Yu. Oganessian gave a lecture on the discovery of superheavy elements in Serbian Academy of Sciences and Arts as its foreign member.

At the "Vinča" Institute of Nuclear Sciences in front of the first ECR ion source of TESLA project



The JINR delegation visiting Mr. B. Đelić, the minister of scientific and technical development of Serbia.



Fig. 1. Important milestones of JINR–Serbia cooperation



Fig. 2. The 10th anniversary of JINR–Serbia cooperation

of the participation of the Republic of Serbia at JINR as an Associate Member (Fig. 2). The main goal of the implementation of the Roadmap was to pave the way for the full membership of Serbia at JINR in 2020, while the participation of Serbian scientists in JINR activities was to be significantly expanded.

In 2025, the State Secretary of the Ministry of Science, Technological Development, and Innovation of the Republic of Serbia introduced Deputy Director of the Vinča Institute of Nuclear Sciences M. Janković as the new coordinator of JINR–Serbia cooperation.

The collaboration on physical phenomena in Josephson structures between the group of Dr. Yu. M. Shukrinov from BLTP JINR and the Serbian scientists started in August 2016 during the visit of Dr. J. Tekić (Laboratory for Theoretical and Condensed Matter Physics, VINS) and Dr. P. Mali (Department of Physics, Faculty of Science, University of Novi Sad) to the Bogoliubov Laboratory of Theoretical Physics. Some years prior to that visit, the work of J. Tekić on the dynamics of the driven Frenkel–Kontorova (FK) model was already attracting the attention of Yu. M. Shukrinov’s group since it was closely related to their work on Josephson junctions (JJs). The FK model provides a good theoretical framework for the description of many phenomena in the systems of JJs, and thus its study can provide important insights into their physics. Since the group of Yu. M. Shukrinov had already been working on the chaotic phenomena in JJs and J. Tekić had a years-long experience

working on the ac-driven FK model with Prof. B. Hu and Prof. O. Braun in Hong Kong (China), the first projects of the collaboration were dedicated to the chaos and dynamics of the ac-driven underdamped FK model and JJs.

That first joint investigation of chaotic behavior in the FK model was the start of a years-long collaboration on the research in fields of nonlinear physics, the physics of JJs, and, recently, time crystals. In the past ten years, this collaboration resulted in many interesting works that are presented here in chronological order.

In Sec. 1, the studies on synchronization, chaos, and hysteresis in the dc + ac driven FK model are presented, where in Subsec. 1.1, we review a comparative study of the overdamped FK model and the 1D stack of intrinsic JJs. That study was further extended in Subsec. 1.2 to the underdamped case where the presence of inertial effects leads to subharmonic mode-locking, chaos, and hysteretic behavior.

The studies of annular Josephson junctions (AJJs) are shown in Sec. 2, where the underdamped dynamics of AJJs under external radiation is investigated in Subsec. 2.1, while the studies on the resonance phenomena and synchronization are presented in Subsec. 2.2.

In Sec. 3, the results of our studies on intrinsic JJs are shown. The manifestation of chaos along the resonance circuit branch of the coupled JJs shunted by resistive, inductive, and capacitive circuit elements is analyzed over wide cross sections of the parameter space.

Chaotic dynamics from coupled magnetic monodomain and Josephson current is demonstrated in Sec. 4.

Unique effects that characterize the dynamics of the JJs with a ferromagnetic interface are presented in Sec. 5. The emergence of Buzdin, Shapiro, and chimera steps is demonstrated in Subsec. 5.1, while the unique properties of Buzdin steps are analyzed in Subsec. 5.2.

In Sec. 6, we show that hybrid JJs can provide a ground for the realization of one of today's most controversial phenomena, time crystals. Namely, the modification of the critical current by the magnetic moment can lead to an intrinsic space-time crystalline order.

## **1. FRENKEL–KONTOROVA MODEL: SYNCHRONIZATION, CHAOS, AND HYSTERESIS**

Externally driven systems exhibit very rich dynamics on both the macroscopic and microscopic levels. One of the models capable of capturing the essence of driven dynamics in these systems is the FK model under external periodic forces [1–4]. The intensive studies of locking and resonance phenomena in charge-density or spin-density wave conductors, vortex lattices, and Josephson-junction arrays biased by external currents have been the main impulse for the theoretical studies of the driven FK model. In this Section, we present our studies on the most interesting phenomena that characterize the dynamics of the FK model and that are particularly relevant for the systems of JJs.

The standard FK model represents a chain of harmonically interacting identical particles subjected to the sinusoidal substrate potential [1, 2]. It can be described by the following Hamiltonian [2]:

$$H = \sum_i \left\{ \frac{1}{2} \left( \frac{du_i}{dt} \right)^2 + \frac{1}{2} (u_{i+1} + u_i - a_0)^2 + \frac{K}{(2\pi)^2} [1 - \cos(2\pi u_i)] \right\}.$$

In this form, the Hamiltonian describes the harmonic chain of interacting particles with unit mass and position  $u_i$ , where  $a_0$  is the equilibrium distance of the interatomic potential, moving in the sinusoidal potential with pinning strength  $K$  and periodicity 1.

When the external dc and ac forces are applied, locking occurs between the frequency of the particles motion over the periodic potential and the frequency of the external ac force [2]. On the macroscopic scale, this effect is characterized by the appearance of a staircase of Shapiro steps [5] in the curve for average velocity as a function of the average external driving force  $\bar{v}(\bar{F})$ . In this staircase, the steps are called harmonic if locking appears at integer multiples of the ac frequency or subharmonic at noninteger rational multiples.

**1.1. Devil's Staircase and the Absence of Chaos in the dc- and ac-Driven Overdamped Frenkel–Kontorova Model.** Motivated by our previous experiences [6, 7] and the numerous theoretical and experimental results in the charge density wave systems and the systems of JJs, our first collaboration project was dedicated to the nonlinear dynamics of the ac-driven overdamped FK model. These systems represent typical examples of dissipative or overdamped physical systems where the inertia is irrelevant on physical grounds and where the long-term behavior is largely independent of how we start up the system.

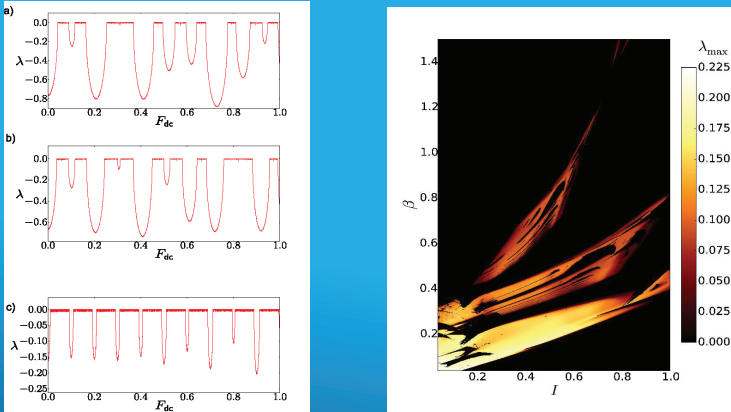
In our paper [8], we have examined in detail the appearance and order of subharmonic Shapiro steps in the overdamped dc+ac driven FK model, particularly, focusing on the signs of chaos. This study is further accompanied by the comparative study of the 1D stacks of intrinsic JJs, irradiated by electromagnetic waves.

The devil's staircase structure arising from the complete mode-locking of an entirely nonchaotic system, the overdamped dc+ac driven FK model with deformable substrate potential, was developed. In Fig. 3, the comparative analysis of chaotic behavior of the largest Lyapunov exponent (LE) in the overdamped FK model and the 1D stacks of intrinsic JJs under external radiation is presented.

The results clearly demonstrate the disappearance of chaotic behavior with increasing dissipation. It is important to point out that the FK model studied here has many degrees of freedom, as do Josephson-junction systems. Thus, the absence of chaos in the overdamped FK model cannot be ascribed to a reduction in the effective dimensionality of the system.

Even though no chaos was found, a hierarchical ordering of the Shapiro steps was made possible through the use of a previously introduced continued

**The comparative analysis on the absence of chaos in  
the dc+ac driven FK model and 1D stack of intrinsic Josephson junctions**



Sokolović I., Mali P., Ođavić J., Radošević S., Medvedeva S.Y., Botha A.E., Shukrinov Y.M., Tekić J.  
**Devil's staircase and the absence of chaos in the dc-and ac-driven  
overdamped Frenkel-Kontorova model**  
Phys. Rev. E. 96, 022210 (2017)

Fig. 3. Demonstration of the absence of chaos in the dc + ac overdamped FK model and the corresponding 1D stack of intrinsic JJs. Left: The results for the FK model: the largest LE as a function of the ac amplitude  $K = 4$ ,  $\nu_0 = 0.2$ ,  $\omega = 12$ , and  $F_{ac} = 1.5$ , 1.8, and 10 in (a), (b), and (c), respectively. Right: Maximum LE as a function of the bias current  $I$  and damping parameter  $\beta$  for the stack of seven junctions. Adopted from [8]

fraction formula. The absence of chaos, deduced here, can be attributed to the overdamped character and the Middleton no-passing rule, which applies on the strictly overdamped systems with convex interatomic interaction, and according to which, the order of particles must be preserved in dynamics. A comparative analysis of a one-dimensional stack of JJs confirmed the disappearance of chaos with increasing dissipation. Other common dynamic features were also identified through this comparison.

**1.2. Inertial Effects in the dc + ac Driven Underdamped Frenkel–Kontorova Model: Subharmonic Steps, Chaos, and Hysteresis.** We further extended our investigations on the inertial effects in the dynamics of the dc + ac driven FK model [9], in particular, on how the mass of particles affects subharmonic steps, chaos, and hysteresis. In our previous work [8] presented in Subsec. 1.1, we have shown that the dc + ac driven overdamped FK model has an interesting property: though entirely nonchaotic, it exhibits the devil's staircase structure arising from the complete mode-locking with the fractal dimension which varies with the system parameters. In this Subsection,

we demonstrate that as the FK model becomes underdamped, new phenomena such as the introduction of new subharmonic steps, the appearance of chaos, and hysteresis, initially nonexistent in the overdamped regime, will start to appear [9]. As the mass increased and the systems transferred from the overdamped to underdamped limit, one of the immediate effects was the emergence of a series of subharmonic steps in the staircase of the average velocity as a function of average driving force in commensurate structures. At certain values of parameters, the subharmonic steps became separated by chaotic windows, while the whole structure retained scaling similar to the original staircase. In Fig. 4, the Lyapunov exponents for the region between the zero and the first harmonic are presented, where more than one positive LE clearly indicates the presence of hyperchaos.

The mass of the particles also determined their sensitivity to the forces governing their dynamics. Depending on their mass, they exhibit three types of dynamics, from dynamical mode-locking with chaotic windows, through to a typical dc response, to essentially a free-particle response. Our analysis of these dynamics in both the upforce and downforce directions showed that

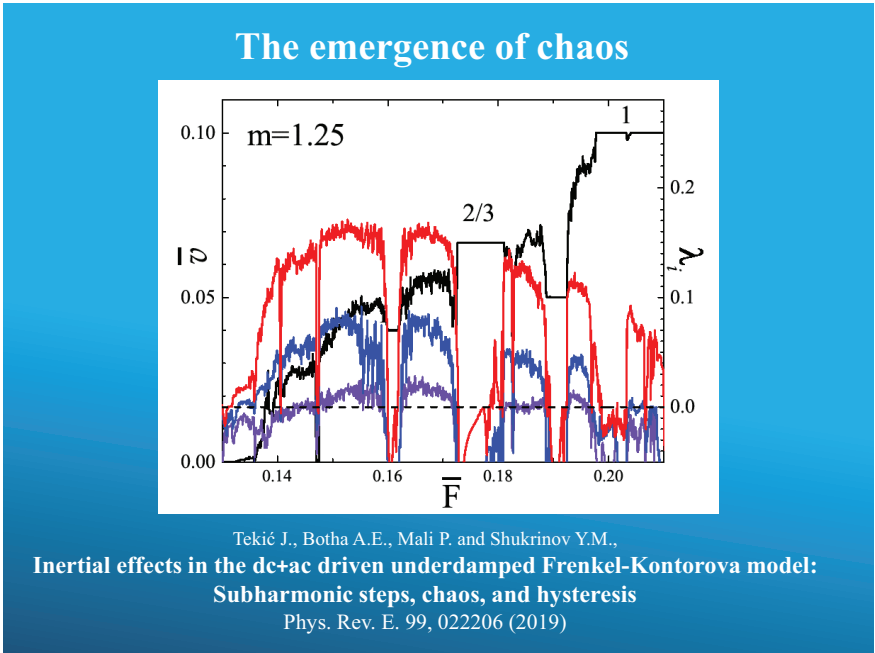


Fig. 4. Demonstration of chaos in the underdamped dc+ac driven FK model. The Lyapunov exponents  $\lambda_i$  as a function of the average driving force  $\bar{F}$  and the corresponding response function  $\bar{v}(\bar{F})$  in the region between the zero and the first harmonic step for  $m = 1.25$  are presented. Different colors of  $\lambda_i$  correspond to different Lyapunov exponents. Adopted from [9]

the system may not only exhibit hysteresis, but also demonstrates that large Shapiro steps may appear in the downforce direction, even in cases for which no dynamical mode-locking occurred in the upforce direction [9].

Our results were presented at the 13th Chaotic Modeling and Simulation International Conference in 2021 [10].

## 2. PHYSICS AND APPLICATIONS OF ANNULAR JOSEPHSON JUNCTIONS

From the fundamental point of view, JJs are excellent devices for studies of nonlinear dynamics of discrete systems, as they represent an experimental realization of the FK model (discrete sine-Gordon model) [1, 2, 11]. At the same time, regarding their applications, JJs are promising devices for the development of various fields, from the generation and detection of electromagnetic radiations in the very low THz range, quantum information technologies [12–15], and superconducting metamaterials [16] to the fields as distant as biology [17].

The idea that a fluxon behaves as a particle-like solitary wave, which can be manipulated and controlled, motivated the creation of a logic circuit by using Josephson fluxon as elementary bits of information [12, 13]. In the creation of new logic elements, particularly important are the long JJs [11] described by a continuous sine-Gordon equation and the Josephson-junction parallel array by its discrete counterpart, i.e., the FK model [1, 2, 11]. However, in long JJs, the motion of fluxon strongly depends on the geometry and boundaries of the junctions, which makes studies of fluxon dynamics very challenging. These problems led to the creation of annular Josephson junctions [18] as ideal systems for the studies of fluxon dynamics, which provide an undisturbed and tunable fluxon motion [19–21].

Here we present the results of our investigation on AJJs. We demonstrate that in an annular array of underdamped JJs under the external radiation, not only the number but also the type of rotating excitations (fluxons or antifluxons) determined the ability of the system to lock with the external radiation. The resonance phenomena in the absence of external radiation have been analyzed across a wide range of currents and voltages for different numbers and types of excitations present in the system.

**2.1. ac-Driven Annular Josephson Junctions: The Missing Shapiro Steps.** One of the most interesting properties of Josephson-junction systems is their ability to exhibit various resonance phenomena. In the absence of any external radiation, the so-called zero-field steps (ZFSs) [22] appear in the current–voltage ( $I$ – $V$ ) characteristic due to resonant motion of fluxons and antifluxons inside the system. If, on the other hand, some external radiation is applied, the  $I$ – $V$  characteristic exhibits the well-known Shapiro steps [5] as a result of the locking with the external frequency. Though the Shapiro steps are today one of the most recognized frequency locking phenomena associated with a wide variety of physical systems, the majority of the works [20, 21, 24]

devoted to AJJs have been focused on the resonance phenomena in the absence of external radiation.

In the study [23], we examined the underdamped dynamics of an annular array of Josephson junctions (AAJJs) under the external radiation. In contrast to previous studies of AJJs, which were mainly focused on the case of one trapped fluxon in a small range of currents and voltages [21, 24], here we considered the Shapiro steps in various cases of circulating excitations (fluxons and antfluxons), in a wide range of currents and voltages in order to get the full picture of dynamical behavior. Surprisingly, our results showed that the ability of the system to lock with some external radiation depends not only on the number but also on the type of excitations.

In Fig. 5, we show the  $I-V$  characteristics of the AAJJ with one trapped fluxon ( $M = 1$ ). Since, in addition to the one trapped fluxon, the additional excitations appear only in the form of fluxon-antifluxon pairs, the system exhibits ZFSs for  $n = 1, 3, 5,$  and  $7$ . In this case, for the applied frequency

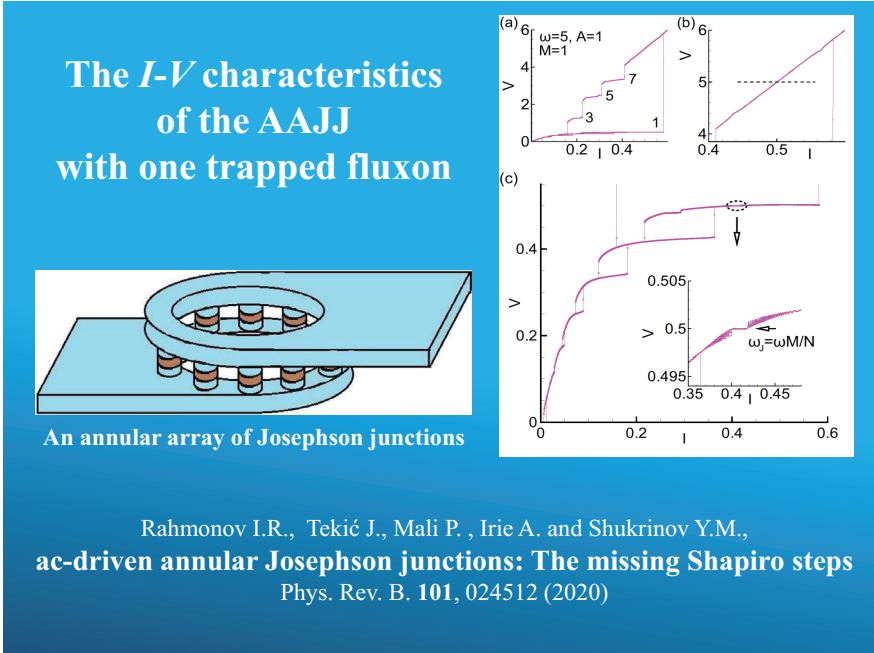


Fig. 5. Absence of the Shapiro steps in the  $I-V$  characteristics of the AAJJ with one trapped fluxon. *a)* The  $I-V$  characteristic of the AAJJs for  $M = 1$ , with the amplitude and frequency of external radiation  $A = 1$  and  $\omega = 5$ , respectively. Numbers 1, 3, 5, and 7 mark the total number of fluxons and antfluxons  $n$ . *b)* The absence of the Shapiro step in the  $I-V$  curve. Dashed line marks where the step should be. *c)* High-resolution plot of the step  $n = 1$ , which exhibits the Shapiro step shown in the inset. Adopted from [23]

of the external radiation  $\omega = 5$ , we would expect to see in Fig. 5, *a* the Shapiro step at  $V = 5$ , as well as other subharmonic steps in the  $I-V$  characteristic. However, as we can see in Fig. 5, *b*, the Shapiro step is absent.

Shapiro steps were observed in the  $I-V$  characteristic only in the system with trapped fluxons or in the system with fluxon–antifluxon pairs. If the trapped fluxons circulate simultaneously with fluxon–antifluxon pairs, there are no Shapiro steps regardless of the amplitude or frequency of the applied external radiation.

**2.2. Resonance Phenomena in an Annular Array of Underdamped Josephson Junctions.** Resonance phenomena in Josephson-junction systems have been an active research topic in science and technology for years. In [25], we have studied the appearance and origin of resonance phenomena in an annular system of underdamped JJs. If no fluxon was trapped in the system, the dynamics was governed by the motion of fluxon–antifluxon pairs. If, on the other hand, trapped fluxons were present, in addition to their motion, the system could also exhibit the simultaneous motion of trapped fluxons and fluxon–antifluxon pairs. Locking between the rotating excitations (fluxons and antifluxons) and the Josephson frequency led to the appearance of ZFSs in the current–voltage characteristics, whose number was determined by the number

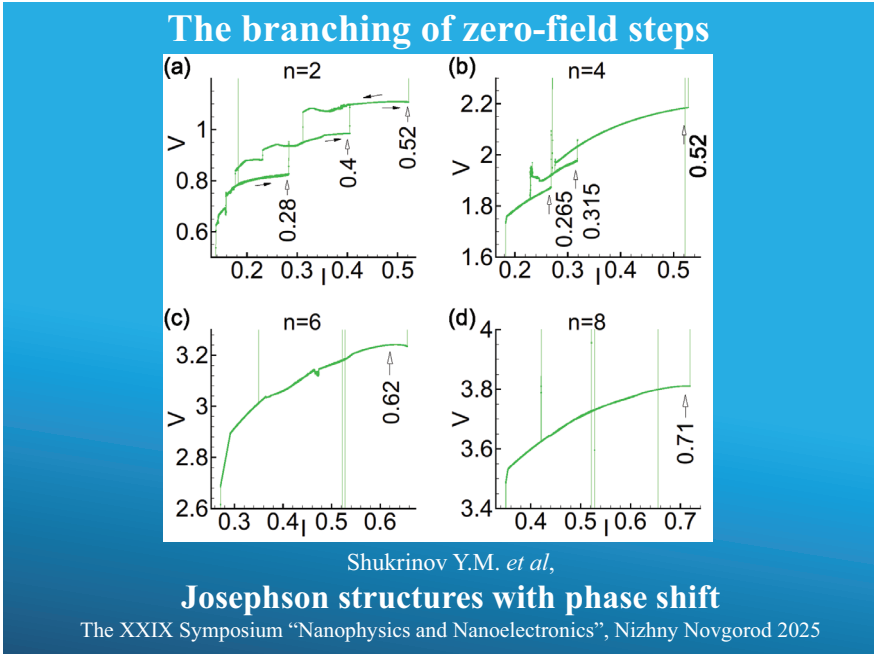


Fig. 6. Branching of the zero-field steps in the AAJJ. High-resolution plot for the ZFSs for  $n = 2, 4, 6$ , and  $8$ , in (a)–(d), respectively. Adopted from [25]

of junctions and the total number of excitations present in the system. The obtained results showed that the branching of ZFSs due to resonance between the rotating excitations and plasma oscillations in their tails appeared only at the lower steps and completely disappeared at the higher steps, as shown in Fig. 6.

A comparative analysis between systems without and those with trapped fluxons showed a correlation between their current–voltage characteristics. From a high-resolution analysis some special features of ZFSs emerge, such as an additional branch due to resonance between the pulsating fluxon and the Josephson frequency. Examination of systems with the same number but different types of excitations further revealed that their dynamics was determined not only by the number, but also by the type of excitations, i.e., systems with the same number but different types of excitations had different  $I$ – $V$  characteristics.

The results of these studies were also presented at the two conferences: The XXII and the XXIV International Symposia “Nanophysics and Nanoelectronics”, Nizhny Novgorod, in 2018 [26] and 2020 [27], respectively.

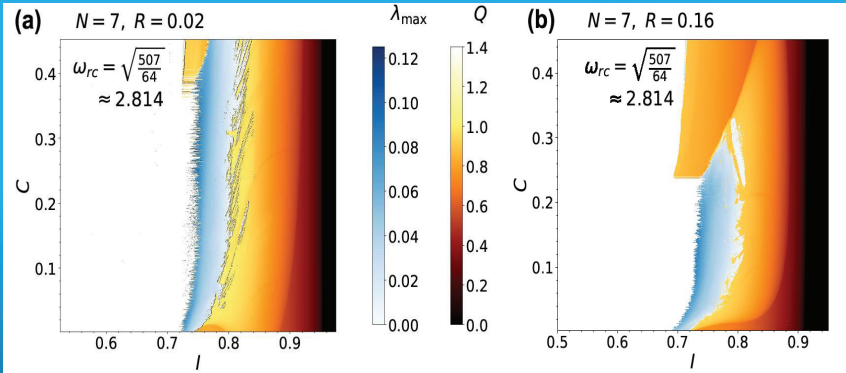
### 3. CHAOS IN THE SYSTEM OF INTRINSIC JOSEPHSON JUNCTIONS

Although single Josephson-junction chaos has been extensively studied, chaos and its control in systems of coupled JJs remain an active area of research. Interest in Josephson-junction chaos started in the early 1980s with the work by B. Huberman et al. [28], who connected the experimentally observed “noise-rise phenomenon” to the deterministic chaos seen in the numerical simulations of the rf-biased single Josephson junction. With the view of practical applications, such as the voltage standard [29], chaos and chaos control in the Josephson junction became an important topic of investigation.

Recently, several different forms of chaos control have also been proposed, not only with the view of suppressing chaos, but also to establish more robust synchronization between the junctions. The JJs could be either in a chaotic state, for applications involving chaos synchronization, or in a regular state, where the junctions might be used, for example, to detect weak electromagnetic signals in the presence of noise, or to unscramble specific types of noise components within a noisy signal [30]. Synchronized JJs in high-temperature superconductors are promising candidates for sufficiently light and compact THz wave generators. While the radiation from a single Josephson junction is extremely weak, systems of synchronized JJs possess practically viable radiation power.

Here we present the results from our work [30] on the appearance of chaos along the resonant branch of a shunted system of intrinsic JJs in a high-temperature superconductor. Detailed analysis of the  $I$ – $V$  characteristics, the electric charge density, and the full spectrum of Lyapunov exponents was performed for two different cases: when the shunting parameters were changed

## The effect of rc-frequency on chaotic behavior



Botha A. E., Shukrinov Yu. M., and Tekić J.

**Chaos along the rc-branch of RLC-shunted intrinsic Josephson junctions**  
 Chaos Soliton Frac. **156**, 111865 (2022)

Fig. 7. Chaos along the resonant branch of a shunted system of RLC-shunted intrinsic JJs. Color renditions of the maximum LE and maximum charge density  $Q$  along the rc-branch as functions of capacitance  $C$  and decreasing dc-bias current  $I$  for  $R = 0.02$  (a) and for  $R = 0.16$  (b). The blue color scale for  $\lambda_{\max}$  has been superimposed over the charge density, with all  $\lambda_{\max} < 0.01$  being rendered transparent. Inductance  $L$  is varied simultaneously with  $C$ , so as to keep  $\omega_{rc} = 2.814$ . Adopted from [30]

so as to maintain a fixed value of the rc-frequency, and when changes in the shunting parameters resulted in different rc-frequencies. Color images of the maximum LE and maximum charge density along the resonant branch as functions of capacitance and decreasing dc-bias current are presented in Fig. 7.

Based on the calculated electrical characteristics of the superconducting layers and various indicators of chaos, such as Lyapunov exponents and Poincaré sections, the regions of current–voltage characteristics with a predominance of chaos, which are determined by the frequency of the resonant circuit, are indicated. The study of metric entropy and the maximum Kaplan–Yorke dimension shows that the sizes of chaotic attractors associated with chaos do not reach a plateau, as in the case of systems with strong damping, but increase without limit with an increase in the number of JJs, demonstrating multidimensional chaos. The results obtained indicate the possibility of controlling chaos in the studied systems.

#### 4. CHAOTIC DYNAMICS FROM COUPLED MAGNETIC MONODOMAIN AND JOSEPHSON CURRENT

The possibility of achieving electric control over the magnetic properties of the magnet via Josephson current and its counterpart, i.e., achieving magnetic control over Josephson current, recently attracted much attention [31–38].

The ordinary (superconductor–insulator–superconductor) Josephson junction cannot exhibit chaos in the absence of an external ac-drive, whereas in the superconductor–ferromagnet–superconductor (SFS) Josephson junction, known as the  $\varphi_0$  junction, the magnetic layer effectively provides two extra degrees of freedom that can facilitate chaotic dynamics in the resulting four-dimensional autonomous system.

Here we present the most interesting findings from our studies [39] on the chaotic dynamics of the Josephson  $\varphi_0$  junction. The results show that, due

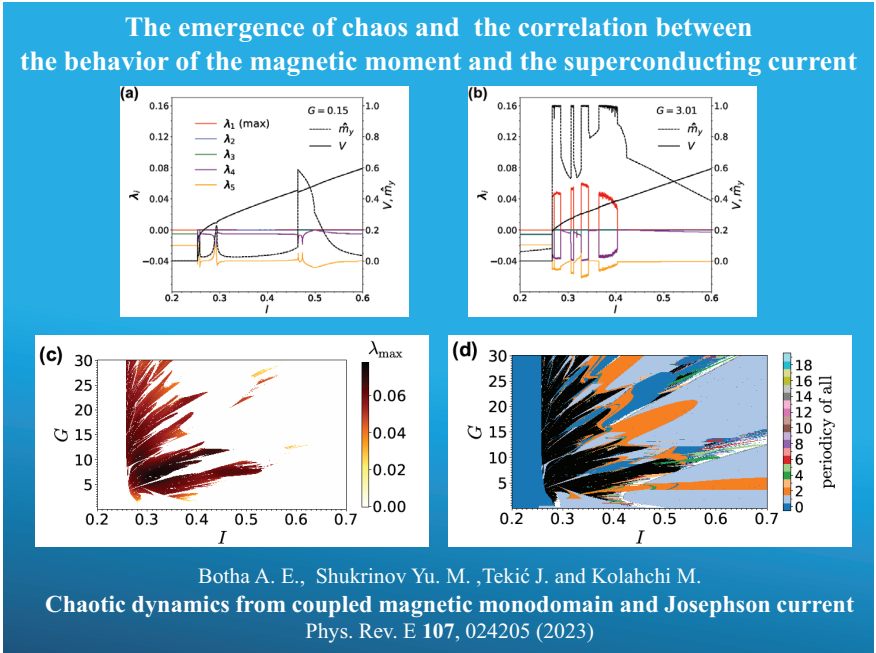


Fig. 8. Chaos, periodicity, and synchronization between the magnetic moment and the superconducting current in the Josephson  $\varphi_0$  junction. *a, b*) The emergence of chaos in the Josephson  $\varphi_0$  junction. Here  $V$  is the time-averaged voltage across the junction, and  $\hat{m}_y$  is the maximum value of the  $y$ -component of the magnetization.  $\lambda_i$  are the Lyapunov exponents. *c*) The chaotic regions as functions of the ratio of Josephson energy to the magnetic energy,  $G$ . *d*) Demonstration of the periodicity and synchronization in the Josephson  $\varphi_0$  junction. Here the regions marked by the blue color corresponding to zero indicate the regions where not all of the state variables have the same periodicity, i.e., are not fully synchronized. Adopted from [39]

to the conservation of magnetic moment magnitude, two of the numerically computed full spectrum Lyapunov characteristic exponents are always zero. The two-dimensional bifurcation diagrams, which are similar to traditional isospike diagrams, display the different periodicities and synchronization properties of the junction over parameter ranges that are experimentally accessible. It was demonstrated that as the bias current  $I$  is reduced, the onset of chaos occurs shortly before the transition to the superconducting state.

In Fig. 8, *a* and *b*, we illustrate the emergence of chaos in the Josephson  $\varphi_0$  junction with increasing ratio  $G$ , of the Josephson energy to the magnetic energy. The full spectrum of Lyapunov exponents can be seen, with two being trivially zero for any  $I$ . In Fig. 8, *c*, we show the regions of chaos, as indicated by positive values of the maximum LE, as functions of  $G$  and the decreasing dc-bias  $I$ . Figure 8, *d* demonstrates the periodicity and synchronization in the Josephson  $\varphi_0$  junction.

To gain insight into this behavior, we apply a recurrence plot (RP) analysis, which is an advanced technique for nonlinear data analysis, used to visualize how often and how close the trajectory of the dynamical system revisits areas in the phase space [40]. In its simplest form, it is a visualization

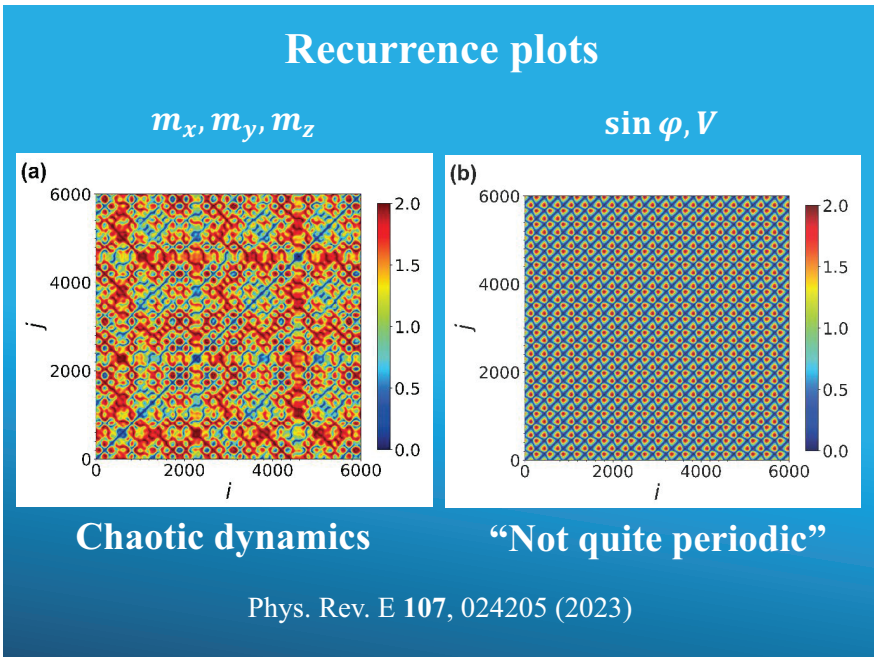


Fig. 9. Demonstration of the chaotic dynamics of the magnetic monodomain and the more regular structure for the junction coordinates using RP technique. Recurrence plots for the magnetic system coordinates ( $m_x, m_y, m_z$ ) (*a*) and the junction coordinates ( $\sin \varphi, V$ ) (*b*). Adopted from [39]

of a square matrix, in which the column and row indices correspond to a certain pair of equally spaced sample times from the system trajectory. In Fig. 9, the recurrence plots of the magnetic system coordinates and junction coordinates are presented.

Our work on the chaotic behavior of the Josephson  $\varphi_0$  junction was also presented at the two International Conferences. In 2023, our study on the nonlinearity, locking, and chaos in anomalous JJs was demonstrated at the International Symposium in Nizhny Novgorod [41]. The paper “Chaos in the  $\varphi_0$  SFS Josephson junction” was presented at the 14th Chaotic Modeling and Simulation International Conference in 2023 [42].

## **5. RESONANCE AND LOCKING IN THE ANOMALOUS JOSEPHSON JUNCTIONS**

The coexistence and mutual influence of superconductivity and magnetism for a long time remains one of the most pressing problems in condensed matter physics [31, 32]. The possibility to combine superconductivity and magnetism in hybrid Josephson structures holds promise to increase the technological applications of superconductors and superconducting nanostructures in the recent rapid development of spintronics and superconducting logic devices [43–46]. Currently, intense activity is focused on identifying combinations of materials and types of superconductor–ferromagnet (SF) structures that enhance device functionality and performance, leading to progress in superconducting spintronics and quantum computation [47–50].

One particular type of anomalous JJs that demonstrates transport properties with disrupting scientific and technological potential is the Josephson SFS  $\varphi_0$  junction. It belongs to a special class of anomalous JJs with a noncentrosymmetric ferromagnetic layer and broken time-reversal symmetry. These properties result in the occurrence of an additional phase shift  $\varphi_0$  proportional to the magnetic moment [36, 51], and the current-phase relation becomes  $I = I_c \sin(\varphi - \varphi_0)$ , where  $I_c$  is the critical current and  $\varphi$  is the superconducting phase difference.

In general, in the studies of Josephson-junction systems driven by external radiation, the influence of the magnetic component of radiation is usually neglected, and the description of the effect is reduced to adding the periodic signal to the bias current. However, contrary to ordinary superconductor–insulator–superconductor (SIS-type) junctions, in the Josephson  $\varphi_0$  junction, the microwave magnetic field generates an additional magnetic precession with the microwave frequency, which might lead to a series of unusual effects.

An interesting aspect of this problem was manifested in the physics of the anomalous Josephson junction, where a direct coupling between the Josephson phase and magnetization is realized. This leads to the Buzdin ferromagnetic resonance with unique properties, various stable magnetic trajectories, and the manifestation of nonlinear properties of the Duffing oscillator. Interesting prospects are opening up in the field of superconducting

spintronics, in particular, based on the reversal of the magnetic moment by a superconducting current, on the manifestation of the properties of the Kapitza pendulum. External radiation leads to the appearance of additional synchronizations in the system (Buzdin and chimera steps), the properties of which depend on the periodic signal and the dynamics of the Josephson junction.

A brief review of our main results of the study of Josephson SFS nanostructures and structures with a nanomagnet was presented at the IX International Scientific Conference “Modern Problems of Physics”, which took place in Dushanbe in 2024 [53]. The dynamics of magnetization in the region of ferromagnetic resonance, the manifestation of bifurcations and hysteresis in the region of magnetic precession synchronization, as well as application in the field of superconductor electronics and spintronics were discussed.

Besides the ferromagnetic resonance and locking, there is an additional effect that the microwave magnetic field may have on the system. Namely, its direct influence on the magnetic moment of the ferromagnetic layer leads to the Kittel ferromagnetic resonance. In Fig. 10, the combination of Buzdin and Kittel ferromagnetic resonances (FMRs) in the  $\varphi_0$  junction with

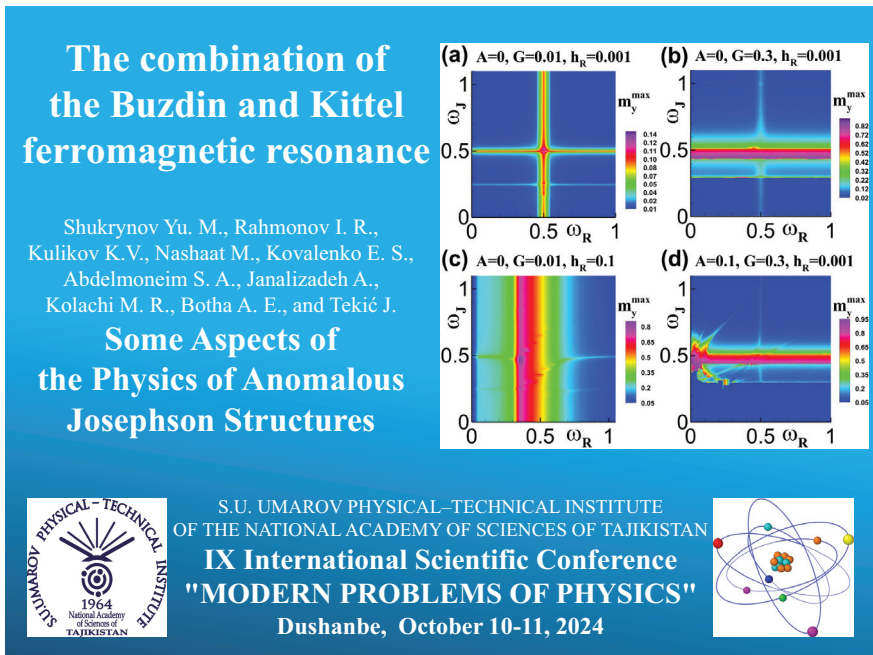


Fig. 10. The competition of the Kittel and Buzdin FMRs at  $r = 0.2$  and different model parameters: *a*) with varying  $\omega_J$  and  $\omega_F$ , respectively; *b*) domination of Buzdin resonance at  $G = 0.3$ ; *c*) domination of Kittel resonance at  $h_R = 0.1$ ; *d*) the effect of both electromagnetic wave components. Adopted from [52]

different types of synchronization, clearly expressed in the dynamics and in the current–voltage characteristics, makes the physics of this system very interesting and opens up a number of new applications.

In the same year, our results on this topic were also presented at the XXVIII International Symposium “Nanophysics and Nanoelectronics” in Nizhny Novgorod [52] and also at the XXXIX International Conference “Meeting on Low Temperature Physics” (FNT-2024) in Chernogolovka [54].

### 5.1. Buzdin, Shapiro, and Chimera Steps in Josephson $\varphi_0$ Junctions.

In this Subsection, we present the results from our investigations [55] of the effects of microwave radiation on the dynamics of the Josephson SFS  $\varphi_0$  junction. For the first time, when not just the electric, but also the magnetic component of external radiation was taken into account, two different mechanisms of locking the Josephson oscillations and the ferromagnetic moment precessions were demonstrated.

Due to the coupling of superconductivity and magnetism in this system, the magnetic moment precession of the ferromagnetic layer caused by the magnetic component of external radiation can lock the Josephson oscillations, which results in the appearance of a particular type of step in the current–

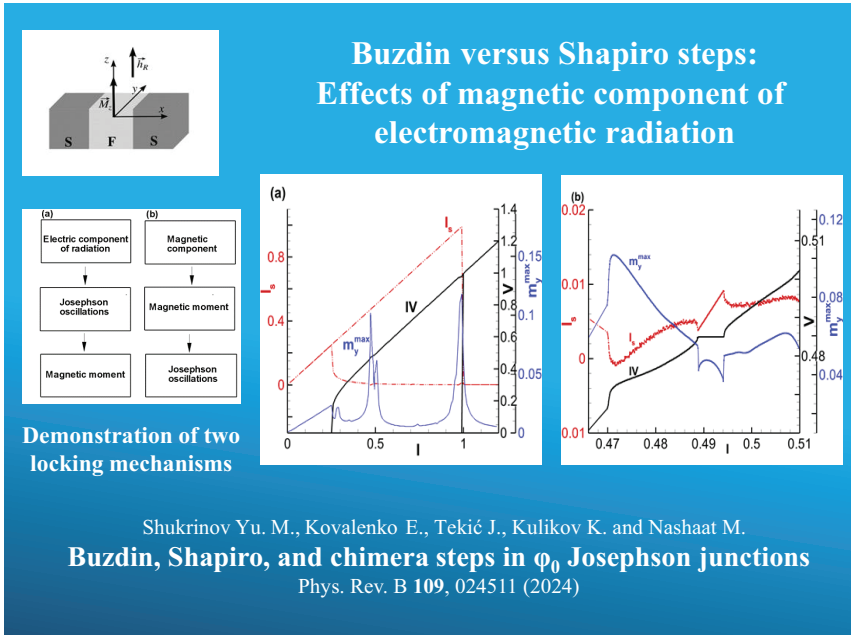


Fig. 11. The manifestation of the Buzdin step in the Josephson  $\varphi_0$  junction. *a*) The average voltage  $V$ , the maximum value of the magnetic component  $m_y^{\max}$ , and the superconducting current  $I_s$  as functions of decreasing biased current  $I$ . *b*) Magnified view of *a*) showing the Buzdin step. Adopted from [55]

voltage characteristics, completely different from the well-known Shapiro steps. We have called these steps the “Buzdin steps” in the case when the system is driven only by the magnetic component and the “chimera steps” in the case when both magnetic and electric components are present.

In Fig. 11, the average voltage, the maximum value of magnetic component, and the superconducting current as functions of decreasing biased current are presented.

The effects of both radiation components are demonstrated in Fig. 12. Unlike the Shapiro steps where the magnetic moment remains constant along the step, here it changes though the system is locked. The spin-orbit coupling substantially contributes to the amplitude, i.e., the size of these steps. Dramatic changes in their amplitudes are also observed at frequencies near the ferromagnetic resonance. Combinations of the Buzdin and Kittel FMRs together with different types of locking pronounced in dynamics and  $I-V$  characteristics make the physics of this system very interesting and open up a series of new applications.

In 2025, we presented our work on physical phenomena in Josephson structures with a phase shift at the XXIX International Symposium “Nanophysics and Nanoelectronics” in Nizhny Novgorod [56]. In particular, we discussed the resonant control of magnetization in a shunted  $\varphi_0$  junction and demonstrated the possibility of effective influence on the resonant properties of the Josephson junction by changing the shunt parameters, the magnitude of magnetization, and controlling its dynamics.

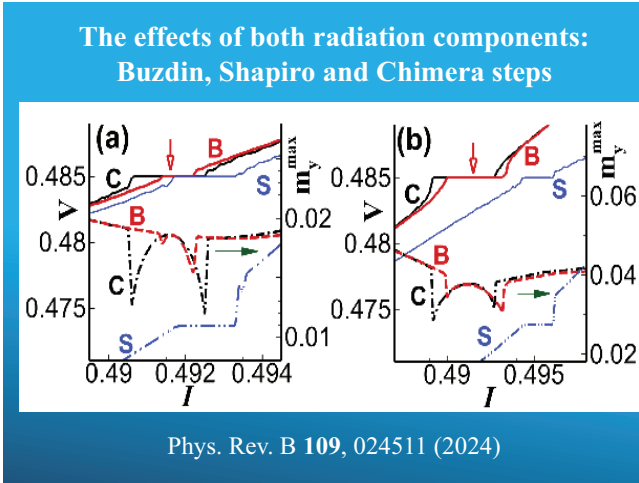
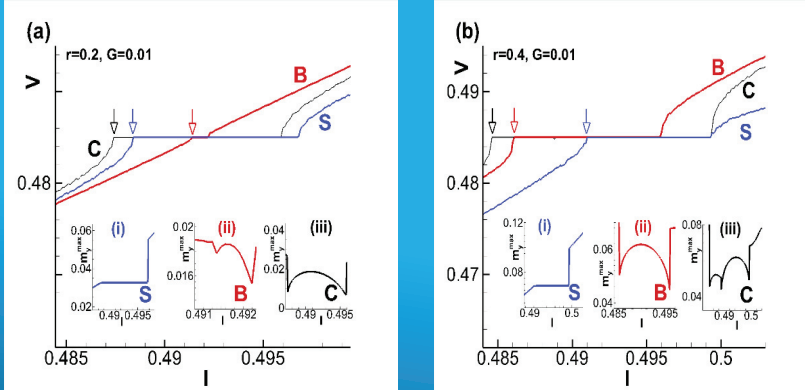


Fig. 12. Demonstration of Buzdin, Shapiro and chimera steps. *a*) Parts of the  $I-V$  characteristics with the Buzdin (label B), Shapiro (label S), and composite (label C) steps at the value of the spin-orbit coupling parameter  $r = 0.2$ . *b*) The same at  $r = 0.4$ . Adopted from [55]

## The interplay of the electric and magnetic components of the external electromagnetic radiation on the constant voltage step



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### Josephson structures with phase shift

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Fig. 13. The effects of both components of the external radiation on the  $I$ – $V$  characteristics. *a*) Shapiro step (blue curve with amplitude of external radiation  $A = 0.05$ ), Buzdin step (red curve with amplitude of magnetic component  $h_R = 1$ ), and chimera step (black curve with  $A = 0.05, h_R = 1$ ) at  $r = 0.2$ . *b*) The same as in (*a*), but with  $r = 0.4$  and  $h_R = 1.7$ . The insets show the corresponding  $m_y^{\max}$  behavior. Adopted from [56]

The interplay of the electric and magnetic components of the external electromagnetic radiation on the constant voltage step in the Josephson  $\varphi_0$  junction is demonstrated in Fig. 13, where we can clearly see the manifestation of all three steps: Shapiro, Buzdin, and chimera ones. The corresponding  $m_y^{\max}$  shows that in the case of Shapiro steps as the electric component of radiation locks the Josephson oscillations, it also locks the precession of the magnetic moment of the ferromagnetic barrier (see inset i) due to the spin-orbit coupling. However, in the case of the Buzdin step, the corresponding  $m_y^{\max}$  demonstrates a bubble-like structure along the step (see inset ii). Similar to the Buzdin step, the corresponding  $m_y^{\max}$  for the chimera step also exhibits a bubble-like structure along the step (see inset iii).

**5.2. Anomalous Josephson Junctions: Specific Features of Buzdin Steps.** To gain a better understanding of this new form of locking, we further focus our research only on Buzdin steps. In this Subsection, we present the results from paper [57], where we have demonstrated the distinctive properties of these steps, their unique origins, and locking mechanisms.

The width of the Buzdin step oscillates with the amplitude of the magnetic component; nevertheless, it exhibits anomalies in the Bessel-like behavior. Additionally, we have performed an analytical analysis that supports the numerical results and shows that the width of the Buzdin step represents a product of two Bessel functions. The comparison between the numerical and analytical results for different values of spin-orbit coupling  $r$  and Gilbert damping  $\alpha$  is presented in Fig. 14, *a* and *b*, respectively.

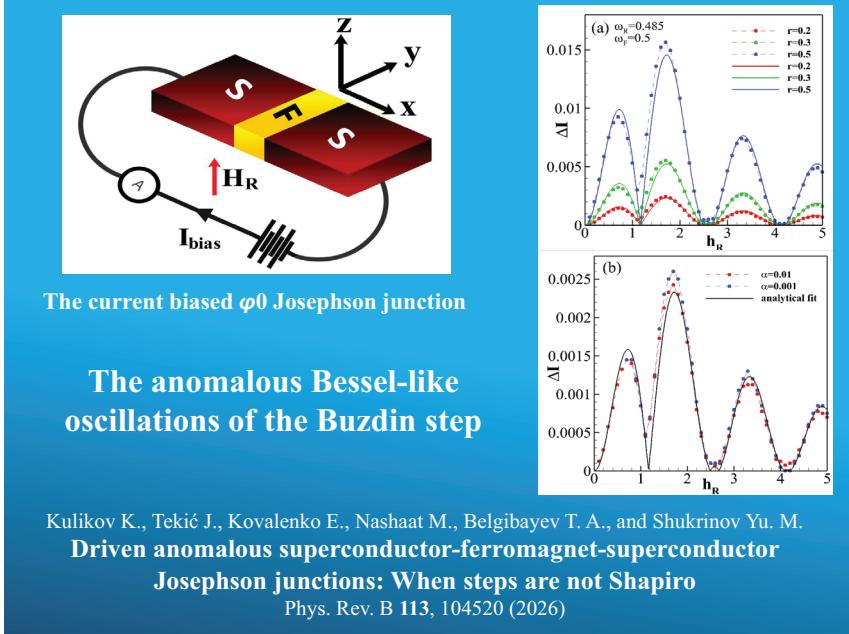
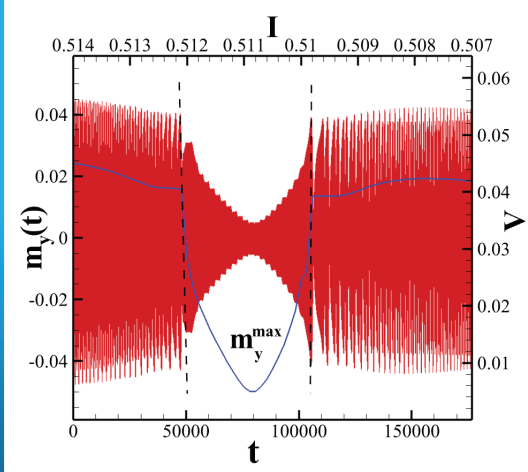


Fig. 14. Demonstration of the unique properties of Buzdin steps. The comparison between the numerical and analytical results for the size of the Buzdin step  $\Delta I$  as a function of the amplitude of the magnetic component  $h_R$  at  $\omega_J = \omega_R \approx \omega_F$  for different values of  $r$  and  $\alpha = 0.01$  (a) and different values of  $\alpha$  and  $r = 0.2$  (b). Dotted (solid) lines correspond to the numerical (analytical) results, respectively. Adopted from [57]

Investigation of the effects that simultaneously appear in the magnetic subsystem reveals the presence of destructive interference and magnetization reorientation that accompany the occurrence of Buzdin steps. Figure 15 shows the correspondence between the dip area in the  $m_y^{\text{max}}(I)$  dependence and the area of significant reduction of the  $m_y$  oscillation amplitude. This interesting phenomena might be a subject for detailed investigations in the future.

The results of this research were also presented in 2025 at the International Workshop “Superconducting and Magnetic Hybrid Structures” at BLTP JINR [58].

## Destructive interference: An abrupt drop in the amplitude of the magnetization precession



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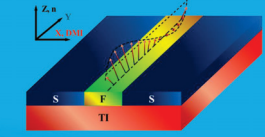
Fig. 15. Demonstration of the sudden drop of the amplitude of magnetic precession. Time dependence of the  $m_y$  component (red line) together with  $m_y^{\max}$  as a function of bias current  $I$  (blue line) at  $G = 0.01$ ,  $\alpha = 0.01$ ,  $r = 0.2$ ,  $\omega_F = 0.5$ ,  $h_R = 1$ ,  $\omega_R = 0.505$ . Adopted from [57]

## 6. INTRINSIC SPACE-TIME CRYSTALLINE ORDER IN A HYBRID JOSEPHSON JUNCTION

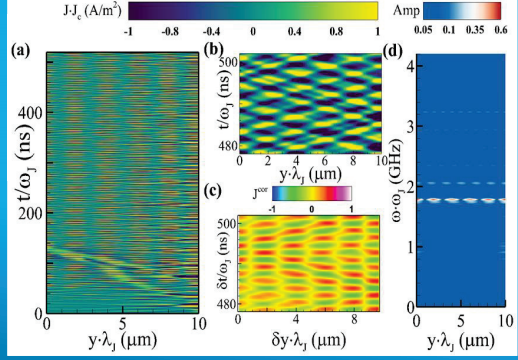
In addition to all previously mentioned possible applications of Josephson  $\varphi_0$  junctions, these systems could also play a significant role in one of the most controversial fields in physics today, time crystals. Since their first proposal by F. Wilczek [59, 60], time crystals (TC) have been causing a great stir in the scientific community. After the initial controversies on their existence [61, 62], they are now considered as a nonequilibrium phase of matter that can operate in the time dimension in a manner akin to the standard crystals in the space dimension. The defining characteristic of these systems is the breaking of discrete or continuous time translation symmetry, leading to a self-sustained and robust time-periodic order.

The published studies on time crystals as well as our knowledge on hybrid JJs, led us to the idea that certain types of these systems could be good candidates for the realization of TC, and in paper [63], we extended our studies to this new interesting field. We demonstrated the emergence of an intrinsic space-time crystalline (STC) order in a long ferromagnetic

## Breaking of the continuous time translational symmetry, and the emergence of space-time crystalline order in the inplane current



Long  $\varphi_0$  Josephson junction on the top of the 3D topological insulator



Nashaat M., Tekić J., and Shukrinov Yu. M.

### Intrinsic space-time crystalline order in a hybrid Josephson junction

Phys. Rev. B 112, 184507 (2025)

Fig. 16. Emergence of the STC order along the Josephson  $\varphi_0$  junction placed on a topological insulator (TI). *a*) Spatio-temporal dependence of the in-plane current  $J(y, t)$  in the Josephson SFS-TI junction for  $c_{\text{ex}} = 0.05$ ,  $D_1 = 1.1$ , and  $D_2 = 0.8$ . *b*) Magnified view of the part in *a*) that demonstrates the STC pattern. *c*) The corresponding spatio-temporal averaged current-current correlation function  $J^{\text{cor}}(\delta y, \delta t)$ . *d*) The Fast-Fourier transform for  $J(y, t)$  shown in *a*). The parameters were fixed to the following values:  $\Omega_F = 1$ ,  $\alpha_g = 0.05$ ,  $k_{\text{an}} = 0.5$ ,  $r = 0.9$ ,  $G = 0.1$ ,  $\beta = 0.00024$ ,  $\tilde{d} = 0.3$ ,  $\Gamma = 0.0762$ , and  $J_{\text{noise}}$  of  $10^{-4}$ . Adopted from [63]

Josephson  $\varphi_0$  junction on a topological insulator without any external periodic modulation (Fig. 16).

The presence of the exchange and Dzyaloshinskii–Moriya interactions in a ferromagnetic layer with broken inversion symmetry internally modulates the critical current due to the coupling between the magnetic moment and the Josephson phase. This breaks the time translation symmetry, leading to the appearance of the STC pattern in the spatio-temporal dependence of the in-plane current, which oscillates with almost twice the ferromagnetic resonance frequency.

In the limit, where the critical current is not modulated internally, the space-time crystalline order does not occur. In this case, only when an external parametric modulation is applied, the system exhibits a typical classical discrete STC order that oscillates at half of the modulation frequency.

Considering the still-pending problem of experimental detection, we demonstrate that a recently developed magnetometry device, which visualizes the supercurrent flow at the nanoscale, can be used to detect STC patterns in hybrid JJs experimentally.

Our results on TC in hybrid JJs were presented at several conferences. In 2025, they were presented at the International Workshop “Superconducting and Magnetic Hybrid Structures” at BLTP JINR in Russia [64] and at the Time Crystals Conference in Cetraro (Italy) [65]. In 2026, they were presented at the South African–German WE-Heraeus Seminar “Nonlinear Dynamics and Anomalous Transport in Low Dimension” in Cape Town (South Africa) [66].

## 7. CONCLUSION AND FUTURE OUTLOOK

On 27 March 2025, one of the State Secretaries of the Ministry of Science, Technological Development, and Innovation of the Republic of Serbia, M. Trajanović, met with the JINR Directorate. The delegation of the Republic of Serbia also included Senior Advisor in the Ministry S. Bogdanović and Deputy Director of the Vinča Institute of Nuclear Sciences M. Janković. JINR was represented by the Institute’s Director G. V. Trubnikov, Vice-Director L. Kostov, Chief Engineer of the Institute B. N. Gikal, JINR University Centre Director D. V. Kamanin, and Head of the International Cooperation Department O.-A. Culicov. The parties discussed prospects for JINR–Serbia cooperation and outlined next steps for implementing joint initiatives and projects (Fig. 17).



Fig. 17. The visit of the delegation of the Serbian Ministry of Science, Technological Development, and Innovation to JINR

Over the past 10 years, our joint work has focused on solving fundamental problems in superconducting electronics and spintronics, in particular, developing fundamentally new methods for controlling magnetization in anomalous Josephson structures. A number of new effects with important practical applications have been predicted, such as the manifestation of Kapitza pendulum properties in an anomalous Josephson junction and the indirect capture of magnetic precession by Josephson oscillations under the influence of an external periodic signal.

We have studied synchronization, chaos, and hysteresis in the FK model controlled by direct and alternating currents, as well as a one-dimensional system of intrinsic JJs. We have examined the role of dissipation in the generation of chaos and have shown that a ladder-like structure (devil's staircase) in the  $I-V$  characteristic could arise even in the absence of chaos. The study of the overdamped FK model was further extended to the underdamped case, where we showed that the presence of inertial effects led to subharmonic mode-locking, chaos, and hysteretic behavior.

Extensive studies of annular JJs were conducted, revealing damped AJJ dynamics under the influence of external radiation. The results of the study of resonance phenomena and locking in AJJs were presented at various international conferences.

The emergence of chaos along the resonant branch of the  $I-V$  characteristic was demonstrated in a system of coupled JJs shunted by resistive, inductive, and capacitive circuit elements. This revealed the complex dynamics of high-temperature superconductors and indicated the possibility of controlling these dynamics. The demonstrated chaotic dynamics arising from the coupling of a magnetic single domain and the Josephson current opens up a number of new avenues in the research and applications of Josephson structures with magnetic barriers.

A number of unique effects characterizing the dynamics of JJs with a ferromagnetic interface were presented. Specifically, the appearance of Buzdin steps, Shapiro steps, and chimera steps in the  $I-V$  characteristics. The unique properties of Buzdin steps were analyzed; in particular, it was found that the dependence of their width on the radiation amplitude was determined by the product of Bessel functions.

Unique synchronization and resonance phenomena were discovered in the Josephson junction in the presence of external electromagnetic radiation by taking into account not only the electric but also the magnetic component of the radiation.

It has been demonstrated that hybrid JJs could provide a basis for realizing one of the most controversial phenomena in modern physics — a time crystal. Specifically, it was shown that modifying the critical current with a magnetic moment could lead to internal spatio-temporal crystalline order.

It is worth noting that in 2023, a group of scientists (Fig. 18) from the Bogoliubov Laboratory of Theoretical Physics, led by Yu. M. Shukrinov, was awarded the JINR First Prize for 2022 for a series of theoretical papers studying the anomalous Josephson effect.



Fig. 18. Scientists from Russia (*left to right, here and after*): Yu. M. Shukrinov, I. R. Rahmonov, K. V. Kulikov, M. Nashaat, and E. S. Kovalenko. Scientists from Serbia: J. Tekić, P. Mali, J. Odavić, and I. Sokolović. Scientists from other countries that participated in the projects: A. E. Botha (South Africa), M. R. Kolahchi (Iran), and T. A. Belgibayev (Kazakhstan)

The JINR–Serbia collaboration developed in parallel with the JINR–South Africa collaboration, where a decisive contribution was made by Prof. A. E. Botha (UNISA). A number of problems in studying the properties of Josephson nanostructures were solved jointly; their results are reflected in preprint D17-2024-16 and published in [67].

Research in superconducting spintronics has expanded rapidly in recent years. Among the most attractive targets for this research are hybrid structures of superconductors and magnets. In particular, the ability to control magnetic precession in a ferromagnetic layer using a superconducting current passing through the structure, as well as the magnetic influence on the superconducting current, opens up broad prospects for a variety of applications.

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International Cooperation Department, and M. Janković, Deputy Director of the Vinča Institute of Nuclear Sciences and the national representative of Serbia in JINR, for organizing and supporting our collaboration and for supporting our joint projects and mutual visits.

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