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ON THE RELATION BETWEEN *x*-DEPENDENCES OF THE HIGHER TWIST CONTRIBUTION TO F_3 AND $g_1^p - g_1^n$

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We compare the higher twist (HT) contribution to the unpolarized structure function F_3 with that one to the nonsinglet combination $g_1^p - g_1^n$ of the polarized proton and neutron structure functions using the assumption that the HT contributions to the Gross-Llewellyn Smith and the Bjorken sum rules are similar. We have found that the relation $\frac{1}{3x}h^{xF_3}(x) \approx \frac{6}{g_A}h^{g_1^p - g_1^n}(x)$ is valid for $x \ge 0.1$ and for $x \ge 0.2$ in the case of LO and NLO QCD approximations, respectively.

Проведено сравнение вкладов высших твистов (ВТ) в неполяризованную структурную функцию F_3 и в несинглетную комбинацию $g_1^p - g_1^n$ поляризованных структурных функций протона и нейтрона. На основе предположения о приближенном равенстве вкладов ВТ в правила сумм Гросса–Ллевеллина Смита и Бьеркена предложено соотношение $\frac{1}{3x}h^{xF_3}(x) \approx \frac{6}{g_A}h^{g_1^p - g_1^n}(x)$, выполняющееся при $x \ge 0,1$ и $x \ge 0,2$ для лидирующего и следующего за лидирующим порядков КХД соответственно.

The structure functions in deep inelastic lepton nucleon scattering are presently a subject of intensive experimental and theoretical investigations. While the leading twist (LT) part of the structure functions related with the parton distributions and their Q^2 -evolution is studied in detail in pQCD, the higher twist corrections ($\sim 1/Q^2$) are of a big interest and under intensive study in the last years. The higher twist effects are especially important in the case of polarized structure functions because the most of the precise data (JLab, HERMES, SLAC) are in the region of $Q^2 \sim 1 \text{ GeV}^2$.

In this note we consider the relation between the HT contributions to the unpolarized structure function F_3 and $g_1^p - g_1^n$ which are pure nonsinglets. As was shown in the paper [1], the Q^2 -evolutions of the F_3 and the nonsinglet part of the g_1 structure functions are identical up to NLO order. Moreover, the x shapes of the F_3 and nonsinglet part of g_1 are also similar¹. By analogy, one could suppose that the HT contributions to F_3 and $g_1^p - g_1^n$ are similar too. Such an assumption was recently used for the first moments of the HT corrections in the Gross-Llewellyn Smith and Bjorken sum rules in the infrared renormalons approach [3]:

$$\operatorname{GLS}(Q^2) = \int_0^1 dx F_3(x, Q^2) = 3\left(\operatorname{GLS}_{pQCD} - \frac{\langle \langle O_1 \rangle \rangle}{Q^2}\right), \quad (1)$$

$$Bjp(Q^{2}) = \int_{0}^{1} dx [g_{1}^{p}(x, Q^{2}) - g_{1}^{n}(x, Q^{2})] = \frac{g_{A}}{6} \left(Bjp_{pQCD} - \frac{\langle \langle O_{2} \rangle \rangle}{Q^{2}} \right), \quad (2)$$

¹This property is intensively used in the phenomenological applications [2].

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where

$$\langle \langle O_1 \rangle \rangle \approx \langle \langle O_2 \rangle \rangle.$$
 (3)

Here GLS_{pQCD} and Bjp_{pQCD} are the leading twist contribution to corresponding sum rules:

$$GLS_{LO} = Bjp_{LO} = 1, (4)$$

$$GLS_{NLO} = Bjp_{NLO} = 1 - \alpha_S(Q^2)/\pi.$$
 (5)

In this note we are going to verify if relation (3) between the lowest moments of the HT contribution can be generalized for the higher twists themselves, namely:

$$\frac{1}{3x}h^{xF_3}(x) \approx \frac{6}{g_A}h^{g_1^p - g_1^n}(x).$$
(6)

To test this relation we will use the values of HT obtained in the QCD analyses of the corresponding structure functions in a model-independent way. In the QCD analyses of DIS data when the higher twist corrections are taken into account, the structure functions are given by

$$xF_3(x,Q^2) = xF_3(x,Q^2)_{\rm LT} + h^{xF_3}(x)/Q^2,$$
 (7)

$$g_1^p(x,Q^2) = g_1^p(x,Q^2)_{\rm LT} + h^{g_1^p}(x)/Q^2,$$
 (8)

$$g_1^n(x,Q^2) = g_1^n(x,Q^2)_{\rm LT} + h^{g_1^n}(x)/Q^2.$$
(9)

In (9) $h^{xF_3}(x)$, $h^{g_1^p}(x)$ and $h^{g_1^n}(x)$ are the dynamical higher twists corrections to xF_3 , g_1^p and g_1^n , which are related to multiparton correlations in the nucleon. They are nonperturbative effects and cannot be calculated without using models. The target mass corrections, which are also corrections of inverse powers of Q^2 , are calculable [4, 5] and effectively belong to the leading twist term. A model-independent determination of $h^{xF_3}(x)$ was done in [6]¹ on the basis of the analysis of CCFR–NuTev (anti-)neutrino deep-inelastic scattering data [7] at $Q^2 \ge 5 \text{ GeV}^2$ and in [8] using the combined set of data [9] different from that of CCFR at $Q^2 \ge 0.5 \text{ GeV}^2$. We consider also the results of [10] where the infrared renormalon model approach for HT contribution was applied in analysis of combined set of IHEP–JINR [11] and CCFR–NuTev data. The values of $h^{g_1^p}(x)$ and $h^{g_1^n}(x)$ in LO² and NLO($\overline{\text{MS}}$) are given in [12], where the results of the analysis of the world data on polarized structure function g_1 [13] at $Q^2 \ge 1 \text{ GeV}^2$, including the precise JLab g_1^n [14] data, are presented. Using these results and taking into account the coefficients in (1) and (2), one could construct the l.h.s. and r.h.s. of Eq. (6).

In Figs. 1–3 we compare the results on HT in the l.h.s. and r.h.s. of Eq. (6). One can see (Fig. 1) that while in the polarized case the values of HT change slightly from LO to NLO approximation, in the unpolarized one the shape of $h^{xF_3}(x)$ depends on the order of pQCD

¹See Table 12 in [6].

²It should be stressed that the LO approach for QCD analysis of polarized structure function g_1 is not reliable enough. See discussion in [15, 16].



Fig. 1. Comparison of the LO (a) and NLO($\overline{\text{MS}}$) (b) results for $\frac{1}{3x}h^{xF_3}(x)$ based on the analysis of the CCFR data [6, 7] (O), and for $\frac{6}{g_A}h^{g_1^p-g_1^n}(x)$ based on the results of [12] (\bullet)



Fig. 2. Comparison of the LO (*a*) and NLO($\overline{\text{MS}}$) (*b*) results for $\frac{1}{3x}h^{xF_3}(x)$ based on the combined data analysis [8, 9] (Δ), and for $\frac{6}{g_A}h^{g_1^p-g_1^n}(x)$ based on the results of [12] (\bullet)

Fig. 3. Comparison of the NLO($\overline{\text{MS}}$) results for $\frac{1}{3x}h^{xF_3}(x)$ based on the combined analysis of IHEP–JINR [10, 11] and CCFR data (dashed lines correspond to upper and lower limits of the infrared renormalon HT contribution), and for $\frac{6}{g_A}h^{g_1^p-g_1^n}(x)$ based on the results of [12] (\bullet)



used, especially for $x \le 0.1$. As seen from Figs. 1–3, equality (6) is approximately valid for $x \ge 0.1$ and $x \ge 0.2$ for the LO and NLO approximations, respectively. It means that the

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higher Mellin moments of the both parts of equation (6) should approximately coincide:

$$\int_0^1 dx \ x^N \frac{1}{3x} h^{xF_3}(x) \quad \approx \quad \int_0^1 dx \ x^N \frac{6}{g_A} h^{g_1^p - g_1^n}(x), \ N - \text{large.}$$
(10)

We would like to mention that equality (3) is suggested in the framework of the infrared renormalon approach, so the violation of equality (4), which is shown in Fig. 1, b, Fig. 2, a, b at x < 0.1, could be due to the contribution of the dynamical twists connected with the nonperturbative structure of the nucleon in this x region.

Finally, it should be noted that there are additional sources of uncertainties which should be taken into account in a more detailed test of Eq. (6): the contribution of $\mathcal{O}(1/Q^4)$; the separation of the twist-3 contribution in the polarized case, which is effectively included in $h^{g_1}(x)$; the Q^2 -dependence of the functions h(x), etc.

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REFERENCES

- 1. Kodaira J. et al. // Phys. Rev. D. 1979. V. 20. P. 627; Nucl. Phys. B. 1979. V. 159. P. 99.
- 2. Kotikov A. V., Peshekhonov D. V. // Eur. Phys. J. C. 1999. V.9. P. 55.
- 3. Kataev A. L. // JETP Lett. 2005. V. 81. P. 608; Pisma ZhETF. 2005. V. 81. P. 744.
- 4. Wandzura S., Wilczek F. // Phys. Lett. B. 1977. V. 72. P. 195.
- Matsuda S., Uematsu T. // Nucl. Phys. B. 1980. V. 168. P. 181; Kawamura H., Uematsu T. // Phys. Lett. B. 1995. V. 343. P. 346; Piccione A., Ridolfi G. // Nucl. Phys. B. 1998. V. 513. P. 301; Blumlein J., Tkabladze A. // Nucl. Phys. (Proc. Suppl.). 1999. V. 79. P. 541; Simula S. et al. // Phys. Rev. D. 2002. V. 65. P. 034017.
- Kataev A. L., Parente G., Sidorov A. V. // Part. Nucl. 2003. V. 34. P. 20; Fiz. Elem. Chast. Atom. Yadra. 2003. V. 34. P. 43.
- 7. Seligman W. G. et al. (CCFR-NuTeV Collab.) // Phys. Rev. Lett. 1997. V. 79. P. 1213.
- 8. Sidorov A. V. // JINR Rapid Commun. 1996. No. [80]. P. 11; hep-ph/9609345.
- Berge P. et al. // Z. Phys. C. 1991. V. 49. P. 187; Ammosov V. V. et al. (SKAT Collab.) // Z. Phys. C. 1986. V. 30. P. 175; Varvell K. et al. (BEBC–WA59 Collab.) // Z. Phys. C. 1987. V. 36. P. 1; Bosetti P. C. et al. // Nucl. Phys. B. 1982. V. 203. P. 362; Barabash L. S. et al. (JINR–IHEP Collab.). JINR, E1-96-308. Dubna, 1996.
- 10. Alekhin S. I. et al. (IHEP-JINR Neutrino Detector Collab.) // Phys. Lett. B. 2001. V. 512. P. 25.
- 11. Sidorov A. V. et al. (IHEP-JINR Neutrino Detector Collab.) // Eur. Phys. J. C. 1999. V. 10. P. 405.
- 12. Leader E., Sidorov A. V., Stamenov D. B. // JHEP. 2005. 06. 033; hep-ph/0503140.

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- Anthony P. L. et al. (SLAC/E142 Collab.) // Phys. Rev. D. 1996. V. 54. P. 6620; Abe K. et al. (SLAC/E154 Collab.) // Phys. Rev. Lett. 1997. V. 79. P. 26; Adeva B. (SMC Collab.) // Phys. Rev. D. 1998. V. 58. P. 112001; Ackerstaff K. et al. (HERMES Collab.) // Phys. Lett. B. 1997. V. 404. P. 383; 1998. V. 442. P. 484; Abe K. et al. (SLAC/E143 Collab.) // Phys. Rev. D. 1998. V. 58. P. 112003; Anthony P. L. et al. (SLAC/E155 Collab.) // Phys. Lett. B. 1999. V. 463. P. 339; 2000. V. 493. P. 19.
- 14. Zheng X. et al. (JLab/Hall A Collab.) // Phys. Rev. Lett. 2004. V. 92. P. 012004.
- 15. Leader E., Sidorov A. V., Stamenov D. B. // Eur. Phys. J. C. 2002. V. 23. P. 479.
- 16. Leader E., Sidorov A. V., Stamenov D. B. // Phys. Rev. D. 2003. V. 67. P. 074017.

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