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PHASE TRANSITIONS IN COMPACT STARS — PROBLEM OF MICRO AND MACRO STABILITY

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Problem of microscopic stability of dense matter in the case of a phase transition to the new, dense phase (for example, quark matter) is discussed. An analysis of the constraints resulting from the observations of $2M_{\odot}$ pulsars and the consequences for a stiffness of a dense phase are presented. This may lead to the instability with respect to reconversion to the basic nuclear phase before reaching maximum mass.

Macroscopic stability is defined as a dynamical stability with respect to radial oscillations (in the case of rotating star — axisymmetric pulsations). The conditions for the stability of the star with a small, dense core of new phase are discussed and the universality of the stability properties of the families of rotating neutron stars is presented.

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INTRODUCTION

Many models of dense matter predict existence of phase transitions at high pressure and density, i.e., conditions typical to the interior of neutron stars. In particular, strangeness of dense matter is postulated through appearance of hyperons and deconfinement of quarks. Phase transitions to other exotic phases of matter, as pion or kaon condensates, were also considered [1]. Observations of neutron stars and measurements of their parameters (mass, radius) seem to be the only possibility to study properties of cold, catalyzed matter at density $\rho \sim 10^{15}$ g $\cdot$ cm^{-3}. Recent determinations of the mass of two pulsars $(2M_{\odot})$ put a constraint on the dense matter properties allowing only for a relatively stiff equation of state (EOS). On the other side, phase transitions and appearance of new particles (for example, hyperons) always result in softening of the EOS. As a consequence, constructing models of neutron stars with phase transition in their center requires strong tuning of the dense matter model.

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In the present paper we discuss two aspects of a stability properties of the neutron stars in the case of a phase transition in the interior. For catalyzed matter the microscopic stability corresponds to the minimum of the thermodynamical potential. If two phases of dense matter are possible, the stable phase minimizes the Gibbs thermodynamical function at a given pressure. The relation between the observational constraint ($M_{\max} > 2M_{\odot}$) and the stiffness of the EOS at high density is discussed in Sec. 1.

The softening of the EOS could lead to the interesting properties of the pulsar evolution. The back-bending phenomenon, a temporary spin-up era as an isolated pulsar looses its energy and angular momentum, was proposed by Glendenning et al. [2] as a signature of a phase transition to quark phase in the center of neutron star. However, if phase transition is connected with relative strong softening, it results in a dynamical destabilization of a star leading to a minicollapse in a timescale of milliseconds. In Sec. 2 we present conditions for dynamical stability of a neutron star with a small core of a new dense phase.

1. MICROSCOPIC STABILITY

Thermodynamically, microscopic stability corresponds to a minimum of the baryon chemical potential μ_b (Gibbs energy per baryon) at given pressure. We consider the possibility of the existence of matter in two different phases: N — normal, preferred in low-density region, and Q — superdense high-density phase. In this case the minimalization of μ_b can lead to a first-order phase transi-

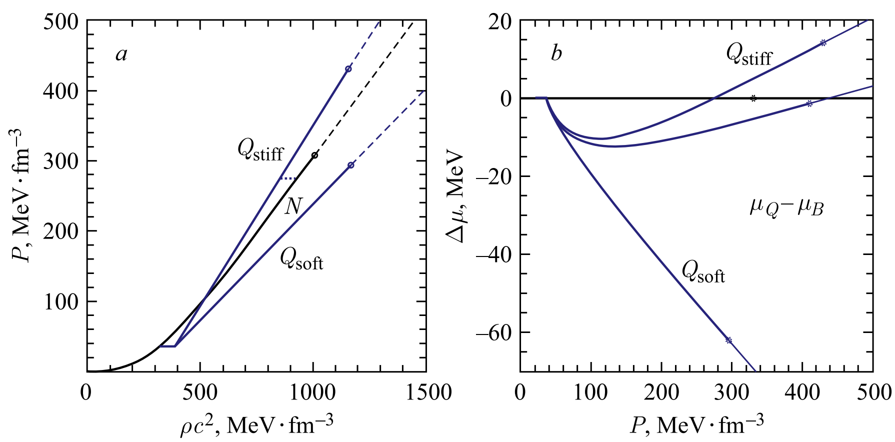

Fig. 1. *a*) EOS with first-order phase transition from N to Q phase. Dotted line at $P \simeq 280$ MeV · fm^{-3} corresponds to the reconfinement $Q \to N$ for stiff Q phase. *b*) Baryon chemical potential with respect to N phase. Negative values — stable Q phase. Dots correspond to the parameters in the center of maximum mass star

tion accompanied by the density jump $\rho_N \to \rho_Q$ which can be described by a parameter $\lambda = \rho_Q/\rho_N$. The condition at transition pressure P_{NQ} reads $\mu_b^N(P_{NQ}) = \mu_b^Q(P_{NQ})$ and is equivalent to the Maxwell construction between N and Q phases. The schematic equation of state (EOS) exhibiting first-order phase transition is presented in Fig. 1, *a*. If we relax the condition of local electric charge neutrality, two phases coexist in a mixed-phase state in a finite pressure range [3].

The phase transition results always in the softening of the EOS. This leads to the decrease of the slope of the function $M(P_c)$ (mass vs. central pressure) for a configuration with central pressure equal to P_{NQ} (see Eq. (1) in Sec. 2). As a result, the maximum mass of a star with phase transition is usually lower than that for N star. Observations of $2M_\odot$ pulsars created some problems for the modeling of phase transitions in the interior of neutron stars. The EOS of Q phase should be sufficiently stiff to overcome the softening effect of phase transition and to obtain the maximum mass of two-phase star larger than $2M_\odot$. The role of the stiffness of a phase Q for the maximum mass of a star is presented in Fig. 2, *a*. To simplify our discussion, we decided to use linear EOS for Q phase. This is a very good approximation of modern EOSs of quark matter [4, 5]. For the two EOSs for dense phase of different stiffness $Q_{\rm soft}$ and $Q_{\rm stiff}$ the phase transition parameters are the same and define the similar $M(R)$ dependence for small core of Q phase in the center of a star (see Eq. (1)). For higher central pressure the stiffness of the dense phase is crucial and gives $\sim 0.4M_\odot$ difference in maximum mass. In high-density region $Q_{\rm stiff}$ EOS is stiffer than N EOS without phase transition, this results in higher value of maximum mass.

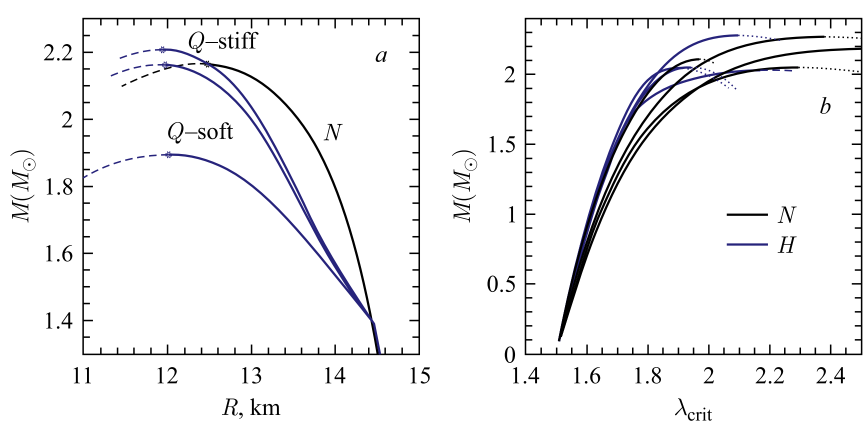

Fig. 2. *a*) Mass–radius relation for N stars and hybrid stars for different stiffness of Q phase. *b*) Critical value of density jump at phase transition. For $\lambda > \lambda_{\rm crit}$ star looses its stability at configuration with $P_c = P_{NQ}$

However, a stiffening of the EOS is associated with the increase of the baryon chemical potential and may lead to the violation, above a certain pressure, of the condition $\mu_b^Q(P) < \mu_b^N(P)$. This means that the stiff Q phase is thermodynamically unstable with respect to the reconversion into the N phase. For complete thermodynamical equilibrium a first-order phase transition back to the N phase occurs — for quark matter this reconversion to N phase is represented by a reconfinement.

In Fig. 1, *b* we present the difference between baryon chemical potentials of three models of Q phase of different stiffness and N phase without phase transition. Negative or positive value of $\delta\mu$ corresponds to microscopically stable or unstable Q phase, respectively. At large pressures $Q_{\rm stiff}$ EOS is unstable with respect to the reconversion into the N phase. If we assume that matter is in a complete equilibrium at pressure $P \simeq 280$ MeV $\cdot$ fm^{-3}, there exists a first-order phase transition from Q to N phase.

Figure 2, *a* presents mass and radius of the considered stars. For "typical" stiffness of Q phase ($Q-$soft) maximum mass is about $0.25M_\odot$ lower than that of N stars. One can reach larger maximum mass increasing stiffness of Q phase ($Q-$stiff), but the matter in the center of hybrid star is then microscopically unstable with respect to reconfinement. Allowing for reconversion of Q phase at high density, we will end up with a similar maximum mass as obtained for N model. The largest stiffness of microscopically stable Q phase gives a maximum mass almost exactly equal to the value for N star (middle Q configurations in Fig. 1, *b* and Fig. 2, *a*). One can conclude that the assumption of the thermodynamical equilibrium results in the very similar maximum masses for hybrid (Q) and neutron stars without deconfinement (N).

2. MACROSCOPIC STABILITY

In this section we discuss macroscopic stability on neutron stars defined as a stability with respect to small perturbation of the density profile. By stability (instability) of an equilibrium configuration we will mean stability (instability) with respect to radial perturbations in the nonrotating case, and with respect to axisymmetric perturbations for rotating configurations.

The well-known stability condition corresponds to the extremum of the gravitational or total baryon mass of a star [6]. In the case of nonrotating star at maximum mass the star becomes unstable with respect to radial oscillations [7] — the fundamental mode of radial pulsations looses its stability. In linear theory the corresponding condition reads: $\omega_0^2(M = M_{\rm max}) = 0$.

Phase transition in the interior of neutron star results in the appearance of a new mode of radial oscillations, which has no counterpart in one-phase star. The main property of this mode is a flow of matter through the pulsating phase boundary transforming the N phase into Q, and vice versa [8]. The compressibility of matter does not play a crucial role and this radial mode exists and has finite frequency even for a star built of the two phases of incompressible fluid.

2.1. Phase Transition in Neutron Star Core. The effect of a phase transition occurring in the center of a star can be studied using a method of a linear response of an equilibrium configuration to a perturbation of the boundary conditions at stellar center. This approach describes how the basic parameters of the star ($X = M_b, I, R$) change their dependence on the central pressure [9]. It could be presented as a relation between derivatives taken at the transition pressure for N and Q configurations:

$$\left(\frac{dX}{dP_c}\right)_Q = \frac{(3-2\lambda+3x_N)(1+x_N)}{(\lambda+3x_N)(\lambda+x_N)}\left(\frac{dX}{dP_c}\right)_N, \tag{1}$$

where λ is density jump and we define relativistic parameter $x_N \equiv P_{NQ}/\rho_N c^2$.

The consequence of formula (1) used for gravitational or baryon masses of a star is the stability condition at the appearance of a small, condensed core of a new phase in the center of a star which reads [10, 11]

$$\lambda < \lambda_{\rm crit} \equiv \frac{3}{2}(1+x_N). \tag{2}$$

The relativistic factor x_N leads to a significant increase of $\lambda_{\rm crit}$, stabilizing stellar configuration with first-order phase transition in the center in GR as compared to the Newtonian theory. The dependence of $\lambda_{\rm crit}$ on the mass of neutron star is presented in Fig. 2, *b* for the set of equations of state compatible with the measured $2M_\odot$ pulsars (for the list of EOSs presented, see [12]).

For weak first-order phase transition (defined by the condition $\lambda < \lambda_{\rm crit}$), a star with a very small core of the new, dense Q phase is dynamically stable — the new mode of radial oscillation has frequency proportional to $\sqrt{3-2\lambda+3x_N}$. In the case of strong phase transition ($\lambda > \lambda_{\rm crit}$), the appearance of an infinitesimally small dense core destabilizes dynamically neutron star and results in a mini-collapse into a new configuration with large core of Q phase. In this case there exists a new branch of stable configurations — "twins".

3. ROTATING STAR

Rigidly rotating stellar configurations are parametrized by two quantities. One of them is the same as for nonrotating star — central pressure P_c or density ρ_c. The second one, frequency Ω or total angular momentum J, describes rigid rotation of a star. Three basic global parameters of rotating star are mass M, baryon mass M_b, and J. The onset of instability can be equivalently formulated

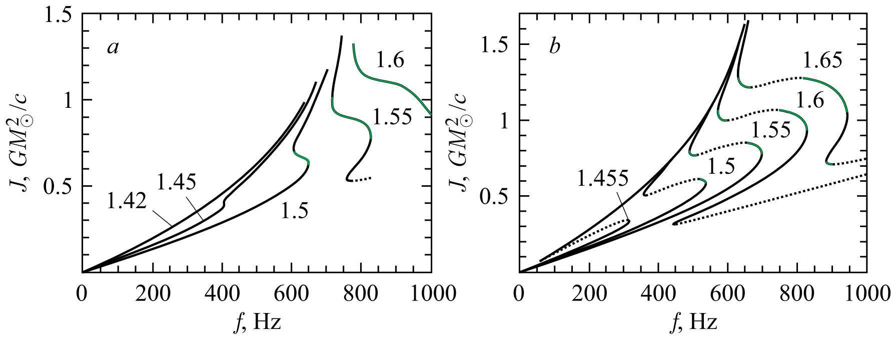

Fig. 3. Total angular momentum J as a function of frequency of rotation for different baryon masses fixed along curves. Dotted fragments correspond to configurations unstable with respect to axisymmetric pulsations

as an extremum of the two of them with third parameter fixed [13]. In Fig. 3 we employ this condition for the total angular momentum J presented at fixed value of baryon mass M_b. This choice corresponds to the physical situation of the evolution of the rotating star as J decreases. Each curve presented in Fig. 3 corresponds to the fixed M_b with central density ρ_c increasing downwards. Figure 3, *a* presents situation corresponding to a weak phase transition, which softens EOS but does not destabilize rotating star. The softening manifests itself as a backbending phenomenon — increase of rotational frequency as the star looses rotational energy and angular momentum [2]. In Fig. 3, *b* the softening of the EOS by phase transition results in the existence of unstable segment of increasing J as central density increases. This instability strip separates two families of stable configurations with large central density (and large compactness) and less compact neutron stars. The study of a large set of EOSs with phase transitions at constant pressure (first order) or through a mixed phase leads us to the conclusion that rotation neither stabilizes nor destabilizes sequences of stationary configurations. If an EOS with a phase transition gives a single family of static neutron stars, then it produces also a single family of rigidly rotating normal stars. If for an EOS one obtains two disjoint families of stable static configurations ("twin neutron stars") of the same baryon mass but different radius, then also for stable rotating configurations these two families are disjoint.

CONCLUSIONS

In the case of phase transition in the interior of neutron star, the $M > 2M_\odot$ constraint results in a very stiff EOS of dense phase and instability at high pressure with respect to reconversion to a phase preferred at low density. The consequence of a requirement of a complete thermodynamic equilibrium of dense matter is then very similar $M_{\max}$ for stars with and without phase transition.

Dynamical destabilization of a family of stellar configurations with phase transition is a general property which does not depend on the rigid rotation of a star. If for static configurations there exist "twin stars", then also for rotating stars the families of very compact stars and stars with larger radius and the same mass are separated by the "instability strip".

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