E2-2001-92

N.P.Konopleva*

RELATIVISTIC PHYSICS AS GEOMETRY

Submitted to the Proceedings of the International Seminar ISHEPP XV,
25–29 September 2000, Dubna, Russia

*Permanent address: All-Russian Scientific Research Institute
of Electromechanics, Horomnyi tup. 4, Moscow 101000;
E-mail: nelly@thsun1.jinr.dubna.su; vniiem@orc.ru

1 Contents

2 Introduction

Just hundred years ago in 1900 D.Hilbert formulated 23 problems which, in his opinion, the mathematicians of XX century would have to solve. Among them the sixth problem pointed to necessity to state the mathematical formulation of the axioms of physics. As particular case of this problem Hilbert considered the possibility of the physical axioms construction according to the model of the axioms of geometry. Thus the sixth Hilbert's problem contained the problem of physics geometrization. For all XX century long this problem formed the strategies of scientific researches in theoretical physics and in some new mathematical topics, especially in geometry. Appearance of the special and general relativity as well as the geometrical gauge field theory can be regarded as consequent stages in the sixth Hilbert's problem solution. According to these physical theories the corresponding new geometries appeared: Minkowski 4D-geometry (for SR), Riemann 4D-geometry and its Cartan's formulation by tetrads (for GR), and after all, the fibre bundle space geometry which is the base of the geometrical gauge field theory. The gauge field theory is successful in explanation of phenomena of particle physics and gravity. Now the problem of today consists in application of the geometrical gauge field theory for relativistic nuclear physics and nuclear structure explanation.

3 Physics axiomatization: what for?

Research of 23 problems from different mathematical topics, which D.Hilbert formulated, could, in his opinion, stimulate further science progress. Among them the sixth problem was named "Mathematical formulation of physical axioms". At first sight it seems unexpected and misunderstood that Hilbert put pure mathematicians attention to the problem of physics axiomatization. Those were the years when the physicists put experiments in first place in science investigation, and physics axioms were really not exist. It is enough to remember Newton's words about he did not contrive hypotheses. But Maxwell already allowed that a theory of new physical phenomena could be in principle constructed before full experimental investigation of it has been fulfilled. In his paper "On the mathematical classification of physical quantities"Maxwell writes that the first stage of physical science development consists in finding out the quantity system which are proposed about that the phenomena under the science consideration depend on them. The second stage consists in finding out a mathematical form of relations between these quantities. After that it is possible to regard this science as mathematical one. Verification of its laws can be realized by theoretical investigation of conditions under which some quantities could be measured as accurately as possible, and also by following experimental realization of these conditions and real measurement of these quantities. Thus Maxwell held that theoretical physics is a mathematics. In his turn Hilbert regarded geometry as physics branch.

What is physics axiomatization necessary for?

D.Hilbert was one of the outstanding German mathematicians of the S.Lie and F.Klein school where new mathematics and, in particular, new geometry were created by using of historical genesis of geometry. It is known that before Euclidean times geometry was experimental science and formed as a sum of experimentally fixed facts and knowledge about extension properties of bodies. Thanks to its axiomatization geometry became the strict mathematical discipline. Euclidean axiom system permitted to obtain many geometrical relations following formal logic rules from the first principles and without use of experiments on each stage. Axiomatization of geometry make trustworthy its theoretical predictions. Therefore when one will experimentally detect that triangle angle sum differs from 2π he will look for mistakes in his measurements but have no doubts in Euclidean geometry. The experiment conditions are, of course, proposed corresponding to just Euclidean geometry.

If physics axiomatization will be done its theoretical predictions will be more trustworthy also. Then some test of statement truth will appear within the theory without experiments. Such theory would be so much trustworthy that it can be used for engineering calculations and correct predictions for the future. Only the theory which is fundamental in such meaning can be the base of the further science progress.

4 VI Hilbert's problem formulation and its role in science knowledge

VI Hilbert's problem formulation begins with note about investigations on geometry foundations are closely tied with the problem of axiomatization of those physical disciplines where mathematics is already now playing the leading role. In the first place it concerns to probability theory and mechanics.

Then Hilbert explains: "To construct the physical axioms according to the model of axioms of geometry, one must first try to encompass the largest possible class of physical phenomena by means of a small number of axioms and then, by adding each subsequent axiom, to arrive at more special theories after which there may arise a classification principle which can make use of the deep theory of infinite Lie groups of transformations. Moreover, as it is done in geometry, the mathematician must bear in mind not only the facts of actual reality, but also all the logically possible theories, and must be particularly careful to obtain the most complete survey of the totality of consequences which follow from the adopted systematization. "

Thus VI Hilbert's problem is not a concrete narrow mathematical one demanding the concrete answer, but it points the way of development for many fields of knowledge basing on the mathematical methods. The Hilbert's problem talks about science investigation strategy in nature study of XX century and close connection between geometry and physics. Just this strategy led to active use of group theory in physics, whereas S.Lie and F.Klein were first who applied group theory in geometry. From this strategy the new fundamental physical theories, as special and general relativity, quantum mechanics, quantum and gauge field theories arose. Among them SR, GR and classical gauge field theory have now purely geometrical forms. That is, relativistic physics of XX century arose thanks to the fact that from the century beginning it was found the right direction to science development and effective methods were used. VI Hilbert's problem solved just this task. It did really stimulate science development in XX century.

However, it must be noted that Hilbert's hopes were only partly realized. When probability theory was axiomatized it was found that it made use not Lie groups but other algebraic methods. Special relativity, quantum mechanics and elementary particle theory make use a classification principle being based on finite Lie group theory, but it is no concern of infinite Lie groups. The classification principle, which is a talk of VI Hilbert's problem, did only arise in a gauge field theory. This principle classifies not elementary particles but interactions between them.

The gauge field theory is fundamental unified theory of all existing in nature interactions including nuclear forces. At the same time this theory can have purely geometrical form like Einstein's gravity theory. But in this case the geometry being in use is more general than Riemann 4D one. Thus the geometrical theory of nuclear forces and geometrical nuclear physics must exist. They will be new relativistic theories of XXI century.

The process of solution of VI Hilbert's problem stimulated development not only physics but geometry also. New geometries arose simultaneously with new physical theories. Just 4D Minkowski geometry arose as geometrical interpretation of SR. For GR 4D pseudoriemannian geometry was developed. Cartan's tetrad formulation of Riemann geometry arose thanks to necessity to unify the Riemann geometry conceptions and Lie-Klein's axiomatics of geometry using Lie group theory. Generalization of Cartan's geometrical approach led to a new geometry of fibre bundle spaces. It was in 50-60th of XX century simultaneously with appearance of the gauge field theory. Fibre bundle space geometry permitted to geometrize the gauge field theory. Thus the most part of modern physics can be at once translated into geometrical language.

5 Geometrical physics and experiment

Modern theoretical physics exploits different geometrical models and methods more and more. Sometimes it looks a fashion which happened to be now and will soon come away. Really geometry use for description of physical phenomena is natural and in order. But as physics as geometry had to go very long way until the geometrical description of physics was realized.

What is at the bottom of deep and indissoluble connection between physics and geometry?

Firstly, geometry can be regarded as physics and as mathematics. Geometry as physics studies the extension properties of material bodies. Its statements can and must be proved by experiments. Geometry as mathematics is only interesting in the logical dependences between its statements and the process of obtaining them from the axioms. Describing by geometry a motion of matter, we unify the space and time into a single extension and unify geometry with physics.

Secondly, the fact is that any physical experiment can not be done without geometry describing the properties of space and time. The great German philosopher of XVIII century I.Kant was first who showed that description of any experiment contains two parts. One of them indicates where and when this event takes place, and other part describes the event in itself. In mechanics all interactions described by notion of force F.

What are space-time relations? Where do they come from?

In Newton's mechanics the space is absolute and has no connection with matter motion. It is only regarded as arena for events. The space is 3D continuous manifold with Euclidean geometry. It is only used for event coordinatization. The absolute time is 1D continuous manifold which has not any connection with matter or space. As a rule the time is regarded as a parameter along moving body trajectory.

The most important question is where does man receive his geometrical (i.e. space-time) ideas from? Kant's answer was: these ideas are given to man with his birth. Thus Euclidean geometry is inborn. But Kant's answer did not satisfy many scientists and philosophers in XVIII-XIX centuries. Poincaré gave other answer. He thought that geometry choice is a result of conventional agreement of scientists and can be done at will.

The connection between physics and geometry may be also determined by Einstein's symbolic formula: $\mathbf{G}=\mathbf{G}_0 + \mathbf{F}$, where $\mathbf{G}$ represents the dynamical geometry, $\mathbf{G}_0$ is the geometry of the "background,"and $\mathbf{F}$ - the forces of interaction. The meaning of this formula right side is that physics and geometry do not separately occur in experiment; only the combination of geometry and physical laws is subject to experimental verification. Just this idea was first expressed by Kant. The fact fixed by Poincaré is really that the decomposition of the sum $\mathbf{G}$ into purely geometrical background $\mathbf{G}_0$ and interaction $\mathbf{F}$ depends on us. But according to Einstein there always exists purely geometrical description of experiment which is equivalent to the sum of right side of formula under consideration. This is ensured by the dynamical geometry $\mathbf{G}$ of the left side of Einstein's symbolic formula.

But let us return to initial question about origin of geometrical ideas of man. Now this question arises twice: concerning background and dynamical geometries. Einstein talks that the dynamical geometry $\mathbf{G}$ describes the real world. It is determined by set of trajectories of test particles which are freely falling in external gravity field. Each particle of them is the point of reading in a local Lorentz frame of reference. This frame of reference is mathematical image of real instruments for measurements in gravity field. Developing the Einstein's ideas one can say that the decomposition of right side formula depends on our choice of the means of measurement. When the man regards himself as a device for measurements he choice Euclidean geometry as $\mathbf{G}_0$. When the frame of references is tied with Earth and oriented to infinite stars we have Newtonian absolute space with Euclidean geometry. Drag-free satellite system in gravity field of Earth realizes local Lorentz frame of reference.

As long as physical phenomena are described as occurring at some place and time, space-time ideas can not be excluded from the description of experiment. But the idea of forces, which produce an interaction, is not so important. The notion of forces can be excluded from the experiment description. Under a force-free description of interactions the theory becomes purely geometrical one. The real curvature of observed particle trajectories is described by means of the concept of connection coefficients of nonholonomic space. It replaces the concept of force. If one and the same phenomenon is described in two different ways, there must exist a "principle of equivalence"which permits the transition from one description to another. But in view of the relation between the form of the theory and the choice of the means of measurement, we must remember that the scheme of an experiment to test the geometrical theory must differ from that to test the ordinary theory of interactions in terms of forces. Geometrical description, which is equivalent to description in terms of forces, always exists. But for experimental verification of the geometrical form of the theory the test bodies and instruments must be correctly chosen. Any physical theory in geometrical form is theory of the test bodies motion.

Thus, every physical theory is based on some postulate about the geometrical properties of space-time, and this postulate finds its expression in the principle of relativity of the theory. In this sense geometry logically precedes experiment. In its turn, the principle of relativity singles out a definite class of frames of reference known as inertial system, in which, by definition the motion of particles is assumed to be rectilinear and the particles themselves are free. The observed deviation of the real trajectories from inertial trajectories and the interaction between the particles are described by means of the concept of a force.

Consequently, if we intend to measure of the interaction intensity or force acting on a particle, we must find a standard straight line. In other words, we need a trajectory which is rectilinear by definition, and we need a fixed procedure for comparing trajectories.

But do free particles and straight lines exist? For theories like Newtonian mechanics, this is a fundamental question. As a model of straight line in mechanics one usually uses light rays. This choice is equivalent to the assumption that the quanta of light - photons - are not subject to mechanical forces, i.e., that their mass is zero. In principle, the straight line can be taken to be the trajectory of any free particle, i.e., undergoing inertial motion. According to Newton the equation of such motion is following: $\dot{p} = 0$. The inertial trajectories are integral curves of this equation. In the aggregate inertial trajectories determine Euclidean geometry used in mechanics.

What is difference between the points of view of Newton and Einstein? A space of classical mechanics is homogeneous,i.e., as a whole it possesses some symmetry. Then all geometrical objects in it are characterized by a set of numbers - the invariants of the various representations of the symmetry group of this space. These numbers describe the properties of the geometrical objects which remain unchanged under transformations that take the space under consideration into itself. In a homogeneous space, it is sufficient to know its group of motions to describe everything that can occur in it: the properties of geometrical objects and the relations between them. This is the meaning of the famous "Erlangen Program"of F.Klein. The use of homogeneous spaces in physics means that the properties of the studied objects (for example, particles) are formulated in terms of the invariants of the representations of the symmetry groups of space-time and internal space. The numeral values of the invariants correspond to integral conserved quantities:

energy, momentum, angular momentum, spin, isospin and so on.

A.Poincaré was the first who used geometrical ideas of F.Klein and S.Lie in physics. He applied them for formulation of SR. P.A.M. Dirac applied these ideas to quantum mechanics. The modern classification of elementary particles is based on the invariants of finite Lie group representations. In these theories all fields (including gravity) are on an equal footing and differ only by the law of propagation and interaction with the currents that produce them.

Einstein's conception of motion makes all trajectories of test bodies inertial. The interaction (or gravity force) is completely eliminated. It is regarded as a manifestation of the dynamical nature of geometry. All the results of measurements then refer directly to the geometrical properties of space-time. In this case, it is no longer necessary to distinguish inertial trajectories as standards. It is sufficient to compare the trajectories of two arbitrary particles or bodies. The distance between them, called the geodesic deviation is proportional to the curvature tensor of space-time. Hence, the geometry of physical space-time becomes experimentally testable. It is sufficient to indicate what physical bodies or processes provide realization of the basic geometrical notions. Such realization always approximate, since they involve idealization. It is well known that Euclidean geometry can be realized by means of solid bodies. The electrodynamics of photons realized the geometry of Minkowski space. To realize Riemann geometry of GR it is necessary to find real objects which can be regarded as a test bodies. Just these bodies motion is free in Einstein's sense.

It was found that the best test bodies are massive ones such as planets, widely separated from one another, and drag-free space probes. Therefore Einstein's theory must well describe the behavior of massive bodies in cosmic space, but its application to elementary particle physics is a problem. It should be noted that the first drag-free probe was only created in 1972. Experimental verification of GR is very complex technical problem and became possible as regular procedure in 50 years after GR creation.

6 Relativistic physics: close-range action hypothesis and geometry

The foundation of relativistic physics is two basic ideas: symmetry principles and close-range action hypothesis. As it arose, the first relativistic physical theory (Maxwell's electrodynamics) did not use symmetry principles. The role of these principles for construction of physical theory was understood later. But close-range action hypothesis was basic in Maxwell theory. Just this idea about an interaction as a process propagating from point to point differs in essence Maxwell's electrodynamics from Newtonian mechanics. All following relativistic physical theories use the close-range action hypothesis also.

In mechanics without gravity separated from one another bodies do not interact between themselves. They are moving in vacuum which does not act to its motion. Interaction does only arise under collisions, i.e., under intersections of trajectories. The only known interaction in mechanics is gravity. But it transmits momentally to any distances and so is a long-distance interaction. Newton wrote that the process of gradual propagation of interaction would be more adequate to physics, but he did not know any equations describing such process. Moreover Newton noted that he proposed the idea of absolute infinite space enveloping the whole universe because he did not only know how to describe the great number of real finite frames of reference used by observers.

Electrodynamics of Maxwell and SR settled the Newton's questions. SR is in essence the first physical theory unified mechanics and electrodynamics. Maxwell equations describe the propagation process of electromagnetic interaction between bodies. The velocity of this propagation occurred finite and equal to light velocity c. Therefore the electromagnetic interaction realizes the close-range action hypothesis.

Can gravity be interaction of close-range action kind and simultaneously described by geometry of space-time? For that a space-time geometry must not be given globally in a whole universe. In this case the space-time as a whole and nonlocal characteristics (for example, length) are only determined from point to point. This is Riemann geometrical point of view. In general Riemann spaces do not possess any degree of homogeneity. The invariants of Riemann geometry are differential invariants of the group of arbitrary continuous transformations of the coordinates. The group of these general covariant transformations has no any invariants like invariants of finite Lie groups describing Klein spaces. Therefore it does not lead to ordinary conservation laws. Consequently any classification like the elementary particle classification taking place in homogeneous spaces of Minkowski or Galiley is not exist in Riemann space. We meet with the same problems when a global internal symmetry becomes local one as it happens in the gauge field theory.

The solution was found thanks to Cartan's formulation of Riemann geometry. The essence of the matter consist in following. 4D Riemann space-time can be regarded as possessing the same symmetry properties that Minkowski space-time but only in the neighborhood of each its point. In fact, a Riemann space can be represented as a manifold whose "points"are Minkowski spaces, these being "interlinked"by Ricci or Christoffel connection coefficients. The geometrical concept of connection coefficients corresponds in physics to the gravitation interaction. If in a similar way we consider a 4D manifold whose points are homogeneous spaces of the representations of internal symmetry group, we obtain an example of a fibre bundle space. The connection coefficients introduced in it correspond to vector-potentials of gauge fields, or multiplets of vector mesons. This geometrical interpretation of gauge fields allows us to consider the trajectories of particles interacting with gauge field as free trajectories in a fiber bundle space. Thus in the description of any interactions mediated by some gauge field, we can get rid of the concept of force and make the theory of such interactions purely geometrical as in GR. This is also true for electrodynamics.

These results were obtained by N.P.Konopleva in 1965/67 and reported in Dubna on International Seminar on Vector Mesons and Electromagnetic interactions in 1969. The geometry of fibre bundle spaces was developed in 50-60th years by mathematicians of Moscow and Kazan state universities and by their schools. It was at the same time when the gauge field theory was formed by Sakurai, Yang and Mills, Utiyama, De Witt, L.Faddeev and V.Popov and others.

An important property of local symmetries is the existence of identity relations between the extremals and their derivatives. These identities can be expressed in the form of conservation laws which are strong, i.e., are satisfied independently of the specific form of the Lagrangian and the equations of motion. Integration of such conservation laws yields invariants having topological sense.

The basis of the theory of gauge fields comprises symmetry principles and close-range action hypothesis, which converts global symmetries into local one. The principle of local gauge invariance reflects a deep relationship between the universality of the various interactions, conservation of the vector currents, and the existence of the interactions themselves. This principle determines the form of all interactions, irrespective of their physical nature, and thereby opens the way to the construction of a consistent unified theory of the elementary particle interactions. At the same time, the principle of local gauge invariance, like Einstein's general principle of relativity, gives the theory such form, which admits a purely geometrical interpretation. As a result, it becomes possible to generalize Einstein's idea that the geometry of space-time must determined by the physical processes and have a dynamical character. Such geometry effectively reflects the influence on a distinguished test particle of all the remaining matter in the world. The possibility to classify elementary particles in terms of the invariants of the finite Lie groups arise again, but this classification becomes local one.

7 Summary: is the geometrical nuclear physics possible?

The geometrical description of interactions makes it possible to axiomatize physics, at least those its topics which are described by the gauge field theory. In this theory different kinds of interactions possess the different local gauge symmetry groups. Such groups, just as general covariant group of coordinate transformations in GR, appertain to infinite Lie's groups. So, in the gauge field theory the classification principle arise which use a deep theory of the infinite Lie groups, as it predicted Hilbert in his VI problem.

Now the geometrical picture corresponds to the transmission of interaction through a medium. Therefore gauge models have naturally led to the notion of the vacuum as a medium. In this vacuum under quantum consideration pairs of particles can be produced in such numbers that a condensate is formed, and polarization effects are so strong that they can completely screen a charge introduced without. The phenomenon of asymptotic freedom can be regarded as antiscreening produced by the dispersion of the vacuum. In description of vacuum in the gauge field theory GR occurs very useful. It can be used for development of nonperturbative quantization methods.

The classical theory of gauge fields is rapidly developing. The nonlinearity of the classical equations of non-Abelian gauge fields has given birth to a new industry among theoreticians. This is the study of particle-like solutions of these equations (solitons, instantons, kinks, monopoles and vortices). Particle-like solutions possess a new type of charge - topological charge, which one can attempt to associate with the new quantum numbers, that characterize the elementary particles. Therefore the gauge field theory puts the relations between quantum and classical physics to a new way.

The gauge field theory describes all interactions known in our days. It is very effective in elementary particle physics, gravity and condensed matter physics. But nuclear physics makes weakly use the gauge field theory. The nuclear forces are described by the gauge field theory as three kinds of forces:

1. universal nuclear forces associated with $U(1)$-symmetry (Sakurai, model of vector dominance);
2. weak nuclear forces associated with $SU(2)$-symmetry (Yang=Mills fields);
3. strong nuclear forces associated with $SU(3)$-symmetry (quantum chromodynamics).

While Sakurai model led to the discovery of the vector mesons ρ, ω, ϕ, $SU(2)$-symmetry led to discovery of the W- and Z-bosons, $SU(3)$- symmetry was responsible for the discovery of the Ω-resonance and the appearance of new models of hadrons known as quark models.

Thus the results of the gauge theory of nuclear forces apply to elementary particle physics but do not for the present concern to a nuclear structure. The problem of XXI century is to construct the nuclear theory on the base of ideas of the gauge field theory. Such nuclear theory will be relativistic and can be done purely geometrical.

Geometrical field theories are frequently regarded as pure mathematics, bearing no direct relation to experiment. This is due, in particular, to the fact that one of the basic concept of such kind theory is the concept of a test body. Such geometrical theories as GR and classical gauge field theory are theories of the motion of test bodies. At the same time attempts to find a physical model of a test body often encounter difficulties, since the basic property of a test body is its ability to feel the influence of an external field without exerting an inverse effect on the field. Is this possible and under what conditions? An affirmative answer to this question would mean that it is possible to specify a class of real physical objects which under certain conditions can play the role of test body, i.e., can move along geodesic trajectories. These physical objects would specify a domain of applicability of geometrical theory of interactions, and in particular, of nuclear geometrical theory. To find such objects is necessary to analyze all experimental results on nuclear properties from new positions.

Acknowledgments. It is a pleasure to thank the Organizers of the International Seminar ISHEPP XV for their support of this investigation.

References

[1] D. Hilbert, Die Grundlagen der Physik, Nach. von der Kön. Ges. der Wiss. zu Göttingen, Math.-Phys. Kl., Heft 3, S.395-407, 1915

[2] A. Poincaré, Derniéres Pensées, Paris, 1913

[3] A. Einstein, Sitz. Preuss. Akad. Wiss., 2, S.1111-1116, 1916

[4] Proc. of the Int. Sem. on Vector Mesons and Electromagnetic Interactions. Ed. A.M.Baldin, JINR, Dubna, 1969

[5] Konopleva N.P., Popov V.N., Gauge fields, Harwood academic publishers, Chur-London-New York, 1981.

[6] Konopleva N.P., Variational formalism for infinite groups and field theory, in: Gravitation and the Theory of Relativity, Kazan State University, 1968, Nos.4-5, p.67

[7] N.P.Konopleva "Geometrical description of interactions", doc. diss., 1969, Lebedev Institute of Physics, Moscow

[8] N.P.Konopleva, Instantons and gravity, Comm. JINR, E2-96-459, Dubna, 1996, p.1-10

[9] N.P.Konopleva, Hyperbolical instantons and gauge fields vacuum structure, Comm. JINR, E2-98-388, Dubna, 1998, p.1-11

[10] N.P.Konopleva, VI Hilbert's problem and S.Lie infinite groups, Comm. JINR, E2-99-301, Dubna, 1999, p.1-8

Received by Publishing Department
on July 10, 2001.

Коноплева Н.П. E2-2001-92
Релятивистская физика как геометрия

Проблема геометризации физики рассматривается как частный случай проблем, о которых говорит шестая проблема Гильберта. Эта проблема Гильберта касается математической формулировки аксиом физики. Показано, что в течение всего XX века данная проблема формировала стратегии научных исследований в теоретической физике и некоторых разделах математики, особенно в геометрии. Появление специальной и общей теорий относительности, так же как и геометрической теории калибровочных полей, можно рассматривать как последовательные стадии решения шестой проблемы Гильберта. Проблемой сегодняшнего дня является применение геометрической теории калибровочных полей к релятивистской ядерной физике и к объяснению структуры ядра.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 2001

Konopleva N.P. E2-2001-92
Relativistic Physics as Geometry

The problem of physics geometrization is considered as a particular case of problems which the sixth Hilbert's problem talks about. This Hilbert's problem concerns the mathematical formulation of physics axioms. It is shown that for all XX century long this problem formed the scientific research strategies in theoretical physics and some mathematical topics, especially in geometry. Appearance of the special and general relativity, as well as the geometrical gauge field theory can be regarded as consequent stages in the sixth Hilbert's problem solution. Now the problem of today consists in application of the geometrical gauge field theory for relativistic nuclear physics and for nuclear structure explanation.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 2001

Макет Т.Е.Попеко

Подписано в печать 19.09.2001
Формат 60 × 90/16. Офсетная печать. Уч.-изд. л. 1,5
Тираж 425. Заказ 52863. Цена 1 р. 80 к.

Издательский отдел Объединенного института ядерных исследований
Дубна Московской области