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COMMENT ON POLARIZED QUARK DISTRIBUTIONS EXTRACTED FROM SIDIS EXPERIMENTS

The extraction of the polarized quark and gluon densities is one of the main tasks of the Semi-Inclusive Deep Inelastic Scattering (SIDIS) experiments with the polarized beam and target. Of a special importance for the SIDIS experiments are the questions of strange quark and gluon contributions to the nucleon spin, and, also the sea quark share as well as the possibility of the broken sea scenario. Indeed, it is known [1] that the unpolarized sea of light quarks is essentially asymmetric, and, thus, the question arises: does the analogous situation occurs in the polarized case, i.e. whether the polarized density $\Delta\bar{u}$ is equal to $\Delta\bar{d}$ or not.

The crucial tests for the polarized quark distributions extracted from the SIDIS data are the sum rules dictated by $SU_f(2)$ and $SU_f(3)$ symmetries. While $SU_f(3)$ symmetry (and, as a consequence, the respective sum rule) is rather approximate (see, for example [2] and refs. therein), $SU_f(2)$ symmetry may be regarded as almost exact as well as the respective sum rule–Bjorken sum rule.

Let us remind that the Bjorken sum rule written in terms of the first moments of the structure functions $\Gamma_1^p(Q^2) \equiv \int_0^1 dx g_1^p(x,Q^2)$ and $\Gamma_1^n(Q^2) \equiv \int_0^1 dx g_1^n(x,Q^2)$ contains Q^2 dependent quantity C_1^{NS} in the right-hand side[1]:

$$\Gamma_1^p \; - \; \Gamma_1^n = \frac{1}{6}\left|\frac{g_A}{g_v}\right| C_1^{NS}(Q^2), \tag{1}$$

$$\begin{aligned} C_1^{NS} &= 1 - \left(\frac{\alpha_s(Q^2)}{\pi}\right) - 3.5833\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 \\ &- 20.2153\left(\frac{\alpha_s(Q^2)}{\pi}\right)^3 - 130\left(\frac{\alpha_s(Q^2)}{\pi}\right)^4 + O(\alpha_s^5). \end{aligned} \tag{2}$$

However (and this is of great importance for what follows) the first moments of polarized quark distributions satisfy the respective form of the Bjorken sum rule without C_1^{NS} in the right-hand side irrespectively in which QCD order they are extracted. Namely, the Bjorken sum rule written in terms of polarized quark distributions reads

$$\begin{aligned} &\Delta q_3 \equiv a_3 = (\Delta_1 u(Q^2) + \Delta_1\bar{u}(Q^2)) - (\Delta_1 d(Q^2) + \Delta_1\bar{d}(Q^2)) \\ &= \left|\frac{g_A}{g_V}\right| = F + D = 1.2670 \pm 0.0035 \quad \textit{in all QCD orders}, \end{aligned} \tag{3}$$

where the notation $\Delta_1 q \equiv \int_0^1 dx \Delta q$ is used to distinguish the local in Bjorken x polarized quark densities $\Delta q(x)$ and their first moments.

Notice that the well known fact of nonrenormalizability (i.e., Q^2 independence) of the quantity Δq_3 directly follows from its definition

$$\frac{s_\mu}{2}\Delta q_3 = \langle ps|A^3_\mu|ps\rangle \tag{4}$$

due to the conservation[2] of the flavour nonsinglet axial vector current A^3_μ. This fact is

[1]See, for example, excellent theoretical overview in [3] and references therein. The $O(\alpha_s^3)$ correction for C_1^{NS} was calculated in [4], and, $O(\alpha_s^4)$ correction was estimated in [5].

[2]It is important to remind that while the first moments of the *nonsinglet* densities Δq_3 ($SU_f(2)$ symmetry) and Δq_8 ($SU_f(3)$ symmetry) *must be* conserved, i.e. are independent of Q^2 (corresponding to the conservation of the non-singlet axial-vector Cabibbo currents), the *singlet* axial charge, $a_0(Q^2)$ depends on Q^2 because of the axial anomaly .

also confirmed by the explicit calculations of the respective nonsinglet anomaly dimension which is just zero [6].

Let us analyse to what extend the results of the polarized SIDIS experiments are in agreement with the sum rule predictions. Such detailed analysis with respect to sum rule based on $SU_f(3)$ symmetry

$$\Delta q_8 \equiv a_8 = F + D,$$

was performed in [2], so that we will concentrate here on the Bjorken sum rule (3) which, using that $\Delta q = \Delta q_V + \Delta\bar{q}$, may be rewritten in the form convenient for analysis:

$$\Delta_1\bar{u} - \Delta_1\bar{d} = \frac{1}{2}\left|\frac{g_A}{g_V}\right| - \frac{1}{2}(\Delta_1 u_V - \Delta_1 d_V) \quad \textit{in all QCD orders.} \qquad (5)$$

Let us first consider the SMC results [7]. SMC has performed two types of analysis on Δq, with broken and unbroken sea scenarios, respectively. Unfortunately, the SMC analysis within the first scenario suffers from too big errors because the full number of measured asymmetries and achieved statistics were not quite sufficient to release the restriction $\Delta\bar{u} = \Delta\bar{d}$. So, let us look at the SMC results for the first moments of polarized quark distributions obtained within the unbroken sea scenario, where the respective table of first moments looks as (see Table 5 of ref. [7])

$\Delta\bar{u}(x) = \Delta\bar{d}(x)$	x	0-0.003	0.003-0.7	0-1
	$\Delta_1 u_V$	0.04 ± 0.04	$0.73 \pm 0.10 \pm 0.07$	$0.77 \pm 0.10 \pm 0.08$
	$\Delta_1 d_V$	-0.05 ± 0.05	$-0.47 \pm 0.14 \pm 0.08$	$-0.52 \pm 0.14 \pm 0.09$
	x	$0. - 0.003$	$0.003 - 0.3$	$0 - 1$
	$\Delta_1\bar{q}$	$0. \pm 0.02$	$0.01 \pm 0.04 \pm 0.03$	$0.01 \pm 0.04 \pm 0.03$

Taking the first moments of valence distributions directly from the table, one gets

$$\Delta_1 u_V - \Delta_1 d_V = 1.3 \pm 0.17 \pm 0.12, \qquad (6)$$

and this result is in a good agreement with the Bjorken sum rule (3), (5) which within the unbroken sea approximation is rewritten as

$$\Delta_1 u_V - \Delta_1 d_V = \Delta_1 u - \Delta_1 d = \left|\frac{g_A}{g_V}\right| = 1.2670 \pm 0.0035.$$

Let us now perform the similar analysis of HERMES results for the first moments of the polarized quark distributions published in Table 1 of ref. [8] which we, for convenience, partially reproduce here

	Measured region	Low-x	Total integral
$\Delta_1 u + \Delta_1\bar{u}$	$0.51 \pm 0.02 \pm 0.03$	0.04	$0.57 \pm 0.02 \pm 0.03$
$\Delta_1 d + \Delta_1\bar{d}$	$-0.22 \pm 0.06 \pm 0.05$	−0.03	$-0.25 \pm 0.06 \pm 0.05$
$\Delta_1 s + \Delta_1\bar{s}$	$-0.01 \pm 0.03 \pm 0.04$	0.00	$-0.01 \pm 0.03 \pm 0.04$
$\Delta_1\bar{u}$	$-0.01 \pm 0.02 \pm 0.03$	0.00	$-0.01 \pm 0.02 \pm 0.03$
$\Delta_1\bar{d}$	$-0.02 \pm 0.03 \pm 0.04$	0.00	$-0.02 \pm 0.03 \pm 0.04$
Δq_3	$0.74 \pm 0.07 \pm 0.06$	0.07	$0.84 \pm 0.07 \pm 0.06$
Δq_8	$0.32 \pm 0.09 \pm 0.10$	0.01	$0.32 \pm 0.09 \pm 0.10$
$\Delta_1 u_V$	$0.52 \pm 0.05 \pm 0.08$	0.03	$0.57 \pm 0.05 \pm 0.08$
$\Delta_1 d_V$	$-0.19 \pm 0.11 \pm 0.13$	−0.03	$-0.22 \pm 0.11 \pm 0.13$

Directly from the table one gets

$$\Delta q_3 \equiv (\Delta_1 u + \Delta_1\bar{u}) - (\Delta_1 d + \Delta_1\bar{d}) = 0.82 \pm 0.06 \pm 0.06, \quad (7)$$

whereas the right-hand side ought to be equal to $|g_A/g_V| = 1.2670 \pm 0.0035$ in accordance with the Bjorken sum rule (3).

Thus, the HERMES distributions do not satisfy the Bjorken sum rule (3). Instead these distributions are rather claimed to be in agreement with the sum rule (see Eq. (13) of ref. [8]) $\Delta q_3 = \int_0^1 \Delta q^{NS} dx = |g_A/g_V| \times C_{QCD}$ (where $\Delta q_{NS}(x,Q^2) \equiv \Delta u(x,Q^2) + \Delta\bar{u}(x,Q^2) - ((\Delta d(x,Q^2) + \Delta\bar{d}(x,Q^2))$, and $C_{QCD} \equiv C_1^{NS}(Q^2)$ is the nonsinglet coefficient function[3] given by Eq. (2)) which is incorrect[4].

To understand what happens let us briefly remind the HERMES procedure of the polarized density extraction from the measured SIDIS asymmetries. To this end the method of purities is used. Within this method the leading order (LO) expression for SIDIS asymmetry

$$A_1^h(x,Q^2) = \frac{\sum_f e_f^2 \Delta q_f(x,Q^2) \int_{0.2}^1 dz D_f^h(z,Q^2)}{\sum_f e_f^2 q_f(x,Q^2) \int_{0.2}^1 dz D_f^h(z,Q^2)},$$

is rewritten via purities $P_f^h(x,Q^2)$ as

$$A_1^h(x,Q^2) = \sum_f \frac{\Delta q_f}{q_f} P_f^h, \quad P_f^h(x,Q^2) \equiv \frac{e_f^2 q_f(x,Q^2) \int_{0.2}^1 dz D_f^h(z,Q^2)}{\sum_f e_f^2 q_f(x,Q^2) \int_{0.2}^1 dz D_f^h(z,Q^2)},$$

so that one can see that the application of the purity method is equivalent to the leading order (LO) QCD analysis.

Thus both SMC and HERMES collaborations use LO QCD analysis to extract polarized distributions from the measured SIDIS asymmetries. However, there are two important distinctions between SMC and HERMES analysis conditions.

First is that SMC asymmetries were measured in the wider region over Bjorken x variable $0.003 \le x_B|_{SMC} \le 0.7$ as compared with HERMES region $0.023 \le x_B|_{HERMES} \le$

[3] The quantity C_{QCD} in Eq. (13) of ref. [8] is namely the nonsinglet coefficient function C_1^{NS} given by Eq. (2) in 4[th] order of QCD expansion, so that at $\alpha_s(2.5\ GeV^2) = 0.35 \pm 0.04$ the right-hand side of Eq. (13) in ref. [8] reads $C_1^{NS}|g_A/g_V| = 1.01 \pm 0.05$ (just as in [8]). For details see [9], section 5.5.4, Eq. (5.22), Appendix A.7, Eq. (A.44), and also [10], section 2.5

[4] Notice that the HERMES result (7) differs by about 2 standard deviations even from this incorrect sum rule whose right-hand side reads $|g_A/g_V| \times C_1^{NS}(2.5\ GeV^2) = 1.01 \pm 0.05$ (just as in [8]).

0.6, and due to that, the uncertainties at the low x extrapolation in the respective first moments of extracted distributions are less in the SMC case.

Second distinction (and we consider it as a most important one) is that SMC analysis is performed at average $Q^2 = 10\,GeV^2$, i.e., when LO QCD is a quite good approximation, whereas HERMES uses LO analysis to extract the polarized distributions from the respective asymmetries measured at relatively low average $Q^2 = 2.5\,GeV^2$ value. So, the inconsistence of HERMES result on Δq_3 with the Bjorken sum rule can serve as a direct indication that LO analysis is not sufficient and NLO analysis is necessary at such conditions.

It is illustrative to show how one can arrive at the incorrect sum rule using the purity method at low average Q^2 value.

Since the application of this method with respect to SIDIS asymmetries is just LO QCD analysis, the first moments of the DIS structure functions $\Gamma_1^{p,n}$ have LO QCD expressions via HERMES distributions:

$$\Gamma_1^p(2.5\,GeV^2) = \frac{1}{2}\sum_{q,\bar{q}} e_q^2\,\Delta_1 q(2.5\;GeV^2), \quad \Gamma_1^n = \Gamma_1^p\Big|_{u\leftrightarrow d}. \tag{8}$$

On the other hand, the exact expression for the physical (independently measurable) quantity $\Gamma_1^p - \Gamma_1^n$ has a form (1), where C_1^{NS} differs essentially from the LO value 1 at so low Q^2.

Now, if one equates (which is actually incorrect) the LO expression for $\Gamma_1^p - \Gamma_1^n$ derived from (8) to the exact expression (1), then one immediately obtains the sum rule (13) of ref. [8]. However, the quantities satisfying this sum rule certainly have nothing in common with the real LO first distribution moments (as well as with the real NLO, NNLO,... ones) which (as well as the real NLO, NNLO,... first moments) satisfy the Bjorken sum rule (3) without any Q^2 dependence in the right-hand side.

In spite of it being almost obvious, it is expedient to show explicitly that the same trick, but in the NLO order (i.e., equating the quantity $\Gamma_1^p - \Gamma_1^n$, expressed via the NLO extracted distributions, to the exact value), would give rise to an error only of the order $O(\alpha_s^2)$.

Indeed, the extraction of the quark distributions from the SIDIS asymmetries in NLO order means that the respective DIS structure functions are expressed via these distributions as

$$g_1^p(x,Q^2) = \frac{1}{2}\sum_{q,\bar{q}} e_q^2\left(\Delta q + \frac{\alpha_s(Q^2)}{2\pi}[C_q\otimes\Delta q + C_g\otimes\Delta g]\right)(x,Q^2).$$

Then, using the explicit values of the first moments of the respective $\overline{\mathrm{MS}}$ Wilson coefficients [6] $M^1(C_q) = -2, \quad M^1(C_g) = 0$, one gets in NLO QCD:

$$M^1[g_1^p] \equiv \Gamma_1^p = \frac{1}{2}\sum_{q,\bar{q}} e_q^2\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)\Delta_1 q, \quad \Gamma_1^n = \Gamma_1^p\Big|_{u\leftrightarrow d}. \tag{9}$$

Substituting this in the left-hand side of Eq. (1) with C_1^{NS} given by Eq. (2) reduced to NLO QCD: $C_1^{NS} = 1 - \alpha_s/\pi$, one can see that the α_s dependent factors $(1 - \alpha_s(Q^2)/\pi)$ cancel out precisely in the left- and right-hand sides, so that one arrives at Eq. (3) without any Q^2 dependence. On the other hand, setting the difference $\Gamma_1^p - \Gamma_1^n$ composed from (9) equal to the left-hand side of (1) and keeping (at will), simultaneously, in the right-hand side the higher in α_s corrections (see Eq. (2)) for C_1^{NS}, one gets instead of the Bjorken sum rule (3) the sum rule with $O\left(\alpha_s^2(Q^2)\right)$ terms in the right-hand side.

Let us now analyse the results of Table 1 of ref. [8] on $\Delta_1\bar{q}$.

First of all notice that these results are rather inconsistent also with Eq. (13) of ref. [8]. Indeed, substitution of $\Delta_1 u_V$ and $\Delta_1 d_V$ taken from the Table 1 of ref. [8] into equivalent of this equation (compare it with equivalent of the Bjorken sum rule (3), Eq. (5))

$$\Delta_1\bar{u} - \Delta_1\bar{d} = \frac{1}{2}\left|\frac{g_A}{g_V}\right| C_1^{NS} - \frac{1}{2}\left(\Delta_1 u_V - \Delta_1 d_V\right)$$

with $|g_A/g_V| \times C_1^{NS}(2.5\ GeV^2) = 1.01 \pm 0.05$ (just as in [8]), immediately gives

$$\Delta_1\bar{u} - \Delta_1\bar{d} = 0.11 \pm 0.10, \qquad (10)$$

where 0.10 is the total error $\sigma_{tot} = \sqrt{\sigma^2_{statistical} + \sigma^2_{systematical}}$. On the other hand, taking the polarized sea distributions directly from the Table 1, one gets[5]

$$\Delta_1\bar{u} - \Delta_1\bar{d} = (-0.01 + 0.02) \pm 0.061 = 0.01 \pm 0.061, \qquad (11)$$

instead of (10).

Despite this inconsistence, one, certainly, can say that Eq. (10) also predicts very small value for $\Delta_1\bar{u} - \Delta_1\bar{d}$ which is comparable with Eq. (11) within the error.

Let us now do some speculations assuming, for a moment, that at least the first moments of the valence quark distributions from the Table 1 of ref. [8] are close to the real ones. Then, substituting values $\Delta_1 u_V$ and $\Delta_1 d_V$ taken from the Table 1 into the Bjorken sum rule written in the form (5), one arrives at rather amazing result:

$$\Delta_1\bar{u} - \Delta_1\bar{d} = 0.235 \pm 0.097, \qquad (12)$$

i.e., the quantity $\Delta_1\bar{u} - \Delta_1\bar{d}$ we are interested in, is not zero as compared with the total error (2.42 standard deviations), and, the polarized sea of light quarks is asymmetric with respect to u and d quark polarized distributions.

Certainly, this is just a speculation based on the above-mentioned assumption. We rather believe that all this is a direct indication that the HERMES data for asymmetries should be properly reanalysed. First, the low x region should be treated more carefully and, second, the NLO QCD procedure is necessary at so low Q^2 to properly extract so tiny quantities as $\Delta_1 s$ and $\Delta_1\bar{u} - \Delta_1\bar{d}$.

Besides, there is a good lesson here for another polarized SIDIS experiments, in particular, for the COMPASS experiment [11]. On the one hand the low x_B boundary should be as small as possible to achieve the maximal accuracy for the first moments. On the other hand, it is extremely desirable to maximally increase the average Q^2 value in order to safely apply the simple LO analysis. Otherwise, while the SIDIS asymmetries are measured at average Q^2 which is still about 2 GeV^2, the LO analysis is not sufficient and NLO analysis is necessary to get reliable polarized distributions consistent with the fundamental restrictions such as the Bjorken sum rule.

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[5]There is some misprint in the forth column of Table 1. For the first moment of d-quark polarized distribution one must read $-0.02 \pm 0.03 \pm 0.04$ instead of $-0.01 \pm 0.03 \pm 0.04$.

Список литературы

[1] New Muon Collaboration (M.Arneodo et al), Phys. Lett. **B** 364 (1995) 107.

[2] E. Leader, D. Stamenov, Phys. Rev. **D** 67 (2003) 037503, hep-ph/0211083.

[3] Spin Muon Collaboration (D. Adams et al), Phys. Rev. **D** 56 (1997) 5330.

[4] S. Larin, J. Vermaseren, Phys. Lett. **B** 259 (1991) 345.

[5] A. Kataev, V. Starchenko, Mod. Phys. Lett. **A** 10 (1995) 235.

[6] M. Gluck, E. Reya, M. Stratmann, W. Vogelsang, Phys. Rev. **D** 53 (1996) 4775.

[7] Spin Muon Collaboration, (B. Adeva et al.), Phys. Lett. **B** 420 (1998) 180-190.

[8] HERMES collaboration (K. Ackerstaff et al), Phys. Lett. **B** 464 (1999) 123.

[9] M. Ruh, Measurement of polarized quark distributions of the Nucleon at HERMES, Ph. D. thesis,Albert-Ludwigs-Universitaet, Freiburg im Byres, 1999 (can be obtained electronically via "HERMES publications" internet page).

[10] M. C. Simani, Flavour decomposition of the nucleon spin at HERMES, Ph. D. thesis, Vrije Universiteit Amsterdam, 2002, (can be obtained electronically via "HERMES publications" internet page).

[11] COMPASS proposal. CERN/SPSLC 96-14, SPSC/p 297, can be obtained electronically via http://wwwcompass.cern.ch/compass/proposal/

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Сисакян А. Н., Шевченко О. Ю., Иванов О. Н. E2-2003-44
О поляризованных кварковых плотностях,
полученных в экспериментах
по полуинклюзивному глубоконеупругому рассеянию

Рассматриваются результаты экспериментов по полуинклюзивному глубоконеупругому рассеянию, касающиеся первых моментов поляризованных кварковых распределений. Обсуждаются возможные причины отклонений от фундаментальных соотношений (таких как правило сумм Бьоркена) и способы улучшения анализа асимметрий, измеренных в полуинклюзивных экспериментах. Проанализирована возможность реализации несимметричного поляризованного кваркового моря.

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Comment on Polarized Quark Distributions Extracted
from SIDIS Experiments

The results of SIDIS experiments concerning the first moments of the polarized quark distributions are considered. The possible reasons of the deviation from the fundamental restrictions such as the Bjorken sum rule and the ways to properly improve the analysis of measured SIDIS asymmetries are discussed. The possibility of broken polarized sea scenario is analysed.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, at the Dzhelepov Laboratory of Nuclear Problems and at the Scientific Center of Applied Research, JINR.

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