E4-2001-32

E.Kuraev, V.Samoilov, O.Shevchenko, A.Sissakian

EXTRACTION OF σ-TERM FROM SCATTERING AND ANNIHILATION CHANNELS DATA

The effects of chiral symmetry violation are caused by the σ operator [7]

$$\hat{\sigma}_{ab} = [Q_5^a[Q_5^b, H]], \tag{1}$$

where Q_5^a is the axial charge of the flavor $SU(3)$ symmetry, and $H = H_0 + H'$ is the hamiltonian.

A fruitful suggestion about the $(3^* \times 3) + (3 \times 3^*)$ character of the part of the hamiltonian H' that violates the left-right symmetry due to the current mass of quarks

$$H' = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s, \tag{2}$$

leads to the operator with diagonal elements

$$\begin{aligned} \hat{\sigma}_\pi = \hat{\sigma}_{11} = \hat{\sigma}_{22} = \hat{\sigma}_{33} = m(\bar{u}u + \bar{d}d), m = m_u = m_d; \\ \hat{\sigma}_K^u = \hat{\sigma}_{44} = \hat{\sigma}_{55} = \frac{1}{2}(m + m_s)(\bar{u}u + \bar{s}s); \\ \hat{\sigma}_K^d = \hat{\sigma}_{66} = \hat{\sigma}_{77} = \frac{1}{2}(m + m_s)(\bar{d}d + \bar{s}s); \\ \hat{\sigma}_\eta = \hat{\sigma}_{88} = \frac{1}{3}(m(\bar{u}u + \bar{d}d) + 4m_s \bar{s}s). \end{aligned} \tag{3}$$

They can be investigated in processes of the type $\pi(K,\eta)N \to \pi(K,\eta)N$; whereas the nondiagonal ones,in processes of the type $\pi N \to \Sigma K; KN \to \Lambda\pi$. The matrix elements of σ operator being averaged over vacuum states σ_0 leads ,under the Partial Conservation of Axial Current (PCAC) hypothesis $\partial^\mu A_\mu^a = f_a m_a^2 \phi^a, \quad a = 1, ..., 8,$ to expressions for the current quark mass in terms of the vacuum condensate. The average over proton states with different 4-momenta can be interpreted in terms of the scalar form factor

$$< P(p')|(\bar{u}u + \bar{d}d)|P(p) >= \sigma((p - p')^2)\bar{u}(p')u(p), \sigma(0) = \sigma_{N\pi},$$

and the corresponding scalar radius of the proton (which turns out to be 3 times as large as its charge radius). It is useful to mention about the relation of the $\sigma_{N\pi}$ term with the intrinsic strangeness in the proton:

$$y = \frac{< N|\bar{s}s|N >}{< N|(\bar{u}u + \bar{d}d)|N >}, \sigma_{N\pi} = \frac{\sigma}{1-y}, \sigma \approx 30MeV.$$

The quantity y can also be extracted from two kaon σ terms:

$$\sigma_{KN}^{u} + \sigma_{KN}^{d} = 2(1+y)\sigma_{N\pi}.$$

The problem of intrinsic strangeness in the proton becomes now actual.It can shed light upon the spin crisis phenomena in the experiments with deep inelastic scattering of leptons on the proton. We will mention as well the problem of anomalous yield of ϕ, η mesons in low-energy proton-antiproton annihilation experiments. Up to now, the theory is absent which will provide the computation of the σ term. Some approaches are known in terms of the Nambu-Jona-Lasinio [4] model (with a scalar σ-meson intermediate state. Using the value of scalar meson mass $m_\sigma = 450MeV$ the value $\sigma_{N\pi} = 70MeV$ was obtained [1]),effective lagrangian[6], and the scalar form factor was computed in terms of the Chiral Perturbation Theory[5]. The results vary in the wide range

$$20MeV < \sigma_{N\pi} < 200MeV.$$

In the planned experiments on DAFNE with measuring $\pi(K)$-nucleon scattering as well as low energy proton-antiproton annihilation, one can obtain the information that can be used to extract the $\sigma_{N\pi}$ term at the so-called CD-point (Cheng-Dashen point in the plane of Mandelstam variables). Complete development of this procedure was done by R.Koch in paper [1].He suggested the method with applying the dispersion relations with integration of imaginary parts along hyperbolas in the Mandelstam plane. The unknown contributions of additional (complex) cuts ,poles and lacking information about unaccessible regions of the scattering and annihilation channels are parameterized by the

[1]M.K.Volkov, private communication.

so-called discrepancy function Δ. The hyperbolas are chosen so as to pass through the CD-point, that, being situated in the nonphysical region nevertheless belongs to the analyticity region of the amplitude. The Koch method permits one to express $\sigma_{N\pi}$ in terms of some function containing, as free parameters one of Mandelstam variables and two parameters specifying the location of hyperbolas. Varying σ in rather a wide interval of possible(positive) values R.Koch founds out the region of stability of Δ. Further, this value of it is used again to extract the σ term with a better accuracy. This value turns out to be stable in varying free parameters but strongly depends on the assumption about pion-pion scattering lengths,which persists in dispersion relations. So,using them from the data of $Ke4$, decay the value $\sigma = 64 \pm 8MeV$ was extracted, and using the Weinberg prediction for the scattering lengths, one obtains $\sigma = 56 \pm 8MeV$.

One way to reduce the error bars can be to take account of the annihilation channel contribution more correctly. In particular, we can push up the parameter of the maximal energy squared t_{max} in the annihilation channel to use the existing experimental data:$t_{max} \sim 5GeV^2$. Besides, we suggest a simplified form for the contribution of the nonphysical region in the annihilation channel using the logarithmic character of the dispersion integral.

Considering the pion-nucleon scattering, one averages this operator over the nucleon states (case $a, b = 1, 2, 3$). To describe the processes, we use the so-called two soft pion Ward identity, which being averaged over nucleon states,on the one hand, leads to the Adler-Weisberger sum rule and on the other hand, it provides some equation for the sigma-term with meson-nucleon scattering amplitude. For the kaon-nucleon amplitude (which can be measured in the threshold region in DAFNE),this relation provides the possibility to extract the kaon sigma term that describes the intrinsic strangeness in the proton.

There is a technical problem: the direct connection of the $\sigma-$ term with the respective amplitudes can be established only at a nonphysical point, and, therefore one must analytically continue the physically observed amplitudes from a physical point to the nonphysical one (just of this nonphysical point,the $\sigma-$ terms in annihilation channel and in the scattering channel coincide, because of the crossing symmetry).

The fundamental basis for our following consideration is the two-axial current identity (AA identity)

$$\begin{aligned} \partial_x^\mu \partial_y^\lambda T\left(A_\mu^a A_\lambda^b\right) &= T\left(\partial_x^\mu A_\mu^a(x) \partial_y^\lambda A_\lambda^b(y)\right) + \delta(x_0 - y_0)\left[A_0^a(x), \partial^\lambda A_\lambda^b(y)\right] \\ &- \partial_x^\mu \left(\delta(x_0 - y_0)\left[A_\mu^a(x), A_0^b(y)\right]\right). \qquad (4) \end{aligned}$$

Averaging this operator identity over different states, one estimates $\sigma-$ terms in different channels with the respective amplitudes. For example, in the annihilation channel, one takes the matrix elements of this operator identity over the states $< \bar{p}p|$ and $|0>$. Then, the left-hand side is just the amplitude of the annihilation of $\bar{p}p$ into two axial-vector mesons (converted with the respective momenta). Within the PCAC hypothesis, the first term in the right-hand side represents the annihilation amplitude of the proton and antiproton into a pair of pseudo-scalar mesons. The second term is just the sigma term in the annihilation channel. The remaining term in the equation is described in terms of the current algebra and is universal in form.

Let us consider the annihilation of a nucleon and an antinucleon into a pair of pseudoscalar mesons

$$N(p_1) + \bar{N}(p_2) \to \pi^a(q_1) + \pi^b(q_2), p_1^2 = p_2^2 = m^2, q_1^2 = q_2^2 = \mu^2. \qquad (5)$$

The invariant amplitudes of the process depend on the Mandelstam variables:

$$\begin{aligned} s = (p_1 - q_1)^2, t = (p_1 + p_2)^2, u = (p_1 - q_2)^2, s + t + u = 2(m^2 + \mu^2); \\ \nu = \frac{s-u}{4m}, \nu^2 = (\omega + \frac{t}{4m})^2, \qquad (6) \end{aligned}$$

where ω is the pion energy in the laboratory frame of the crossed channel of pion-nucleon scattering.

The σ-term can be expressed through the invariant scattering amplitude at a nonphysical point $t = 0, \nu = 0$. Keeping in mind that this point is located outside the analyticity domain of the scattering amplitude,the following procedure was accepted. First, we calculate σ_{CD},-the value of the σ-term at the unphysical CD point $(t = 2\mu^2, \nu = 0)$ **and then** extrapolate it to the point $t = 0, \nu = 0$. The extrapolation of invariant amplitudes to the CD point can be performed by using the dispersion relations written along some path in the plane of Mandelstam variables (the hyperbolas).

This procedure permits one to avoid the unknown values of spectral functions and to choose the hyperbolas close to the physical regions of scattering and annihilation channels. The hyperbola that passes through the CD point is of the form:

$$(\nu^2 - \nu_0^2)(t - t_0) = (t_0 - 2\mu^2)\nu_0^2. \tag{7}$$

Putting $\omega = \mu$ and then excluding ν^2, we obtain the cubic equation for t, $((\mu + \frac{t}{4m})^2 - \nu_0^2)(t - t_0) = \nu_0^2(t_0 - 2\mu^2)$ that has a single real root t_{th}. The cut in the ν^2-plane starts from $\nu_{th}^2 = (\mu + \frac{t_{th}}{4m})^2$ up to infinity. Using the new variables $\nu^2 = \nu_0^2 + \frac{\nu_0^2(t_0 - 2\mu^2)}{t - t_0}$, one obtains

$$\int\limits_{\nu_0^2}^{\nu_t^2} \frac{d\nu'^2}{\nu'^2 - \nu^2} = \int\limits_{4\mu^2}^{\infty} \frac{(t - t_0)dt'}{(t' - t)(t' - t_0)},$$
$$\nu_t^2 = \nu_0^2 + \frac{\nu_0^2(t - 2\mu^2)}{4\mu^2 - t_0}. \tag{8}$$

In this way we obtain the dispersion representation for the function F "along the hyperbola":

$$F(\nu^2, a) = F_N(\nu^2, a) + \frac{1}{\pi} \int\limits_{\nu_{th}^2}^{\infty} d\nu'^2 \frac{ImF(\nu'^2, a)}{\nu'^2 - \nu^2}$$

$$+ \quad \frac{1}{\pi} \int\limits_{4\mu^2}^{\infty} \frac{(t-t_0)dt'}{(t'-t_0)(t'-t)} ImF(t',a) \tag{9}$$

+ a complex cut and complex poles, where a denotes the hyperbola parameters t_0, ν_0^2. We choose $F = (D^+(\nu^2, a) - A^+(0,a))/\nu^2$ with

$$D^+ = A^+ + \nu B^+, \tag{10}$$

$$\frac{m^2 - (t/4)}{4\pi} A^+ = f_+^0(t) + \frac{5}{2}[3m^2\nu^2 - (m^2 - (t/4))(\mu^2 - (t/4))]f_+^2(t);$$
$$\frac{1}{4\pi} B^+(\nu, t) = (15m\nu/\sqrt{6}) f_-^2(t) + ..., \tag{11}$$

where the variables ν^2, t are related by the hyperbola equation. The functions $f^0(t), f^2(t)$ are the experimentally measured scattering amplitudes. The ellipses denote the contribution of higher partial waves, F_N denotes the contribution of the nucleon pole on the real axis. Applying this relation to the sigma-term at the CD point:

$$\sigma = \sigma_{N\pi} = \frac{f_\pi^2}{2} \left[A^+(\nu = 0, t = 2\mu^2) - \frac{g^2}{m} \right] \tag{12}$$

we obtain the master equation:

$$\begin{aligned} \sigma \quad &= \quad [ReD^+(\nu^2, a) - \frac{\nu^2}{\pi} \int\limits_{\nu_{th}^2}^{\infty} d\nu'^2 \frac{ImD^+(\nu'^2, a)}{\nu'^2(\nu'^2 - \nu^2)} \\ &- \quad \frac{t - 2\mu^2}{\pi} \int\limits_{4\mu^2}^{t_{max}} dt' \frac{ImD^+(t', a)}{(t' - 2\mu^2)(t' - t)}](f_\pi^2/2) - (t - 2\mu^2)\Delta(t, a), \\ & \quad f_\pi = 131.9 MeV. \end{aligned} \tag{13}$$

Here the extra parameter t_{max} is introduced. This parameter in the Koch analysis was chosen as $t_{max} \approx 1 GeV^2$.

The discrepancy function Δ takes account of the contributions of complex poles and cuts and the contribution of the experimentally inaccessible region of the annihilation channel $t > t_{max}$. The main assumption is that this function is smooth and can be obtained by matching different choices of hyperbola parameters.

Further procedure is as follows. Inserting the approximate value of the sigma term into the left-hand side of the master equation and using the experimental data and the set of hyperbolas, one's convinced in the validity of the assumption about the flatness of the discrepancy function. Taking this value to the right-hand side of the equation, one extracts an exact value of the sigma term. This procedure was applied in a series of papers to extract the sigma term from the data in the scattering channel [2]. It is necessary to note that the relevant values of amplitudes in the annihilation channel were obtained from the corresponding values in the scattering channel using the extrapolation procedure.

In the modern experiments, the data in the annihilation channel were obtained [8]. So, there appears the possibility to use them directly. Besides, we discuss below the possibility to apply the method presented above to the annihilation channel.

Since the sigma term is defined at the unphysical point $\nu = t = 0$,the value of the sigma term in the annihilation channel is the same as the one in the scattering channel: $\sigma_{N\pi}(0) = \sigma_{\bar{p}p}(0)$.

Using the results of R.Koch we must take into account two important corrections. First is a more accurate estimation of the contribution of annihilation channel. Keeping in mind that the main contribution comes from the nonphysical region of annihilation channel $4\mu^2 < t < 4m^2$ and the logarithmical character of the dispersion integral in this region, we add the contribution of the region $t_{max} < t < 4m^2$ in the form:

$$\Delta\sigma_{ann} = 2f_\pi^2 a_0^0 \sin^2\delta_0^0 \ln\frac{4m^2}{t_{maxM}} \approx 28MeV.$$

Here we use [9]:$a_0^0 = 0.26\mu^{-1}, \delta_0^0(1GeV) = 36Deg.$

The second correction arises from continuation from the CD-point where Koch $\sigma_{N\pi}$ term was calculated to the point $t = \nu^2 = 0$. This continuation from the CD point can be performed with the help of dispersion relations and the two-particle unitarity condition [3]. It is important that a two-pion intermediate state gives the most considerable contribution. The result is:

$$\sigma_0 = \sigma - \Delta\sigma_{CD}, \Delta\sigma_{CD} = \sigma(2\mu^2) - \sigma(0) = \frac{3\mu^3}{8\pi^2}(\frac{g_A}{f_\pi})^2 J(\frac{\mu}{2m}), \tag{14}$$

with

$$J(z) = \int_4^{1/z^2} dx \frac{\arctan(y)}{x^{3/2}\sqrt{1 - xz^2}}, y = \frac{\sqrt{(1 - xz^2)(x - 4)}}{z(x - 2)}. \tag{15}$$

We note that the main contribution comes from the two-pion intermediate state, and this results in

$$\Delta\sigma_{CD} = \frac{3\mu^3}{16\pi}\frac{g_A^2}{f_\pi^2} \approx 14MeV. \tag{16}$$

We estimate the final value as:

$$\sigma_{\bar{p}p}(0) = \sigma_{N\pi}(2\mu^2) - \Delta\sigma_{CD} + \delta\sigma_{ann} \approx 78MeV. \tag{17}$$

Acknowledgements: We thank M.G.Sapoznikov for drawing our attention to this problem and the participants of N.N.Bogoliubov Laboratory of Theoretical Physics, JINR, seminar for critical comments. We are grateful to M.Nagy and M.Volkov for their attention to this work and the help.

References

[1] R. Koch, Z. Phys. C 15 (1982) 161.

[2] R. Muller, G. Hite and R. Jacob, C. Phys. C 8 (1981) 199, R. Koch, Z. Phys. C 15 (1982) 161.

[3] H. Pagels and W. Pardee, Phys. Rev, D4 (1971) 3335.

[4] M.K.Volkov et al., Phys.Lett.B235(1990),331.

[5] J.Gasser et al., hep-ph/0002283.

[6] U-G.Meissner, hep-ph/0001066.

[7] E.Reya, Rev.Mod.Phys.46,1974,545.

[8] A.Alberico et al., Phys.Lett.B432(1998),427.

[9] G.Höhler, F.Kaiser, R.Koch and E.Pietarinnen, Handbook of pion-nucleon scattering.Physics Data 12-1(1979).

Received by Publishing Department
on February 28, 2001.

Кураев Э. и др. E4-2001-32
Извлечение σ-члена из данных каналов рассеяния
и аннигиляции

Метод «дисперсионных соотношений на гиперболах», развитый в работах Р.Коха для извлечения нарушающего киральную симметрию $\sigma_{N\pi}$-члена из экспериментальных данных по пион-нуклонному рассеянию, применен для оценки $\sigma_{\bar{p}p}$. С учетом поправок вклада от нефизической области канала аннигиляции и от продолжения в точку Чена–Дашена получено значение $\sigma_{\bar{p}p}(0)=78$ МэВ.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна, 2001

Kuraev E. et al. E4-2001-32
Extraction of σ-Term from Scattering
and Annihilation Channels Data

The «dispersion relation on hyperbolas» method by R.Koch, developed for extraction of the chiral symmetry violating $\sigma_{N\pi}$-term from the experimental data on pion-nucleon scattering is applied to estimate $\sigma_{\bar{p}p}$. Two corrections: one arising from nonphysical region of annihilation channel and other continuation to the Cheng–Dashen point are taken into account resulting in $\sigma_{\bar{p}p}(0)=78$ MeV.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna, 2001

Макет Т.Е.Попеко

Подписано в печать 28.03.2001
Формат 60 × 90/16. Офсетная печать. Уч.-изд. листов 1,08
Тираж 360. Заказ 52565. Цена 1 р. 30 к.

Издательский отдел Объединенного института ядерных исследований
Дубна Московской области